Measuring and Controlling
Interest Rate and Credit Risk

Second Edition

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STEVEN V. MANN
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FJF

To the memory of Karl T. Hieber, a man whose business talents were exceeded only by his love, compassion, and generosity for his family.

SVM

To Dr. Brad Smith of the Souper Bowl of Caring and Dr. Ned Pruitt of the Medical College of Georgia

MC

To Lloyd Cole and the Commotions, for writing "Rattlesnakes"

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Preface

The second edition of *Measuring and Controlling Interest Rate and Credit Risk* is a significant revision of the first edition. The first edition focused solely on interest rate risk. In the second edition, coverage is expanded to include credit risk. Coverage on credit risk includes the types of credit risk, measuring credit risk, credit derivatives (features and valuation), and using structured credit products to control credit risk. But the changes to the book are more extensive then just the addition of credit risk coverage. We have included numerous illustrations of the analytical concepts covered in the book using screens provided by Bloomberg.

We are grateful to several individuals for their assistance in this project. We thank Richard Pereira for coauthoring Chapter 17 ("Credit Derivative Valuation"). Chapter 14 ("Controlling Interest Rate Risk of an MBS Derivative Portfolio") is adapted from a coauthored chapter Frank Fabozzi published in *CMO Portfolio Management* (Frank J. Fabozzi Associates, 1994). We thank his coauthors, Michael Schumacher and Daniel Dektar, for allowing him to adapt the chapter for this book.

With respect to Moorad Choudhry, the views, thoughts and opinions expressed in this book represent those in his individual private capacity, and should not be taken to represent those of JPMorgan Chase Bank, or to Moorad Choudhry as an employee, officer or representative of JPMorgan Chase Bank.

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CHAPTER 1
Introduction

The goal of this book is to describe how to measure and control the interest rate and credit risk of a bond portfolio or trading position. In this chapter we provide an overview of measurement and control for these two types of risk. This overview will provide a roadmap for the chapters to follow.

The objectives of this chapter are to:
1. Explain two approaches to measuring interest rate risk—the full valuation approach and the duration approach.
2. Explain what is meant by the duration of a bond or bond portfolio.
3. Explain why the measurement of yield volatility is important in measuring interest rate risk.
4. Briefly describe what the value at risk approach is.
5. Describe what is involved in controlling interest rate risk.
6. Explain the different forms of credit risk: credit default risk and credit spread risk.
7. Briefly describe how credit ratings measure credit default risk and what downgrade risk is.
8. Identify what credit derivatives can be used to control credit risk.

INTEREST RATE RISK

The value of a bond changes in the opposite direction to the change in interest rates.[1] For a long bond position, the position's value will decline

[1] There are some products in the mortgage-backed and asset-backed securities market where the price change is in the same direction as the change in interest rates. An example is an interest-only security.

if interest rates rise, resulting in a loss. For a short bond position, a loss will be realized if interest rates fall.

Measuring Interest Rate Risk

A manager wants to know more than simply when a position will realize a loss. To control interest rate risk, a manager must be able to quantify what will result. The fundamental relationship is that the potential dollar loss of a position resulting from an adverse interest rate change is:

Potential dollar loss of a position
= Value of position after adverse rate change
– Current market value of position

Full Valuation Approach to Risk Measurement

The key to measuring the potential dollar loss of a position is how good the estimate is of the value of the position after an adverse rate change. A *valuation model* is used to determine the value of a position after an adverse rate move. Consequently, if a reliable valuation model is not used, there is no way to measure the potential dollar loss. Because valuation models are essential in the measurement of risk, we describe the principles of valuation and two commonly used valuation models in Chapter 2.

The approach to measuring the potential dollar loss whereby the value of the position after the adverse rate change is estimated from a valuation model is referred to as the *full valuation model*.[2] Given a valuation model, the dollar loss for specific scenarios can be determined. Analyzing interest rate risk in this manner is referred to as *scenario analysis*. The manager can then assess the likelihood or probability of each scenario occurring and any unacceptable outcomes can be modified by using the tools described in this book.

Duration Approach to Risk Measurement

An alternate approach is to estimate the potential dollar loss for any rate change by approximating the sensitivity of a position to a rate change. For example, suppose that a trader has a \$20 million long position in a bond whose value changes by approximately 4% for a 100-basis-point change in rates. Then the manager knows that for a 100-basis-point increase in rates, the value of the position will decline by approximately \$800,000 (\$20 million × 0.04). For a 25-basis-point rise in rates, the position will change in value by approximately 1% or \$200,000.

[2] *RiskMetrics™ —Technical Document*, JP Morgan, Third Edition, 1995, p. 14.

Duration is a measure of the approximate sensitivity of a bond's value to rate changes. More specifically, it is the approximate percentage change in value for a 100-basis-point change in rates.[3] Consequently, duration can be used to approximate the potential dollar loss. For example, if the market value of a bond held is $20 million and if its duration is 4, then the potential dollar loss for a 25-basis-point (0.0025) change in rates is:

$$\$20{,}000{,}000 \times 4 \times 0.0025 = \$200{,}000$$

For a 5-basis-point (0.0005) change in rates, the potential dollar loss is:

$$\$20{,}000{,}000 \times 4 \times 0.0005 = \$40{,}000$$

Using duration to approximate the potential dollar loss is referred to as the *duration approach to risk management*. The advantage of the duration approach over the full valuation approach is that the latter requires more computational time to obtain the new value under each scenario analyzed. The duration approach allows the manager to quickly estimate the effect of an adverse rate change on the potential dollar loss.

A drawback of the duration approach is that duration is only a first approximation as to how sensitive the value of a bond or bond portfolio is to rate changes. Thus, the potential dollar loss of a position is only an approximation, whereas the full valuation approach provides the precise amount of the loss. However, for both approaches, it is essential to have a good valuation model. The duration measure is obtained from a valuation model, so if the valuation model does not do a good job of valuing a security, the duration measure will not be useful. Consequently, when we say that the full valuation approach gives a precise amount of the potential dollar loss, we mean precise given the valuation model used.

In Chapter 3, we take a close look at duration. We will examine how it is measured and its limitations. We will see that the approximation provided by duration can be improved by introducing another parameter called convexity. Together, duration and convexity do a more effective job of estimating the sensitivity of a position to adverse rate changes. Both duration and convexity are referred to as parameters of a valuation model. Consequently, estimating the sensitivity of the value of a portfolio or position to adverse rate changes is referred to as the *parametric*

[3] Similarly, the duration of a liability is the approximate percentage change in the value of the liability for a 100-basis-point change in rates.

approach. Since the full valuation approach does not use parameters, it is also sometimes called the *nonparametric approach*.

Our discussion of the limitations of duration and convexity in Chapter 3 will lead us to the conclusion that the duration and convexity of a portfolio of bonds with different maturities does not tell the whole story about interest rate risk. Another important source of interest rate risk for a portfolio of bonds is how the yield curve changes. In Chapter 2 we describe the yield curve and its role in valuation. In Chapter 4, we discuss several measures of yield curve risk.

Yield Volatility

When measuring interest rate risk, another dimension to consider is how volatile interest rates are. For example, as we will explain in Chapter 3, all other factors equal, the higher the coupon rate, the lower the duration. Thus a 10-year high-yield corporate bond has a lower duration than a current coupon 10-year Treasury note since the former has a higher coupon rate. Does this mean that a 10-year high-yield corporate bond has less interest rate risk than a current coupon 10-year Treasury note? Consider also that a 10-year Swiss government bond has a lower coupon rate than a current coupon 10-year U.S. Treasury note and therefore a higher duration. Does this mean that a 10-year Swiss government bond has greater interest rate risk than a current coupon 10-year U.S. Treasury note? The missing link is the relative volatility of rates which we shall refer to as *yield volatility*.

The greater the expected yield volatility, the greater the interest rate risk for a given duration and current value of a position. In the case of high-yield corporate bonds, while their durations are less than current coupon Treasuries of the same maturity, the yield volatility on junk bonds is greater than that of current coupon Treasuries. For the 10-year Swiss government bond, while the duration is greater than for a current coupon 10-year U.S. Treasury, the yield volatility of 10-year Swiss bonds is considerably less than that of 10-year Treasury U.S. bonds.

Consequently, to measure the exposure of a portfolio or position to rate changes it is necessary to measure yield volatility. This requires an understanding of the fundamental principles of probability distributions. This topic is covered in Chapter 5. The measure of yield volatility is the standard deviation of yield changes. In Chapter 6, we show how to estimate yield volatility. As we will see, depending on the underlying assumptions, there could be a wide range for the yield volatility estimate.

Value at Risk

A framework that ties together the price sensitivity of a bond position to rate changes and yield volatility is the *value-at-risk* (VaR) framework.

Risk in this framework is defined as the maximum estimated loss in market value of a given position that is expected to happen a certain percentage of times. JP Morgan has been the major force in promoting VaR. Its VaR system is *RiskMetrics*™.

We will discuss the interest rate VaR framework in Chapter 5 after we have discussed duration, yield volatility, and probability distributions. What is critical to understand is that measures of duration and yield volatility are not precise, therefore, there could be considerable variation in the VaR of a position.

A VaR measure can be computed for a single bond position or a bond portfolio. Measurement of the risk of a portfolio of bonds or the risk of several trading positions in different assets is more complicated. This measurement involves the correlation between the yields or prices of these assets. For this reason, we describe correlation analysis in Chapter 7 and explain how correlation affects the risk of a portfolio.

Tracking Error

For a bond portfolio manager whose benchmark is a bond market index such as the Lehman Brothers Aggregate Bond Index or the Salomon Smith Barney Broad-Investment Grade Index, risk can be measured in terms of tracking error—the standard deviation of the difference in the return on a portfolio and the return on the benchmark. A forward-looking tracking error allows a manager to determine the future exposure of a bond portfolio relative to a benchmark. Moreover, the reasons for the tracking error can be explained in terms of risk factors—systematic and nonsystematic risk factors.

One of the major risk systematic risk factors is the term structure risk factor. Tracking error due to the term structure risk factor indicates how the duration and yield curve exposure mismatch relative to a benchmark affect will affect tracking error. Tracking error and tracking error due to the term structure risk factor are discussed in Chapter 4.

Controlling Interest Rate Risk

Once the interest rate risk of a bond portfolio or position is measured, the next step in risk management is to alter the risk exposure to an acceptable level. This is the control phase of risk management.

To control the interest rate risk of a position or portfolio, a position must be taken in another instrument or instruments. We shall refer to an instrument that is used to control the risk of a position as a *risk control instrument*. These instruments include derivative instruments and cash market instruments. The former includes futures, forwards, options, swaps,

caps, and floors. They are referred to as derivative instruments because their value is derived from some underlying price, index, or interest rate.

Typically, when cash market instruments are used the instruments of choice are Treasury securities or stripped Treasuries (i.e., zero-coupon Treasuries). In the case of positions in mortgage-backed securities, certain types of mortgage strips (i.e., interest-only and principal-only securities) and certain collateralized mortgage obligation (CMO) products are used. Typically, these products are created from mortgage passthrough securities. These mortgage products are referred to as *mortgage derivative products* because they derive their value from mortgage passthrough securities.

With the advent of derivative instruments, risk management, in its broadest sense, assumes a new dimension. Risk managers can achieve new degrees of freedom. It is now possible to alter the interest rate sensitivity of a bond portfolio, bond position, or asset/liability position economically and quickly. Derivative instruments offer risk and return patterns that previously were either unavailable or too costly to achieve.

In Chapters 9, 10, 11, and 12 we describe these derivative instruments for controlling interest rate risk. Chapter 9 describes futures, forward contracts, and forward rate agreements. Chapter 10 describes interest rate swaps and swaptions. Chapters 11 and 12 cover interest rate options and related products. Chapter 11 focuses on exchange-traded options. Chapter 12 looks at over-the-counter options, interest caps, and interest rate floors.

The selection of the specific instrument or instruments to use involves determining which risk control instruments are the most appropriate to employ given the investment objectives. A key factor in this decision is the correlation between the yield movement of a potential risk control instrument and the yield movement of the bonds whose interest rate risk the manager seeks to control. In addition, it may be necessary to estimate the relationship between yield movements using regression analysis. Correlation and regression analyses are covered in Chapter 8.

Once the appropriate risk control instrument or instruments are selected, the appropriate position (i.e., long or short) and the amount of the position must be determined. The potential outcome of the risk control strategy can then be assessed prior to its implementation. We will explain how this is done in Chapters 13 and 14 using derivative instruments.

CREDIT RISK

Credit risk includes credit default risk and credit spread risk. The former form of credit risk is the risk that an issuer of debt (obligor) is unable to meet its financial obligations resulting in an investor incurring a loss equal to the amount owed by the obligor less any recovery amount. *Credit spread risk* is the risk of financial loss or the underperformance of a portfolio resulting from changes in the level of credit spreads used in the marking-to-market of a product. *Downgrade risk* is a form of credit spread risk because the anticipating or actual downgrading of an issue or issuer will result in an increase in the credit spread.

Measuring Credit Risk

There are various ways that the two forms of credit risk can be measured. They include the use of credit ratings, rating transition tables, credit VaR, and tracking error due to credit risk.

Rating agencies in the United States—Fitch Ratings, Moody's, and Standard & Poor's—assess the credit default risk of an issuer or issue and express their view in the form of a rating. A *rating transition table*, also called a *rating migration table*, is a table that shows how ratings at the beginning of some time period change at the end of a time period. Rating migration tables, produced periodically by the three rating agencies, can be used to gauge downgrade risk.

Credit VaR performs the same function as interest rate VaR. There are various vendors of credit VaR. Similarly, tracking error can be used to measure exposure to credit risk. Tracking error due to quality risk (one type of systematic risk) and due to nonsystematic risk (specifically, issuer-specific and issue-specific risk) can be used to assess the potential risk exposure of a portfolio due to credit risk.

The approaches to measuring credit risk are covered in Chapter 15. Also discussed in that chapter are default rates and default loss rates and recent empirical evidence regarding their historical values.

Controlling Credit Risk

Credit derivatives can be used to control the two forms of credit risk. Credit derivatives include credit default swaps, total return swaps, credit options, and credit forwards. The most popular type of credit derivative is the credit default swap. Each of these instruments is explained in Chapter 16, along with an explanation of how they can be used to control credit risk. The basics of valuing credit derivatives is the subject of Chapter 17.

In Chapter 18 we show how credit derivatives can be combined with securitization techniques to create structured products, used by banks to manage credit risk and regulatory capital. These structures include synthetic collateralized debt obligations and credit-linked notes.

KEY POINTS

1. To control interest rate risk, a manager must be able to quantify the potential dollar loss of a position resulting from an adverse interest rate change.

2. The key to measuring the potential dollar loss of a position is having a good valuation model that can be used to determine what the value of a position is after an adverse rate change.

3. The full valuation approach to measuring the potential dollar loss of a position after the adverse rate change uses a valuation model.

4. Scenario analysis is used to estimate the dollar loss for various interest rate scenarios.

5. The duration approach is an alternative approach for estimating the potential dollar loss for any adverse rate change.

6. The duration of a position is the approximate percentage change in the position's value for a 100-basis-point change in rates.

7. A good valuation model is needed to obtain the duration estimate.

8. The advantage of the duration approach over the full valuation approach is that it allows the manager to quickly estimate the effect of an adverse rate change on the potential dollar loss.

9. A drawback of the duration approach is that duration is only a first approximation of how sensitive the value of a bond or bond portfolio is to rate changes.

10. The duration approach to risk management is referred to as the parametric approach, while the full valuation approach is called the nonparametric approach.

11. Measurement of the interest rate risk of a position must take into account expected yield volatility.

12. The greater the expected yield volatility, the greater the interest rate risk of a position for a given duration and current value of a position.

13. Yield volatility is measured by the standard deviation of yield changes.

14. The value-at-risk framework ties together the price sensitivity of a bond position to rate changes and yield volatility.

15. In the value-at-risk framework, *risk* is defined as the maximum estimated loss in market value of a given position that is expected to happen a certain percentage of times.

16. Tracking error is the standard deviation of the difference between the return on a portfolio and return on a benchmark.

17. Tracking error is the most common measure used by bond portfolio managers in assessing performance versus a bond market index.

18. Yield curve risk of a bond portfolio can be assessed by computing the tracking error due to the term structure risk factor.

19. The control phase of risk management involves altering the risk exposure to an acceptable level.

20. To control the interest rate risk of a position or portfolio, a position must be taken in one or more risk control instruments.

21. Risk control instruments include derivative instruments (futures, forwards, options, swaps, caps, and floors) and cash market instruments.

22. Derivative instruments allow a risk manager to alter the interest rate sensitivity of a bond portfolio or position or an asset/liability position economically and quickly.

23. A key factor in selecting the risk control instrument to employ is the correlation between the yield movements of the bond, whose risk is sought to be controlled, and the candidate risk control instrument.

24. Once the appropriate risk control instrument (or instruments) is selected, the appropriate position (i.e., long or short) and the amount of the position must be determined.

25. Credit default risk and credit spread risk are forms of credit risk.

26. Downgrade risk is related to credit spread risk.

27. Credit rating can be used to gauge credit default risk.

28. Rating transition tables produced by rating agencies can be used to gauge downgrade risk.

29. Value-at-risk can be computed for credit risk and there are several vendors that provide credit VaR systems.

30. Tracking error due to credit risk can be computed to measure the credit risk exposure of a bond portfolio relative to a benchmark.

31. Credit derivatives can be used to control credit risk, the most popular credit derivative being credit default swaps.

32. Synthetic collateralized debt obligations and credit-linked notes can be used to manage a bank's exposure to credit risk.

CHAPTER 2

Valuation

Valuation is the process of determining the fair value of a financial asset. The fundamental principle of valuation is that the value of any financial asset is the present value of the expected cash flow. This principle applies regardless of the financial asset. In this chapter, we will explain the general principles of bond valuation and discuss two valuation methodologies.

The objectives of this chapter are to:

1. Discuss the process involved in valuing a bond.
2. Explain the situations in which determination of a bond's cash flow is complex.
3. Explain why a bond should be viewed as a package of zero-coupon securities.
4. Explain the difference between the Treasury yield curve and the Treasury spot rate curve and how the theoretical spot rate curve for Treasury securities can be constructed from the Treasury yield curve.
5. Demonstrate how the Treasury spot rate curve can be used to value any Treasury security.
6. Explain how credit risk should be introduced into the term structure.
7. Explain how to compute and interpret the nominal spread.
8. Explain how to compute and interpret the zero-volatility spread.
9. Describe what is meant by the option-adjusted spread.
10. Explain why the volatility assumption is critical in the valuation of bonds with embedded options.
11. Explain the binomial method for valuation.
12. Explain the Monte Carlo method for valuing mortgage-backed securities.

ESTIMATING CASH FLOW

Cash flow is simply the cash that is expected to be received each period from an investment. In the case of a bond, it does not make any difference whether the cash flow is interest income or repayment of principal.

The cash flow for only a few types of bonds are simple to project. Noncallable Treasury securities have a known cash flow. For a Treasury coupon security, the cash flow is the coupon interest payments every six months up to the maturity date and the principal payment at the maturity date. For any bond in which neither the issuer nor the investor can alter the repayment of the principal before its contractual due date, the cash flow can easily be determined assuming that the issuer does not default. The difficulty in determining the cash flow for bonds arises under the following circumstances: (1) either the issuer or the investor has the option to change the contractual due date of the repayment of the principal; (2) the coupon payment is reset periodically based on some reference rate; or (3) the investor has an option to convert the bond to common stock.

Most non-Treasury securities include a provision in the indenture that grants the issuer or the bondholder the right to change the scheduled date or dates when the principal repayment is due. Assuming that the issuer does not default, the investor knows that the principal amount will be repaid, but does not know when that principal will be received. Because of this, the cash flow is not known with certainty.

A key factor determining whether either the issuer of the bond or the investor would exercise an option is the level of interest rates in the future relative to the bond's coupon rate. Specifically, for a callable bond, if the prevailing market rate at which the issuer can refund an issue is sufficiently below the issue's coupon rate to justify the costs associated with refunding the issue, the issuer is likely to call the issue. Similarly, for a mortgage loan, if the prevailing refinancing rate available in the mortgage market is sufficiently below the loan's mortgage rate so that there will be savings by refinancing after considering the associated refinancing costs, then the homeowner has an incentive to refinance. For a putable bond, if the rate on comparable securities rises such that the value of the putable bond falls below the value at which it must be repurchased by the issuer, then the investor will put the issue.

What this means is that to properly estimate the cash flow of a bond it is necessary to incorporate into the analysis how interest rates can change in the future and how such changes affect the expected cash flow. As we will see later, this is done in valuation models by introducing a parameter that reflects the volatility of interest rates.

DISCOUNTING THE CASH FLOW

Once the cash flow for a bond is estimated, the next step is to determine the appropriate interest rate to use to discount the cash flow. The minimum interest rate that an investor should require is the yield available in the marketplace on a default-free cash flow. In the United States this is the yield on a U.S. Treasury security. The premium over the yield on a Treasury security that the investor should require should reflect the risks associated with realizing the estimated cash flow.

The traditional practice in valuation has been to discount every cash flow of a bond by the same interest rate (or discount rate). For example, consider the following three hypothetical 10-year Treasury securities: a 6% coupon bond, a 4% coupon bond, and a zero-coupon bond. Since the cash flow of all three securities is viewed as default free, the traditional practice is to use the same discount rate to calculate the present value of all three securities and the same discount rate for the cash flow for each period.

The fundamental flaw of the traditional approach is that it views each security as the same package of cash flows. The proper way to view a bond is as a package of zero-coupon instruments. Each cash flow should be considered a zero-coupon instrument whose maturity value is the amount of the cash flow and whose maturity date is the date of the cash flow. Thus, a 10-year 4% coupon bond should be viewed as 20 zero-coupon instruments. The reason that this is the proper way is because it does not allow a market participant to realize an arbitrage profit. This will be demonstrated later in this chapter.

By viewing any financial asset in this way, a consistent valuation framework can be developed. For example, under the traditional approach to the valuation of bonds, a 10-year zero-coupon bond would be viewed as the same financial asset as a 10-year 4% coupon bond. Viewing a financial asset as a package of zero-coupon instruments means that these two bonds would be viewed as different packages of zero-coupon instruments and valued accordingly.

To properly value a bond, it is necessary to determine the theoretical rate that the U.S. Treasury would have to pay to issue a zero-coupon instrument for each maturity. Another name used for the zero-coupon rate is the *spot rate*. As explained later, the spot rate can be estimated from the Treasury yield curve.

SPOT RATES AND THEIR ROLE IN VALUATION

The key to the valuation of any security is the estimation of its cash flow and the discounting of each cash flow by an appropriate rate. The starting point for the determination of the appropriate rate is the theoretical spot rate on default-free securities. Since Treasury securities are viewed as default-free securities, the theoretical spot rates on these securities are the benchmark rates.

The Treasury Yield Curve

The graphical depiction of the relationship between the yield on Treasury securities of different maturities is known as the *yield curve*. The Treasury yield curve is typically constructed from on-the-run Treasury issues. Treasury bills are zero-coupon securities. Treasury notes and bonds are coupon securities. Consequently, the Treasury yield curve is a combination of zero-coupon securities and coupon securities.

In the valuation of securities what is needed is the rate on zero-coupon default-free securities or, equivalently, the rate on zero-coupon Treasury securities. However, there are no zero-coupon Treasury securities issued by the U.S. Department of the Treasury with a maturity greater than one year. The goal is to construct a theoretical rate that the U.S. government would have to offer if it issued zero-coupon securities with a maturity greater than one year.

There are zero-coupon Treasury securities with a maturity greater than one year that are created by government dealer firms—Treasury STRIPS (i.e., securities issued as part of the Treasury's *S*eparate *T*rading of *R*egistered *I*nterest and *P*rincipal *S*ecurities program). It would seem logical that the observed yield on Treasury STRIPS could be used to construct an actual spot rate curve rather than go through the procedure we will describe. There are three problems with using the observed rates on Treasury STRIPS. First, the liquidity of the Treasury STRIPS market is not as great as that of the Treasury coupon market. Thus, the observed rates on Treasury STRIPS reflect a premium for liquidity. Second, there are maturity sectors of the Treasury STRIPS market that attract specific investors who may be willing to trade off yield in exchange for an attractive feature associated with that particular maturity sector, thereby distorting the term structure relationship. For example, certain foreign governments may grant investors preferential tax treatment on zero-coupon bonds. As a result, these foreign investors invest heavily in long-maturity Treasury STRIPS, driving down yields in that maturity sector. Finally, the tax treatment of stripped Treasury securities is different from that of Treasury coupon securities. Specifically, the accrued interest on Treasury STRIPS is taxed even though no cash is received by the investor. Thus they are negative cash flow securities to taxable entities, and, as a result, their yield reflects this tax disadvantage.

EXHIBIT 2.1 **Maturity and Yield to Maturity for 20 Hypothetical Treasury Securities**

Period	Years	Yield to Maturity (%)	Price ($)	Spot Rate (%)	Discount Function
1	0.5	3.00	—	3.0000	0.9852
2	1.0	3.30	—	3.3000	0.9678
3	1.5	3.50	100.00	3.5053	0.9492
4	2.0	3.90	100.00	3.9164	0.9254
5	2.5	4.40	100.00	4.4376	0.8961
6	3.0	4.70	100.00	4.7520	0.8686
7	3.5	4.90	100.00	4.9622	0.8424
8	4.0	5.00	100.00	5.0650	0.8187
9	4.5	5.10	100.00	5.1701	0.7948
10	5.0	5.20	100.00	5.2772	0.7707
11	5.5	5.30	100.00	5.3864	0.7465
12	6.0	5.40	100.00	5.4976	0.7222
13	6.5	5.50	100.00	5.6108	0.6979
14	7.0	5.55	100.00	5.6643	0.6764
15	7.5	5.60	100.00	5.7193	0.6551
16	8.0	5.65	100.00	5.7755	0.6341
17	8.5	5.70	100.00	5.8331	0.6134
18	9.0	5.80	100.00	5.9584	0.5895
19	9.5	5.90	100.00	6.0863	0.5658
20	10.0	6.00	100.00	6.2169	0.5421

Constructing the Theoretical Spot Rate Curve for Treasuries

A default-free theoretical spot rate curve can be constructed from the observed Treasury yield curve. There are several approaches that are used in practice. The approach that we describe below for creating a theoretical spot rate curve is called *bootstrapping*. To explain this approach, we use the price, annualized yield (yield to maturity), and maturity for the 20 hypothetical Treasury securities shown in Exhibit 2.1.

Throughout the analysis and illustrations to come, it is important to remember that the basic principle is that the value of the Treasury coupon security should be equal to the value of the package of zero-coupon Treasury securities that duplicates the coupon bond's cash flow.

Consider the 6-month Treasury bill in Exhibit 2.1. Since a Treasury bill is a zero-coupon instrument, its annualized yield of 3.00% is equal to the spot rate. Similarly, for the 1-year Treasury, the cited yield of

3.30% is the 1-year spot rate. Given these two spot rates, we can compute the spot rate for a theoretical 1.5-year zero-coupon Treasury. The price of a theoretical 1.5-year Treasury should equal the present value of the three cash flows from the 1.5-year coupon Treasury, where the yield used for discounting is the spot rate corresponding to the cash flow. Since all the coupon bonds are selling at par, the yield to maturity for each bond is the coupon rate. Using $100 as par, the cash flow for the 1.5-year coupon Treasury is as follows:

0.5 years	$0.035 \times \$100 \times 0.5 = \$1.75$
1.0 years	$0.035 \times \$100 \times 0.5 = \$1.75$
1.5 years	$0.035 \times \$100 \times 0.5 + \$100 = \$101.75$

The present value of the cash flow is then:

$$\frac{1.75}{(1+z_1)^1} + \frac{1.75}{(1+z_2)^2} + \frac{101.75}{(1+z_3)^3}$$

where

z_1 = one-half the annualized 6-month theoretical spot rate
z_2 = one-half the 1-year theoretical spot rate
z_3 = one-half the 1.5-year theoretical spot rate

Since the 6-month spot rate and 1-year spot rate are 3.00% and 3.30%, respectively, we know that z_1 is 0.0150 and z_2 is 0.0165. We can compute the present value of the 1.5-year coupon Treasury security as follows:

$$\frac{1.75}{(1+z_1)^1} + \frac{1.75}{(1+z_2)^2} + \frac{101.75}{(1+z_3)^3} = \frac{1.75}{(1.015)^1} + \frac{1.75}{(1.0165)^2} + \frac{101.75}{(1+z_3)^3}$$

Since the price of the 1.5-year coupon Treasury security is par, the following relationship must hold:

$$\frac{1.75}{(1.015)^1} + \frac{1.75}{(1.0165)^2} + \frac{101.75}{(1+z_3)^3} = 100$$

We can solve for the theoretical 1.5-year spot rate to find that z_3 is 1.75265%. Doubling this yield we obtain 3.5053%, which is the theoretical 1.5-year spot rate. That rate is the rate that the market would apply to a 1.5-year zero-coupon Treasury security if, in fact, such a security existed.

Given the theoretical 1.5-year spot rate, we can obtain the theoretical 2-year spot rate. The present value of the cash flow of the 2-year Treasury is

$$\frac{1.95}{(1+z_1)^1} + \frac{1.95}{(1+z_2)^2} + \frac{1.95}{(1+z_3)^3} + \frac{101.95}{(1+z_4)^4}$$

where z_4 is one-half the 2-year theoretical spot rate. Since the 6-month spot rate, 1-year spot rate, and 1.5-year spot rate are 3.00%, 3.30%, and 3.5053%, respectively, then z_1 is 0.0150, z_2 is 0.0165, and z_3 is 0.0175265. Therefore, the present value of the 2-year coupon Treasury security is

$$\frac{1.95}{(1.015)^1} + \frac{1.95}{(1.0165)^2} + \frac{1.95}{(1.0175265)^3} + \frac{101.95}{(1+z_4)^4}$$

Since the price of the 2-year coupon Treasury security is par, the following relationship must hold:

$$\frac{1.95}{(1.015)^1} + \frac{1.95}{(1.0165)^2} + \frac{1.95}{(1.0175265)^3} + \frac{101.95}{(1+z_4)^4} = 100$$

Solving for the theoretical 2-year spot rate, we find that z_4 is 1.9582%. Doubling this yield, we obtain the theoretical 2-year spot rate of 3.9164%.

One can follow this approach sequentially to derive the theoretical 2.5-year spot rate from the calculated values of z_1, z_2, z_3, and z_4 (the 6-month, 1-year, 1.5-year, and 2-year rates), and the price and coupon of the bond with a maturity of 2.5 years. Further, one could derive theoretical spot rates for the remaining 15 semiannual rates. The spot rates thus obtained are shown in the next-to-the-last column of Exhibit 2.1. They represent the term structure of default-free spot rate for maturities up to ten years at the particular time to which the bond price quotations refer.

The Discount Function

The term structure is represented by the spot rate curve. We also know that the present value of \$1 to be received *n* periods from now when discounted at the spot rate for period *n* is

$$\frac{\$1}{\left[1+\left(\frac{\text{Spot rate for period } n}{2}\right)\right]^n}$$

EXHIBIT 2.2 Determination of the Theoretical Price of an 8% 10-Year Treasury

Period	Years	Cash Flow ($)	Spot Rate (%)	Discount Function	Present Value ($)
1	0.5	4.00	3.0000	0.9852	3.9409
2	1.0	4.00	3.3000	0.9678	3.8712
3	1.5	4.00	3.5053	0.9492	3.7968
4	2.0	4.00	3.9164	0.9254	3.7014
5	2.5	4.00	4.4376	0.8961	3.5843
6	3.0	4.00	4.7520	0.8686	3.4743
7	3.5	4.00	4.9622	0.8424	3.3694
8	4.0	4.00	5.0650	0.8187	3.2747
9	4.5	4.00	5.1701	0.7948	3.1791
10	5.0	4.00	5.2772	0.7707	3.0828
11	5.5	4.00	5.3864	0.7465	2.9861
12	6.0	4.00	5.4976	0.7222	2.8889
13	6.5	4.00	5.6108	0.6979	2.7916
14	7.0	4.00	5.6643	0.6764	2.7055
15	7.5	4.00	5.7193	0.6551	2.6205
16	8.0	4.00	5.7755	0.6341	2.5365
17	8.5	4.00	5.8331	0.6134	2.4536
18	9.0	4.00	5.9584	0.5895	2.3581
19	9.5	4.00	6.0863	0.5658	2.2631
20	10.0	104.00	6.2169	0.5421	56.3828
				Total	115.2619

For example, the present value of $1 five years from now using the spot rate for ten periods in Exhibit 2.2, 5.2772%, is

$$\frac{\$1}{\left[1+\left(\frac{0.052772}{2}\right)\right]^{10}} = 0.7707$$

This value can be viewed as the time value of $1 for a default-free cash flow to be received in five years. Equivalently, it shows the price of a zero-coupon default-free security with a maturity of five years and a maturity value of $1.

The last column of Exhibit 2.1 shows the time value of $1 for each period. The set of time values for all periods is called the *discount function*.

Applying the Spot Rates to Value a Treasury Coupon Security

To demonstrate how to use the spot rate curve, suppose that we want to price an 8% 10-year Treasury security. The price of this issue is the present value of the cash flow, where each cash flow is discounted at the corresponding spot rate. This is illustrated in Exhibit 2.2.

The third column shows the cash flow for each period. The fourth column shows the spot rate curve. The discount function is shown in the next-to-the-last column. Multiplying the value in the discount function column by the cash flow gives the present value of the cash flow. The sum of the present values is equal to \$115.2619. This is the theoretical price of this issue.

Why Treasuries Must be Valued Based on Spot Rates

The value of a Treasury security is determined by the spot rates, not the yield-to-maturity of a Treasury coupon security of the same maturity. We will use an illustration to demonstrate the market forces that will assure that the actual market price of a Treasury coupon security will not depart significantly from its theoretical price.

To demonstrate this, consider the 8% 10-year Treasury security. Suppose that this Treasury security is priced based on the 6% yield to maturity of the 10-year maturity Treasury coupon security in Exhibit 2.1. Discounting each cash flow of the 8% 10-year Treasury security at 6% gives a present value of \$114.88.

The question is, could this security trade at \$114.88 in the market? Let's see what would happen if the 8% 10-year Treasury traded at \$114.88. Suppose that a dealer firm buys this issue at \$114.88 and strips it. By stripping this issue, the dealer firm creates 20 zero-coupon instruments guaranteed by the U.S. Treasury. How much can the 20 zero-coupon instruments be sold for by the dealer firm? Expressed equivalently, at what yield can each of the zero-coupon instruments be sold? The answer is in Exhibit 2.1. The yield at which each zero-coupon instrument can be sold is the spot rate shown in the next-to-the-last column.

We can use Exhibit 2.2 to determine the proceeds that would be received per \$100 of par value of the 8% 10-year issue stripped. The last column shows how much would be received for each coupon sold as a zero-coupon instrument. The total proceeds received from selling the zero-coupon Treasury securities created would be \$115.2619 per \$100 of par value of the Treasury issue purchased by the dealer. Since the dealer purchased the issue for \$114.88, this would result in an arbitrage profit of \$0.3819 per \$100 of the 8% 10-year Treasury issue purchased.

To understand why the dealer has the opportunity to realize this arbitrage profit, consider the \$4 coupon payment in four years. By buying the 10-year Treasury bond priced to yield 6%, the dealer effectively pays a price based on 6% (3% semiannual) for that coupon payment, or, equivalently, \$3.1577.[1] Under the assumptions of this illustration, however, investors were willing to accept a lower yield to maturity (the 4-year spot rate), 5.065% (2.5325% semiannual), to purchase a zero-coupon Treasury security with four years to maturity. Thus investors were willing to pay \$3.2747. (See Exhibit 2.2.) On this one coupon payment, the dealer realizes a profit equal to the difference between \$3.2747 and \$3.1577 (or \$0.117). From all the cash flows, the total profit is \$0.3819. In this instance, coupon stripping results in the sum of the parts being greater than the whole.

Suppose that, instead of the observed yield to maturity from Exhibit 2.1, the yields that investors want are the same as the theoretical spot rates that are shown in the exhibit. As can be seen in Exhibit 2.2, if we use these spot rates to discount the cash flows, the total proceeds from the sale of the zero-coupon Treasury securities would be equal to \$115.2619, making coupon stripping uneconomic since the proceeds from stripping would be the same as the cost of purchasing the issue.

In our illustration of coupon stripping, the price of the Treasury security is less than its theoretical price. Suppose instead that the price of the Treasury coupon security is greater than its theoretical price. In this case, investors can create a portfolio of zero-coupon Treasury securities such that the cash flow of the portfolio replicates the cash flow of the mispriced Treasury coupon security. By doing so, the investor will realize a yield higher than the yield on the Treasury coupon security. For example, suppose that the market price of the 10-year Treasury coupon security we used in our illustration is \$116. An investor could buy 20 outstanding zero-coupon stripped Treasury securities with a maturity value identical to the cash flow shown in the third column of Exhibit 2.2. The cost of purchasing this portfolio of stripped Treasury securities would be \$115.1880. Thus, an investor is effectively purchasing a portfolio of stripped Treasury securities that has the same cash flow as an 8% 10-year Treasury coupon security at a cost of \$115.1880 instead of \$116.[2]

It is the process of coupon stripping (when the market price is less than the theoretical price) and reconstituting (when the market price is greater than the theoretical price) that will prevent the market price of Treasury securities from departing significantly from their theoretical price.

---

[1] This is determined as follows: $\$1/(1.03)^8$.

[2] The New York Federal Reserve stands ready to strip a coupon Treasury security (i.e., exchange the requisite portfolio of strips for a coupon Treasury security delivered to them) or reconstitute a coupon Treasury security (i.e., exchange a coupon Treasury security for the requisite portfolio of strips) during normal market hours.

CREDIT SPREADS AND THE VALUATION OF NON-TREASURY SECURITIES

The Treasury spot rates can be used to value any default-free security. For a non-Treasury security, the theoretical value takes more effort to determine. The value of a non-Treasury security is found by discounting the cash flows by the Treasury spot rates plus a yield spread which reflects the additional risks (e.g., default risk, liquidity risks, the risk associated with any embedded options, and so on).

The spot rate used to discount the cash flow of a non-Treasury security can be the Treasury spot rate plus a constant credit spread. For example, suppose the 6-month Treasury spot rate is 1.30% and the 10-year Treasury spot rate is 4.60%. Also suppose that a suitable credit spread is 60 basis points. Then a 1.90% spot rate is used to discount a 6-month cash flow of a non-Treasury bond and a 5.20% spot rate is used to discount a 10-year cash flow. (Remember that when each semiannual cash flow is discounted, the discount rate is one-half the spot rate: 0.95% for the 6-month spot rate and 2.60% for the 10-year spot rate.)

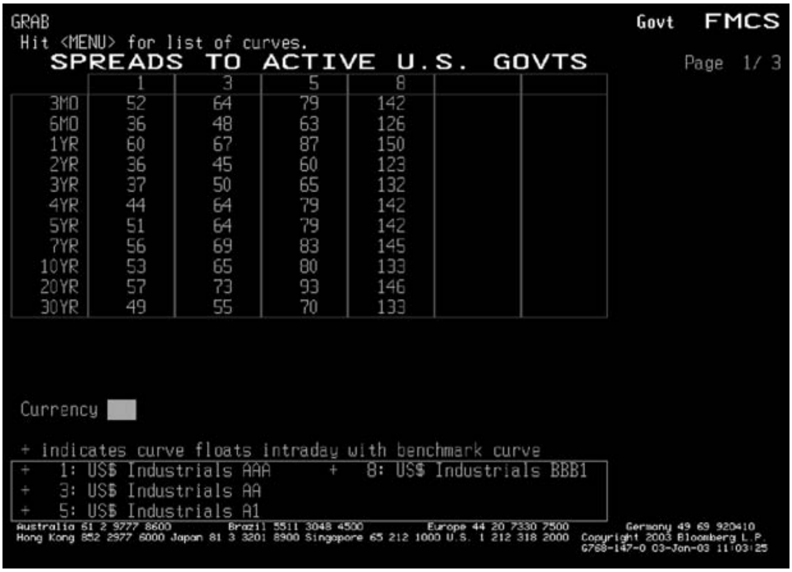
The drawback of this approach is that there is no reason to expect the credit spread to be the same regardless of when the cash flow is expected to be received in the future. Consequently, the credit spread may vary with a bond's term to maturity. In other words, there is a *term structure of credit spreads*.

Dealer firms typically estimate the term structure of credit spreads for each credit rating and market sector. Typically, the lower the credit rating, the steeper the term structure of credit spreads.

When the relevant credit spreads for a given credit rating and market sector are added to the Treasury spot rates, the resulting term structure is used to value the bonds of issuers with that credit rating in that particular market sector. The term structure is referred to as the *benchmark spot rate curve* or *benchmark zero-coupon rate curve*.

As an illustration, Exhibit 2.3 presents a Bloomberg screen (function FMCS) containing the term structure of credit spreads for four sectors of the corporate bond market as of January 3, 2003. These sectors include AAA (1), AA (3), A1 (5), and BAA1 (8) industrial bonds. This term structure of credit spreads is somewhat atypical in that credit spreads generally increase with maturity. Exhibit 2.4 presents a Bloomberg graph of these credit spreads as a function of maturity.

EXHIBIT 2.3 Credit Spreads of Corporate Bonds by Maturity and Credit Rating



GRAB Govt FMCS

Hit <MENU> for list of curves.

SPREADS TO ACTIVE U.S. GOVTS Page 1/ 3

	1	3	5	8		
3MO	52	64	79	142		
6MO	36	48	63	126		
1YR	60	67	87	150		
2YR	36	45	60	123		
3YR	37	50	65	132		
4YR	44	64	79	142		
5YR	51	64	79	142		
7YR	56	69	83	145		
10YR	53	65	80	133		
20YR	57	73	93	146		
30YR	49	55	70	133		

Currency

+ indicates curve floats intraday with benchmark curve

+ 1: US$ Industrials AAA + 8: US$ Industrials BBB1

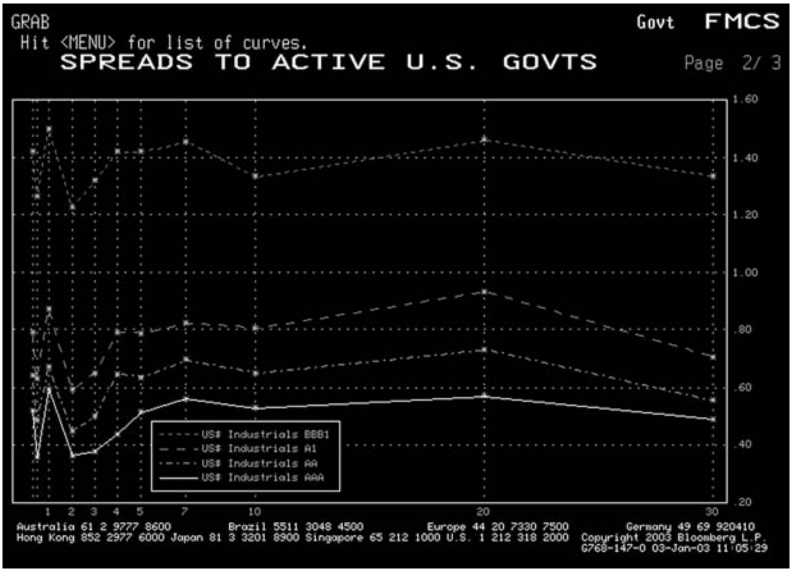
+ 3: US$ Industrials AA

+ 5: US$ Industrials A1

Australia 61 2 9777 8600 Brazil 5511 3048 4500 Europe 44 20 7330 7500 Germany 49 69 920410
Hong Kong 852 2977 6000 Japan 81 3 3201 8900 Singapore 65 212 1000 U.S. 1 212 318 2000 Copyright 2003 Bloomberg L.P.
G768-147-0 03-Jan-03 11:03:25

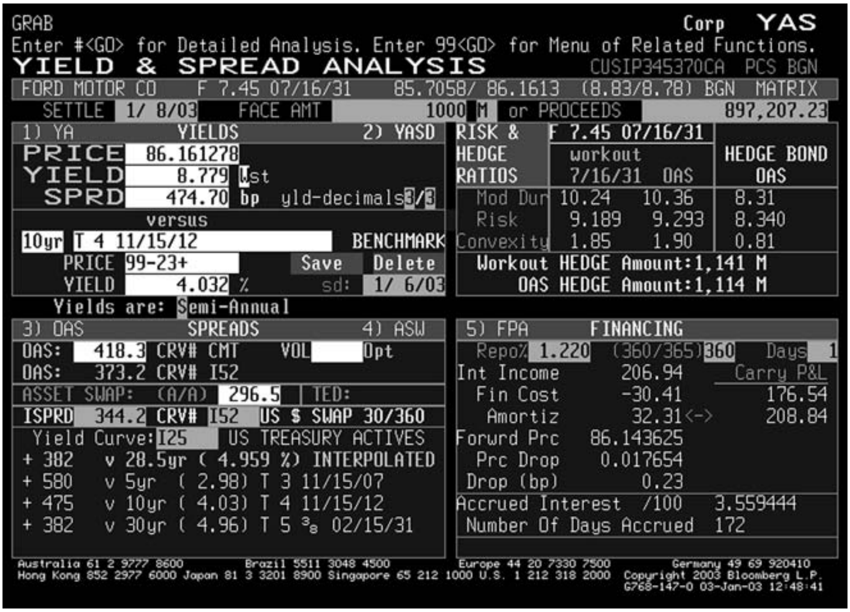
Source: Bloomberg Financial Markets

EXHIBIT 2.4 Term Structure of Credit Spreads



Source: Bloomberg Financial Markets

EXHIBIT 2.5 **Bloomberg Yield and Spread Analysis Screen for Ford Motor Co. Bond**



Source: Bloomberg Financial Markets

YIELD SPREAD MEASURES RELATIVE TO A SPOT RATE CURVE

Traditional yield spread analysis for a non-Treasury security involves calculating the difference between the risky bond's yield and the yield on a comparable maturity benchmark Treasury security. As an illustration, let's use a 7.45% coupon bond issued by Ford Motor Co. that matures on July 16, 2031. Bloomberg's Yield & Spread Analysis screen (function YAS) is presented in Exhibit 2.5. The yield spreads against the benchmark U.S. Treasury yield curve appear in a box at the bottom left-hand corner of the screen. Using a settlement date of January 8, 2003, the yield spread is 382 basis points versus the interpolated 28.5 year benchmark Treasury yield. The yield spread is simply the difference between the yields to maturity of these two yields (8.779% – 4.959%). This yield spread measure is referred to as the *nominal spread*.

The nominal spread measure has several drawbacks. For the present, the most important is that the nominal spread fails to account for the term structure of spot rates for both bonds (e.g., non-Treasury and Treasury). Moreover, as we will see later in the chapter when we discuss bonds with embedded options, the nominal spread does not take into consideration the fact that expected interest rate volatility may alter the non-Treasury bond's expected future cash flows. We will focus here only on the first drawback and pose an alternative spread measure that incorporates the spot rate curve. Later, we will discuss another spread measure for bonds with embedded options—the option-adjusted spread (OAS).

Zero-Volatility Spread

The *zero-volatility spread*, also referred to as the *Z-spread* or *static spread,* is a measure of the spread that the investor would realize over the entire Treasury spot rate curve if the bond were held to maturity. Unlike the nominal spread, it is not a spread at one point on the yield curve. The Z-spread is the spread that will make the present value of the cash flows from the non-Treasury bond, when discounted at the Treasury rate plus the spread, equal to the non-Treasury bond's market price plus accrued interest. A trial-and-error procedure is used to compute the Z-spread.

To illustrate how this is done, consider the following two 5-year bonds:

Issue	Coupon	Price	Yield to Maturity
Treasury	5.055%	100.0000	5.0550%
Non-Treasury	7.000%	101.9576	6.5348%

The nominal spread for the non-Treasury bond is 147.98 basis points. Let's use the information presented in Exhibit 2.6 to determine the Z-spread. The third column in Exhibit 2.6 shows the cash flows for the 7% 5-year non-Treasury issue. The fourth column is a hypothetical Treasury spot rate curve that we will employ in this example. The goal is to determine the spread that, when added to all the Treasury spot rates, will produce a present value for the non-Treasury bond equal to its market price of $101.9576.

Suppose we select a spread of 100 basis points. To each Treasury spot rate shown in the fourth column of Exhibit 2.6, 100 basis points are added. So, for example, the 1-year (period 2) spot rate is 5.33% (4.33% plus 1%). The spot rate plus 100 basis points is used to calculate the present values as shown in the fifth column.[3] The total present value in the fifth column is $104.110. Because the present value is not equal to the non-Treasury issue's price of ($101.9576), the Z-spread is not 100 basis points. If a spread of 120 basis points is tried, it can be seen from the next-to-the-last column of

[3] The discount rate used to compute the present value of each cash flow in the third column is found by adding the assumed spread to the spot rate and then dividing by 2.

EXHIBIT 2.6 **Determination of the Z-Spread for a 7% 5-Year Non-Treasury Issue Selling at $101.9576 to Yield 6.5347%**

Period	Years	Cash Flow ($)	Spot Rate (%)	Present Value ($) Assuming a Spread of 100 bp	120 bp	150 bp
1	0.5	3.50	4.20	3.4113	3.4080	3.4030
2	1.0	3.50	4.33	3.3207	3.3142	3.3045
3	1.5	3.50	4.39	3.2793	3.2222	3.2081
4	2.0	3.50	4.44	3.1438	3.1315	3.1133
5	2.5	3.50	4.51	3.0553	3.0405	3.0184
6	3.0	3.50	4.54	2.9708	2.9535	2.9278
7	3.5	3.50	4.58	2.8868	2.8672	2.8381
8	4.0	3.50	4.73	2.7921	2.7705	2.7384
9	4.5	3.50	4.90	2.6942	2.6708	2.6360
10	5.0	103.50	5.11	76.6037	75.8643	74.7699
			Total	104.110	103.243	101.958

Exhibit 2.6 that the present value is $103.243; again, because this is not equal to the non-Treasury issue's price, 120 basis points is not the Z-spread. The last column of Exhibit 2.6 shows the present value when a 150-basis-point spread is used. The present value of the cash flows is equal to the non-Treasury issue's price. Accordingly, 150 basis points is the Z-spread, compared to the nominal spread of 147.98 basis points.

What does the Z-spread represent for this non-Treasury security? Since the Z-spread is relative to the benchmark Treasury spot rate curve, it represents a spread required by the market to compensate for all the risks of holding the non-Treasury bond versus a Treasury security with the same maturity. These risks include the non-Treasury's credit risk, liquidity risk, and the risks associated with any embedded options.

Divergence Between Z-Spread and Nominal Spread

Generally, the divergence is a function of the term structure's shape and the security's characteristics. Among the relevant security characteristics are coupon rate, term to maturity, and type of principal repayment provision—nonamortizing versus amortizing. The steeper the term structure, the greater will be the divergence. For standard coupon-paying bonds with a bullet maturity (i.e., a single payment of principal), the Z-spread and the nominal spread will usually not differ significantly. For monthly-pay amortizing securities the divergence can be substantial in a steep yield curve environment.

Z-Spread Relative to Any Benchmark

A Z-spread can be calculated relative to any benchmark spot rate curve in the same manner. The question arises: What does the Z-spread mean when the benchmark is not the Treasury spot rate curve (i.e., default-free spot rate curve)? This is especially true in Europe, where swaps curves are commonly used as a benchmark for pricing. When the Treasury spot rate curve is the benchmark, we indicated that the Z-spread for non-Treasury issues captured credit risk, liquidity risk, and any option risks. When the benchmark is the spot rate curve for the issuer, for example, the Z-spread reflects the spread attributable to the issue's liquidity risk and any option risks.

Accordingly, when a Z-spread is cited, it must be cited relative to some benchmark spot rate curve. This is essential because it indicates the credit and sector risks that are being considered when the Z-spread is calculated. While Z-spreads are typically calculated in the United States using Treasury securities as the benchmark interest rates, this is usually not the case elsewhere. Vendors of analytical systems such as Bloomberg commonly allow the user to select a benchmark.

VALUATION METHODOLOGIES

Our discussion of bond valuation has thus far been limited to bonds in which neither the issuer nor the bondholder has the option to alter a bond's cash flows. Now we look at how to value bonds with embedded options. The methodology described here is used to value options, caps, and floors in Chapter 12.

There are two main approaches to the valuation of bonds with embedded options: (1) the binomial lattice method, or simply binomial method, and (2) the Monte Carlo simulation method. There are two things that are common to both methods. First, each begins with an assumption as to the statistical process that is assumed to generate the term structure of interest rates. Second, each method is based on the principle that arbitrage profits cannot be generated. By this it is meant that the model will correctly price the on-the-run issues; or, equivalently, the model is calibrated to the market.

It is important to understand that the user of any valuation model is exposed to *modeling risk*. This is the risk that the output of the model is incorrect because the assumptions, upon which it is based, are incorrect. Consequently, it is imperative that the results of a valuation model be stress-tested for modeling risk by altering the assumptions.

Option-Adjusted Spread

What an investor seeks to do is to buy securities whose value is greater than their price. A valuation model allows an investor to estimate the theoretical value of a security, which at this point would be sufficient to determine the fairness of the price of the security. That is, the investor can say that a particular bond is 1-point cheap or 2-points cheap, and so on.

A valuation model need not stop here, however. Instead, it can convert the divergence between the price observed in the market for the security and the theoretical value derived from the valuation model into a yield spread measure. This step is necessary since many market participants find it more convenient to think about yield spreads than price differences.

The *option-adjusted spread* (OAS) was developed as a measure of the yield spread that can be used to convert dollar differences between value and price. Thus, basically, the OAS is used to reconcile value with market price. But what is it a "spread" over? As we shall see, when we describe the two valuation methodologies, the OAS is a spread over the issuer's spot rate curve or benchmark. The spot rate curve itself is not a single curve, but a series of spot rate curves that allow for changes in rates and cash flows. The reason that the resulting spread is referred to as "option-adjusted" is because the cash flows of the security, whose value is sought, are adjusted to reflect any embedded options.

Binomial Method[4]

The binomial method is a popular technique for valuing callable and putable bonds. To illustrate this, we start with the on-the-run yield curve for the particular issuer whose bonds we want to value. The starting point is the Treasury's on-the-run yield curve. To obtain a particular issuer's on-the-run yield curve, an appropriate credit spread is added to each on-the-run Treasury issue. The credit spread need not be constant for all maturities. For example, the credit spread may increase with maturity.

In our illustration, we use the hypothetical on-the-run issues for an issuer shown in Exhibit 2.7. Each bond is trading at par value (100) so the coupon rate is equal to the yield to maturity. We will simplify the illustration by assuming annual-pay bonds. Using the bootstrapping methodology, the spot rates are those shown in the last column of Exhibit 2.7.

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[4] The model described in this section was presented in Andrew J. Kalotay, George O. Williams, and Frank J. Fabozzi, "A Model for the Valuation of Bonds and Embedded Options," *Financial Analysts Journal*, May–June 1993, pp. 35–46.

EXHIBIT 2.7 On-the-Run Yield Curve and Spot Rates for an Issuer

Maturity (yrs)	Yield to Maturity (%)	Market Price ($)	Spot Rate (%)
1	3.5	100	3.5000
2	4.2	100	4.2147
3	4.7	100	4.7345
4	5.2	100	5.2707

Binomial Interest Rate Tree

Once we allow for embedded options, consideration must be given to interest rate volatility. This can be done by introducing a *binomial interest rate tree*. This tree is nothing more than a graphical depiction of the 1-period or short rates over time based on some assumption about interest rate volatility. How this tree is constructed is illustrated below.

Exhibit 2.8 shows an example of a binomial interest rate tree. In this tree, each node (bold circle) represents a time period that is equal to one year from the node to its left. Each node is labeled with an N, representing node, and a subscript that indicates the path that the 1-year rate took to get to that node. L represents the lower of the two 1-year rates and H represents the higher of the two 1-year rates. For example, node N_{HH} means to get to that node the following path for 1-year rates occurred: The 1-year rate realized is the higher of the two rates in the first year and then the higher of the 1-year rates in the second year.[5]

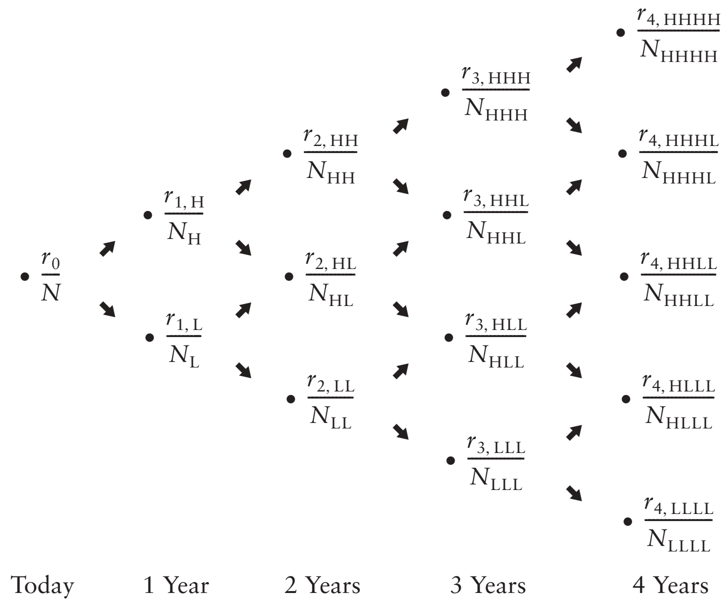
Look first at the point denoted by just N in Exhibit 2.8. This is the root of the tree and is nothing more than the current 1-year spot rate, or equivalently the current 1-year rate, which we denote by r_0. What we have assumed in creating this tree is that the 1-year rate can take on two possible values the next period and the two rates have the same probability of occurring. One rate will be higher than the other. It is assumed that the 1-year rate can evolve over time based on a random process called a *lognormal random walk* with a certain volatility.

We use the following notation to describe the tree in the first year:

σ = assumed volatility of the 1-year rate
$r_{1,\mathrm{L}}$ = lower 1-year rate one year from now
$r_{1,\mathrm{H}}$ = higher 1-year rate one year from now

[5] Note that N_{HL} is equivalent to N_{LH} in the second year, that in the third year N_{HHL} is equivalent to N_{HLH} and N_{LHH}, and that N_{HLL} is equivalent to N_{LLH}. We have simply selected one label for a node rather than clutter up the figure with unnecessary information.

EXHIBIT 2.8 **4-Year Binomial Interest Rate Tree**



The relationship between $r_{1,L}$ and $r_{1,H}$ is as follows:

$$r_{1,H} = r_{1,L}(e^{2\sigma})$$

where e is the base of the natural logarithm 2.71828.

For example, suppose that $r_{1,L}$ is 4.4448% and σ is 10% per year, then

$$r_{1,H} = 4.4448\%(e^{2 \times 0.10}) = 5.4289\%$$

In the second year, there are three possible values for the 1-year rate, which we will denote as follows:

$r_{2,LL}$ = 1-year rate in second year assuming the lower rate in the first year and the lower rate in the second year
$r_{2,HH}$ = 1-year rate in second year assuming the higher rate in the first year and the higher rate in the second year
$r_{2,HL}$ = 1-year rate in second year assuming the higher rate in the first year and the lower rate in the second year or equivalently the lower rate in the first year and the higher rate in the second year

The relationship between $r_{2,LL}$ and the other two 1-year rates is as follows: $r_{2,HH} = r_{2,LL}(e^{4\sigma})$ and $r_{2,HL} = r_{2,LL}(e^{2\sigma})$. So, for example, if $r_{2,LL}$ is 4.6958% and assuming once again that σ is 10%, then

$$r_{2,HH} = 4.6958\%(e^{4 \times 0.10}) = 7.0053\%$$

and

$$r_{2,HL} = 4.6958\%(e^{2 \times 0.10}) = 5.7354\%$$

In the third year there are four possible values for the 1-year rate, which are denoted as follows: $r_{3,HHH}$, $r_{3,HHL}$, $r_{3,HLL}$, and $r_{3,LLL}$, and whose first three values are related to the last as follows:

$$\begin{aligned} r_{3,HHH} &= r_{3,LLL}(e^{6\sigma}) \\ r_{3,HHL} &= r_{3,LLL}(e^{4\sigma}) \\ r_{3,HLL} &= r_{3,LLL}(e^{2\sigma}) \end{aligned}$$

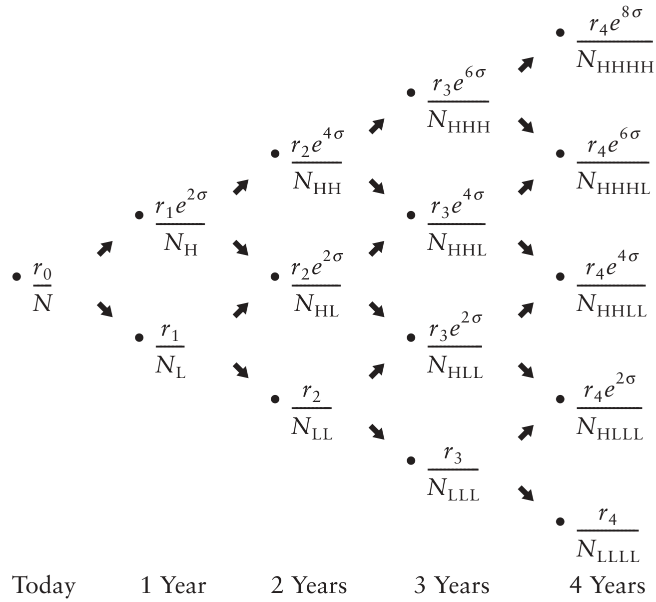
Exhibit 2.8 shows the notation for a 4-year binomial interest rate tree. We can simplify the notation by letting r_t be the 1-year rate t years from now for the lower rate since all the other short rates t years from now depend on that rate. Exhibit 2.9 shows the interest rate tree using this simplified notation.

It can be shown that the standard deviation of the 1-year rate is equal to $r_0\sigma$. The standard deviation is a statistical measure of volatility, and we will discuss this measure and its estimation in Chapter 7. It is important to understand that the process that we assumed generates the binomial interest rate tree (or equivalently the short rates), implies that volatility is measured relative to the current level of rates. For example, if σ is 10% and the 1-year rate (r_0) is 4%, then the standard deviation of the 1-year rate is 4% × 10% = 0.4% or 40 basis points. However, if the current 1-year rate is 12%, the standard deviation of the 1-year rate would be 12% × 10% or 120 basis points.

Determining the Value at a Node

To find the value of the bond at a node, we first calculate the bond's value at the two nodes to the right of the node we are interested in. For example, in Exhibit 2.9, suppose we want to determine the bond's value at node N_H. The bond's value at nodes N_{HH} and N_{HL} must be determined. Hold aside for now how we get these two values because, as we will see, the process involves starting from the last year in the tree and working backwards to get the final solution we want, so these two values will be known.

EXHIBIT 2.9 **4-Year Binomial Interest Rate Tree with 1-Year Rates***



* r_t equals forward 1-year lower rate

Effectively what we are saying is that if we are at some node, then the value at that node will depend on the future cash flows. In turn, the future cash flows depend on (1) the bond's value one year from now and (2) the coupon payment one year from now. The latter is known. The former depends on whether the 1-year rate is the higher or lower rate. The bond's value depending on whether the rate is the higher or lower rate is reported at the two nodes to the right of the node that is the focus of our attention. So, the cash flow at a node will be either (1) the bond's value if the short rate is the higher rate plus the coupon payment, or (2) the bond's value if the short rate is the lower rate plus the coupon payment. For example, suppose that we are interested in the bond's value at N_H. The cash flow will be either the bond's value at N_{HH} plus the coupon payment, or the bond's value at N_{HL} plus the coupon payment.

To get the bond's value at a node, we follow the fundamental rule for valuation: The value is the present value of the expected cash flows. The appropriate discount rate to use is the 1-year rate at the node. Now there are two present values in this case: the present value if the 1-year rate is the higher rate and one if it is the lower rate. Since it is assumed that the probability of both outcomes is equal, an average of the two present values is computed. This is illustrated in Exhibit 2.10 for any node assuming that the 1-year rate is r_* at the node where the valuation is sought and letting:

V_H = bond's value for the higher 1-year rate
V_L = bond's value for the lower 1-year rate
C = coupon payment

Using our notation, the cash flow at a node is either:

$$V_H + C \text{ for the higher 1-year rate}$$

$$V_L + C \text{ for the lower 1-year rate}$$

The present value of these two cash flows using the 1-year rate at the node, r_*, is:

$$\frac{V_H + C}{(1 + r_*)} = \text{Present value for the higher 1-year rate}$$

$$\frac{V_L + C}{(1 + r_*)} = \text{Present value for the lower 1-year rate}$$

Then, the value of the bond at the node is found as follows:

$$\text{Value at a node} = \frac{1}{2}\left[\frac{V_H + C}{(1 + r_*)} + \frac{V_L + C}{(1 + r_*)}\right]$$

EXHIBIT 2.10 **Calculating a Value at a Node**

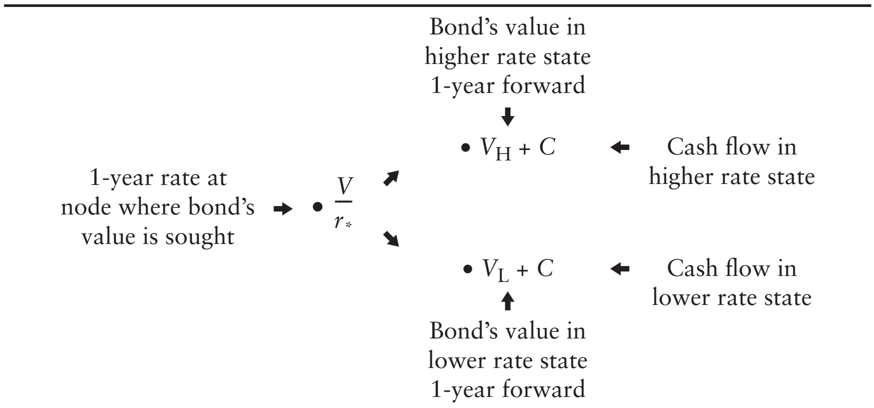
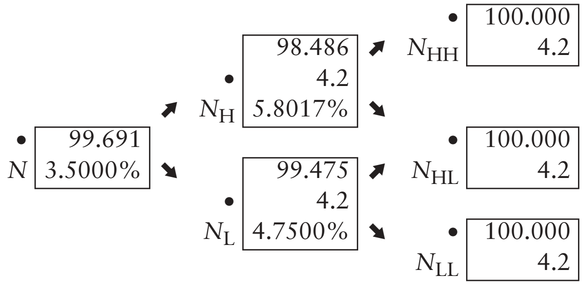


EXHIBIT 2.11 **The 1-Year Rates for Year 1 Using the 2-Year 4.2% On-the-Run Issue: First Trial**



Constructing the Binomial Interest Rate Tree

To see how to construct the binomial interest rate tree, let's use the assumed on-the-run yields we used earlier. We will assume that volatility, σ, is 10% and construct a 2-year tree using the 2-year bond with a coupon rate of 4.2%.

Exhibit 2.11 shows a more detailed binomial interest rate tree with the cash flow shown at each node. We'll see how all the values reported in the exhibit are obtained. The root rate for the tree, r_0, is simply the current 1-year rate, 3.5%.

In the first year there are two possible 1-year rates, the higher rate and the lower rate. What we want to find is the two 1-year rates that will be consistent with the volatility assumption, the process that is assumed to generate the short rates, and the observed market value of the bond. There is no simple formula for this. It must be found by an iterative process (i.e., trial-and-error). The steps are described and illustrated below.

Step 1: Select a value for r_1. Recall that r_1 is the lower 1-year rate. In this first trial, we *arbitrarily* selected a value of 4.75%.

Step 2: Determine the corresponding value for the higher 1-year rate. As explained earlier, this rate is related to the lower 1-year rate as follows: $r_1 e^{2\sigma}$. Since r_1 is 4.75%, the higher 1-year rate is 5.8017% (= 4.75% $e^{2 \times 0.10}$). This value is reported in Exhibit 2.11 at node N_H.

Step 3: Compute the bond value's one year from now. This value is determined as follows:

3a. Determine the bond's value two years from now. In our example, this is simple. Since we are using a 2-year bond, the bond's value is its maturity value ($100) plus its final coupon payment ($4.2). Thus, it is $104.2.

3b. Calculate the present value of the bond's value found in 3a for the higher rate in the second year. The appropriate discount rate is the higher 1-year rate, 5.8017% in our example. The present value is \$98.486 (= \$104.2/1.058017). This is the value of V_H that we referred to earlier.

3c. Calculate the present value of the bond's value found in 3a for the lower rate. The discount rate assumed for the lower 1-year rate is 4.75%. The present value is \$99.475 (= \$104.2/1.0475) and is the value of V_L.

3d. Add the coupon to both V_H and V_L to get the cash flow at N_H and N_L, respectively. In our example, we have \$102.686 for the higher rate and \$103.675 for the lower rate.

3e. Calculate the present value of the two values using the 1-year rate r_*. At this point in the valuation, r_* is the root rate, 3.50%. Therefore,

$$\frac{V_H + C}{1 + r_*} = \frac{\$102.686}{1.035} = \$99.213$$

and

$$\frac{V_L + C}{1 + r_*} = \frac{\$103.675}{1.035} = \$100.169$$

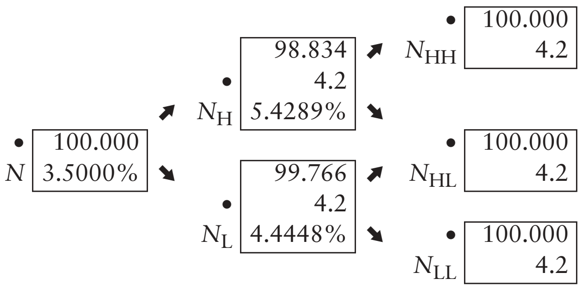
Step 4: Calculate the average present value of the two cash flows in Step 3. This is the value we referred to earlier as

$$\text{Value at a node} = \frac{1}{2}\left[\frac{V_H + C}{(1 + r_*)} + \frac{V_L + C}{(1 + r_*)}\right]$$

In our example, we have

$$\text{Value at a node} = \frac{1}{2}[\$99.213 + \$100.169] = \$99.691$$

EXHIBIT 2.12 **The 1-Year Rates for Year 1 Using the 2-Year 4.2% On-the-Run Issue**



Step 5: Compare the value in Step 4 to the bond's market value. If the two values are the same, then the r_1 used in this trial is the one we seek. This is the 1-year rate that would then be used in the binomial interest rate tree for the lower rate and to obtain the corresponding higher rate. If, instead, the value found in step 4 is not equal to the market value of the bond, this means that the value r_1 in this trial is not the 1-year rate that is consistent with (1) the volatility assumption, (2) the process assumed to generate the 1-year rate, and (3) the observed market value of the bond. In this case, the five steps are repeated with a different value for r_1.

When r_1 is 4.75%, a value of \$99.691 results in Step 4 which is less than the observed market price of \$100. Therefore, 4.75% is too large and the five steps must be repeated trying a lower rate for r_1.

Let's jump right to the correct rate for r_1 in this example and rework steps 1 through 5. This occurs when r_1 is 4.4448%. The corresponding binomial interest rate tree is shown in Exhibit 2.12.

Step 1: In this trial we select a value of 4.4448% for r_1, the lower 1-year rate.

Step 2: The corresponding value for the higher 1-year rate is 5.4289% (=4.4448%$e^{2 \times 0.10}$).

Step 3: The bond's value one year from now is determined as follows:

3a. The bond's value two years from now is \$104.2, just as in the first trial.

3b. The present value of the bond's value found in *3a* for the higher 1-year rate, V_H, is \$98.834 (= \$104.2/1.054289).

3c. The present value of the bond's value found in *3a* for the lower 1-year rate, V_L, is \$99.766 (= \$104.2/1.044448).

3d. Adding the coupon to V_H and V_L, we get \$103.034 as the cash flow for the higher rate and \$103.966 as the cash flow for the lower rate.

3e. The present value of the two cash flows using the 1-year rate at the node to the left, 3.5%, gives

$$\frac{V_H + C}{1 + r_*} = \frac{\$103.034}{1.035} = \$99.550$$

and,

$$\frac{V_L + C}{1 + r_*} = \frac{\$103.966}{1.035} = \$100.450$$

Step 4: The average present value is \$100, which is the value at the node.

Step 5: Since the average present value is equal to the observed market price of \$100, r_1 or $r_{1,L}$ is 4.4448% and $r_{1,H}$ is 5.4289%.

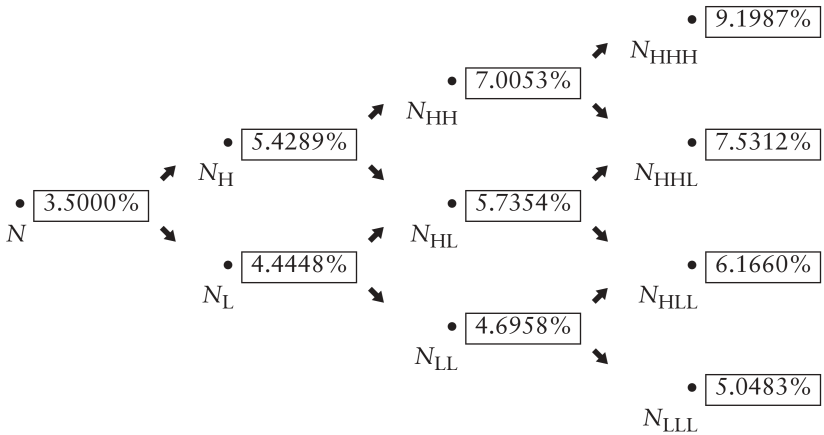
We can "grow" this tree for one more year by determining r_2. We would use the 3-year on-the-run issue, the 4.7% coupon bond, to get r_2. The same five steps are used in an iterative process to find the 1-year rates in the tree two years from now. Our objective is to find the value of r_2 that will produce a bond value of \$100 (since the 3-year on-the-run issue has a market price of \$100) and is consistent with (1) a volatility assumption of 10%, (2) a current 1-year rate of 3.5%, and (3) the two rates one year from now of 4.4448% (the lower rate) and 5.4289% (the higher rate). We will not describe how to complete the tree using the 3-year and 4-year on-the-run issues. Exhibit 2.13 shows the binomial interest rate tree for the on-the-run issues in Exhibit 2.7.

Valuing an Option-Free Bond with the Tree

Now consider an option-free bond of this issuer with three years remaining to maturity and a coupon rate of 6.5%. The value of this bond can be calculated by discounting the cash flow at the spot rates in Exhibit 2.7 as shown below:

$$\frac{\$6.5}{(1.035)^1} + \frac{\$6.5}{(1.042147)^2} + \frac{\$6.5}{(1.047345)^3} + \frac{\$100 + \$6.5}{(1.052707)^4} = \$104.643$$

EXHIBIT 2.13 **Binomial Interest Rate Tree for Valuing Up to a 4-Year Bond for Issuer (10% Volatility Assumed)**



An option-free bond that is valued using the binomial interest rate tree should have the same value as discounting by the spot rates.

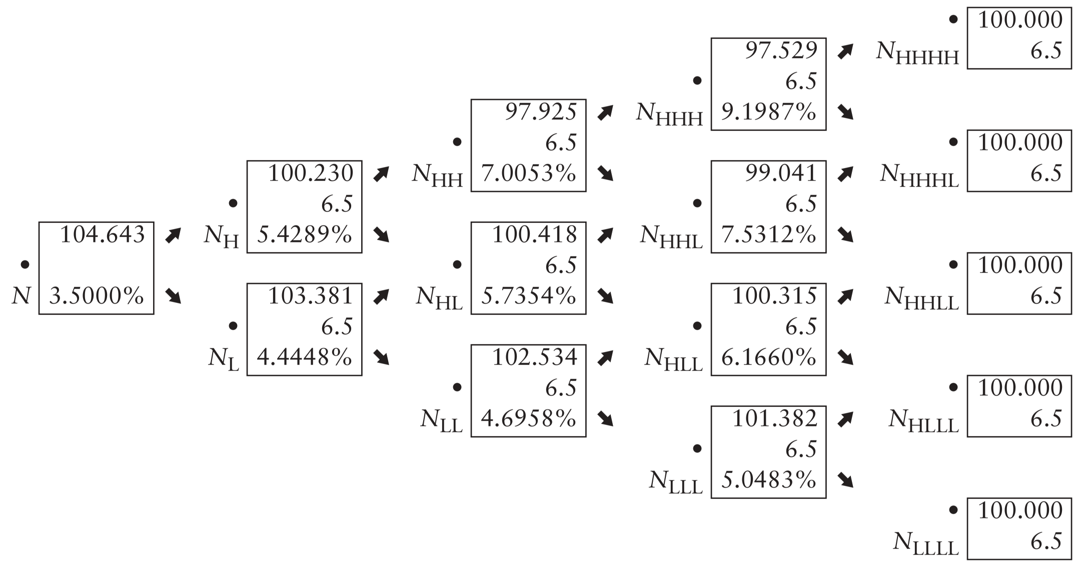
Exhibit 2.13 is the binomial interest rate tree that can be used to value any bond for this issuer with a maturity up to four years. To illustrate how to use the binomial interest rate tree, consider once again the 6.5% option-free bond with three years remaining to maturity. Also assume that the issuer's on-the-run yield curve is the one in Exhibit 2.7, hence the appropriate binomial interest rate tree is the one in Exhibit 2.13. Exhibit 2.14 shows the various values in the discounting process, and produces a bond value of $104.643.

This value is identical to the bond value found when we discounted with the spot rates. This clearly demonstrates that the valuation model is consistent with the standard valuation model for an option-free bond.

Valuing a Callable Corporate Bond

Now we will demonstrate how the binomial interest rate tree can be applied to value a callable bond. The valuation process proceeds in the same fashion as in the case of an option-free bond, but with one exception: When the call option may be exercised by the issuer, the bond value at a node must be changed to reflect the lesser of its values if it is not called (i.e., the value obtained by applying the recursive valuation formula described above) and the call price.

EXHIBIT 2.14 Valuing an Option-Free Bond with Four Years to Maturity and a Coupon Rate of 6.5% (10% Volatility Assumed)



For example, consider a 6.5% bond with four years remaining to maturity that is callable in one year at \$100. Exhibit 2.15 shows two values at each node of the binomial interest rate tree. The discounting process explained above is used to calculate the first of the two values at each node. The second value is the value based on whether the issue will be called. For simplicity, let's assume that this issuer calls the issue if it exceeds the call price of \$100. Then, in Exhibit 2.15 at nodes N_L, N_H, N_{LL}, N_{HL}, N_{LLL}, and N_{HLL}, the values from the recursive valuation formula are \$101.968, \$100.032, \$101.723, \$100.270, \$101.382, and \$100.315. These values exceed the assumed call price (\$100), and therefore the second value is \$100 rather than the calculated value. It is the second value that is used in subsequent calculations. The root of the tree indicates that the value for this callable bond is \$102.899.

The question that we have not addressed in our illustration, which is nonetheless important, is the circumstances under which the issuer will call the bond. A detailed explanation of the call rule is beyond the scope of this chapter. Basically, it involves determining when it would be economic for the issuer on an after-tax basis to call the issue.

The bond valuation framework presented here can be used to analyze other embedded options such as put options, caps and floors on floating-rate notes, and interest sensitive structured notes.

Volatility and the Theoretical Value

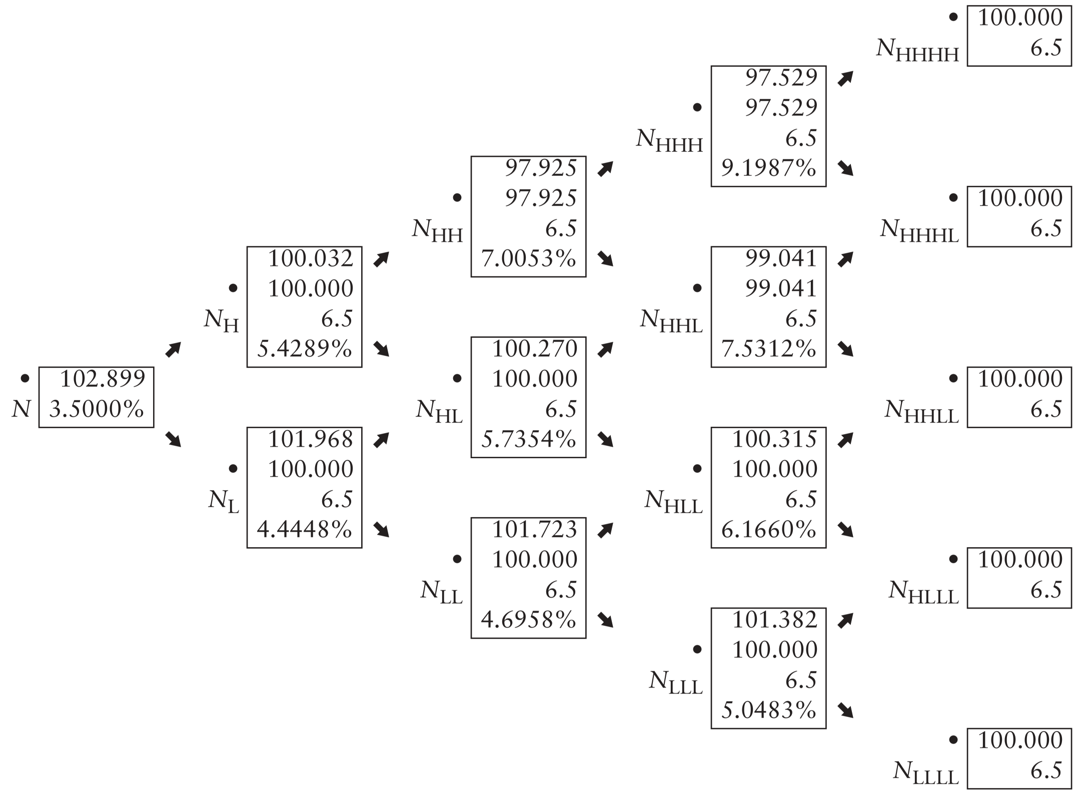
In our illustration, interest rate volatility was assumed to be 10%. The volatility assumption has an important impact on the theoretical value. More specifically, the higher the expected volatility, the higher the value of an option. The same is true for an option embedded in a bond. Correspondingly, this affects the value of the bond with an embedded option.

For example, for a callable bond, a higher interest rate volatility assumption means that the value of the call option increases, and, since the value of the option-free bond is not affected, the value of the callable bond must be lower. For a putable bond, higher interest rate volatility means that its value will be higher.

To illustrate this, suppose that a 20% volatility is assumed rather than 10%. The value of the hypothetical callable bond is \$102.108 if volatility is assumed to be 20% compared to \$102.899 if volatility is assumed to be 10%. The hypothetical putable bond at 20% volatility has a value of \$106.010 compared to \$105.327 at 10% volatility.

In the construction of the binomial interest rate, it was assumed that volatility is the same for each year. The methodology can be extended to incorporate a term structure of volatility.

EXHIBIT 2.15 Valuing a Callable Bond with Four Years to Maturity, a Coupon Rate of 6.5%, and Callable in 1-Year at 100 (10% Volatility Assumed)



Option-Adjusted Spread

Suppose the market price of the 3-year 6.5% callable bond is \$102.218 and the theoretical value assuming 10% volatility is \$102.899. This means that this bond is cheap by \$0.681 according to the valuation model. Bond market participants prefer to think not in terms of a bond's price being cheap or expensive in dollar terms but rather in terms of a yield spread—a cheap bond trades at a higher yield spread and an expensive bond at a lower yield spread.

The OAS is the constant spread that, when added to all the short-term rates on the binomial interest rate tree, will make the theoretical value equal to the market price. In our illustration, if the market price is \$102.218, the OAS would be the constant spread added to every rate in Exhibit 2.13 that will make the theoretical value equal to \$102.218. The solution in this case would be 35 basis points.

As with the value of a bond with an embedded option, the OAS will depend on the volatility assumption. For a given bond price, the higher the interest rate volatility assumed, the lower the OAS for a callable bond and the higher the OAS for a putable bond. For example, if volatility is 20% rather than 10%, it can be demonstrated that the OAS would be –11 basis points.

This illustration clearly demonstrates the importance of the volatility assumption. Assuming volatility of 10%, the OAS is 35 basis points. At 20% volatility, the OAS declines and, in this case, is negative and therefore overvalued.

Monte Carlo Method

The second method for valuing bonds with embedded options is the Monte Carlo simulation or, simply, the Monte Carlo method. This method involves simulating a sufficiently large number of potential interest rate paths in order to assess the value of a security along these different paths. This method is the most flexible of the two valuation methodologies for valuing interest rate sensitive instruments, where the history of interest rates is important. Mortgage-backed securities are commonly valued using this method. Some dealers use Monte Carlo simulation to value callable and putable bonds.

Interest Rate History and Path-Dependent Cash Flows

For some fixed-income securities and derivative instruments, the periodic cash flows are *path-dependent*. This means that the cash flow received in one period is determined not only by the current interest rate level, but also by the path that interest rates took to get to the current level.

In the case of mortgage passthrough securities (or simply, passthroughs), prepayments are path-dependent because this month's prepayment rate depends on whether there have been prior opportunities to refinance since the underlying mortgages were originated. Unlike passthroughs, the decision as to whether a corporate issuer will elect to refund an issue when the current rate is below the issue's coupon rate is not dependent on how rates evolved over time to the current level.

Moreover, in the case of adjustable-rate mortgages (ARMs), prepayments are not only path-dependent but the periodic coupon rate depends on the history of the reference rate upon which the coupon rate is determined. This is because ARMs have periodic caps and floors as well as a lifetime cap and floor. For example, an ARM whose coupon rate resets annually could have the following restriction on the coupon rate: (1) the rate cannot change by more than 200 basis points each year and (2) the rate cannot be more than 500 basis points from the initial coupon rate.

Pools of passthroughs are used as collateral for the creation of collateralized mortgage obligations (CMOs). Consequently, for CMOs, there are typically two sources of path dependency in a CMO tranche's cash flows. First, the collateral prepayments are path-dependent as discussed above. Second, the cash flow to be received in the current month by a CMO tranche depends on the outstanding balances of the other tranches in the deal. Thus, we need the history of prepayments to calculate these balances.

Valuing Mortgage-Backed Securities[6]

Conceptually, the valuation of passthroughs using the Monte Carlo method is simple. In practice, however, it is very complex. The simulation involves generating a set of cash flows based on simulated future mortgage refinancing rates, which in turn imply simulated prepayment rates.

Valuation modeling for CMOs is similar to valuation modeling for passthroughs, although the difficulties are amplified because the issuer has sliced and diced both the prepayment and interest rate risk into smaller pieces called *tranches*. The sensitivity of the passthroughs comprising the collateral to these two risks is not transmitted equally to every tranche. Some of the tranches wind up more sensitive to prepayment and interest rate risk than the collateral, while some of them are much less sensitive.

[6] Portions of the material in this section are adapted from Frank J. Fabozzi and Scott F. Richard, "Valuation of CMOs," Chapter 6 in Frank J. Fabozzi (ed.), *CMO Portfolio Management* (Summit, N.J.: Frank J. Fabozzi Associates, 1994).

The typical model used to generate random interest rate paths takes as input today's term structure of interest rates and a volatility assumption. The term structure of interest rates is the theoretical spot rate (or zero-coupon) curve implied by today's Treasury securities. The volatility assumption determines the dispersion of future interest rates in the simulation. The simulations are normalized so that the average simulated price of a zero-coupon Treasury bond equals today's actual price.

Each model has its own model of the evolution of future interest rates and its own volatility assumptions. Typically, there are no significant differences in the interest rate models of dealer firms and vendors, although their volatility assumptions can be significantly different.

The random paths of interest rates should be generated from an arbitrage-free model of the future term structure of interest rates. By arbitrage-free it is meant that the model replicates today's term structure of interest rates, an input of the model, and that for all future dates there is no possible arbitrage within the model. We will explain how this is done later.

The simulation works by generating many scenarios of future interest rate paths. In each month of the scenario, a monthly interest rate and a mortgage refinancing rate are generated. The monthly interest rates are used to discount the projected cash flows in the scenario. The mortgage refinancing rate is needed to determine the cash flow because it represents the opportunity cost the mortgagor is facing at that time.

If the refinancing rates are high relative to the mortgagor's original coupon rate (i.e., the rate on the mortgagor's loan), the mortgagor will have less incentive to refinance, or even a positive disincentive (i.e., the homeowner will avoid moving in order to avoid refinancing). If the refinancing rate is low relative to the mortgagor's original coupon rate, the mortgagor has an incentive to refinance.

Prepayments are projected by feeding the refinancing rate and loan characteristics, such as age, into a prepayment model. Given the projected prepayments the cash flow along an interest rate path can be determined.

To make this more concrete, consider a newly issued mortgage passthrough security with a maturity of 360 months. Exhibit 2.16 shows N simulated interest rate path scenarios. Each scenario consists of a path of 360 simulated 1-month future interest rates. Just how many paths should be generated is explained later. Exhibit 2.17 shows the paths of simulated mortgage refinancing rates corresponding to the scenarios shown in Exhibit 2.16. Assuming these mortgage refinancing rates, the cash flow for each scenario path is shown in Exhibit 2.18.

EXHIBIT 2.16 Simulated Paths of 1-Month Future Interest Rates

	Interest Rate Path Number						
Month	**1**	**2**	**3**	**…**	***n***	**…**	***N***
1	$f_1(1)$	$f_1(2)$	$f_1(3)$	…	$f_1(n)$	…	$f_1(N)$
2	$f_2(1)$	$f_2(2)$	$f_2(3)$	…	$f_2(n)$	…	$f_2(N)$
3	$f_3(1)$	$f_3(2)$	$f_3(3)$	…	$f_3(n)$	…	$f_3(N)$
t	$f_t(1)$	$f_t(2)$	$f_t(3)$	…	$f_t(n)$	…	$f_t(N)$
358	$f_{358}(1)$	$f_{358}(2)$	$f_{358}(3)$	…	$f_{358}(n)$	…	$f_{358}(N)$
359	$f_{359}(1)$	$f_{359}(2)$	$f_{359}(3)$	…	$f_{359}(n)$	…	$f_{359}(N)$
360	$f_{360}(1)$	$f_{360}(2)$	$f_{360}(3)$	…	$f_{360}(n)$	…	$f_{360}(N)$

Notation:

$f_t(n)$ = 1-month future interest rate for month t on path n

N = number of interest rate paths

EXHIBIT 2.17 Simulated Paths of Mortgage Refinancing Rates

	Interest Rate Path Number						
Month	**1**	**2**	**3**	**…**	***n***	**…**	***N***
1	$r_1(1)$	$r_1(2)$	$r_1(3)$	…	$r_1(n)$	…	$r_1(N)$
2	$r_2(1)$	$r_2(2)$	$r_2(3)$	…	$r_2(n)$	…	$r_2(N)$
3	$r_3(1)$	$r_3(2)$	$r_3(3)$	…	$r_3(n)$	…	$r_3(N)$
t	$r_t(1)$	$r_t(2)$	$r_t(3)$	…	$r_t(n)$	…	$r_t(N)$
358	$r_{358}(1)$	$r_{358}(2)$	$r_{358}(3)$	…	$r_{358}(n)$	…	$r_{358}(N)$
359	$r_{359}(1)$	$r_{359}(2)$	$r_{359}(3)$	…	$r_{359}(n)$	…	$r_{359}(N)$
360	$r_{360}(1)$	$r_{360}(2)$	$r_{360}(3)$	…	$r_{360}(n)$	…	$r_{360}(N)$

Notation:

$r_t(n)$ = mortgage refinancing rate for month t on path n

N = number of interest rate paths

EXHIBIT 2.18 Simulated Cash Flow on Each of the Interest Rate Paths

	Interest Rate Path Number						
Month	**1**	**2**	**3**	**...**	***n***	**...**	***N***
1	$C_1(1)$	$C_1(2)$	$C_1(3)$	...	$C_1(n)$	...	$C_1(N)$
2	$C_2(1)$	$C_2(2)$	$C_2(3)$	...	$C_2(n)$	...	$C_2(N)$
3	$C_3(1)$	$C_3(2)$	$C_3(3)$	...	$C_3(n)$	...	$C_3(N)$
t	$C_t(1)$	$C_t(2)$	$C_t(3)$	...	$C_t(n)$	...	$C_t(N)$
358	$C_{358}(1)$	$C_{358}(2)$	$C_{358}(3)$	...	$C_{358}(n)$	...	$C_{358}(N)$
359	$C_{359}(1)$	$C_{359}(2)$	$C_{359}(3)$	...	$C_{359}(n)$	...	$C_{359}(N)$
360	$C_{360}(1)$	$C_{360}(2)$	$C_{360}(3)$	...	$C_{360}(n)$	...	$C_{360}(N)$

Notation:
$C_t(n)$ = cash flow for month t on path n
N = number of interest rate paths

Given the cash flow on an interest rate path, its present value can be calculated. The discount rate for determining the present value is the simulated spot rate for each month on the interest rate path plus an appropriate spread. The spot rate on a path can be determined from the simulated future monthly rates. The relationship that holds between the simulated spot rate for month T on path n and the simulated future 1-month rates is

$$z_T(n) = \{[1 + f_1(n)][1 + f_2(n)]\ldots[1 + f_T(n)]\}^{1/T} - 1$$

where

$z_T(n)$ = simulated spot rate for month T on path n
$f_j(n)$ = simulated future 1-month rate for month j on path n

Consequently, the interest rate path for the simulated future 1-month rates can be converted to the interest rate path for the simulated monthly spot rates as shown in Exhibit 2.19.

Therefore, the present value of the cash flow for month T on interest rate path n discounted at the simulated spot rate for month T plus some spread is

$$PV[C_T(n)] = \frac{C_T(n)}{[1 + z_T(n) + K]^T}$$

EXHIBIT 2.19 Simulated Paths of Monthly Spot Rates

	Interest Rate Path Number						
Month	**1**	**2**	**3**	**...**	***n***	**...**	***N***
1	$z_1(1)$	$z_1(2)$	$z_1(3)$	...	$z_1(n)$	...	$z_1(N)$
2	$z_2(1)$	$z_2(2)$	$z_2(3)$	...	$z_2(n)$	...	$z_2(N)$
3	$z_3(1)$	$z_3(2)$	$z_3(3)$	...	$z_3(n)$	...	$z_3(N)$
t	$z_t(1)$	$z_t(2)$	$z_t(3)$	...	$z_t(n)$	...	$z_t(N)$
358	$z_{358}(1)$	$z_{358}(2)$	$z_{358}(3)$	...	$z_{358}(n)$	...	$z_{358}(N)$
359	$z_{359}(1)$	$z_{359}(2)$	$z_{359}(3)$	...	$z_{359}(n)$	...	$z_{359}(N)$
360	$z_{360}(1)$	$z_{360}(2)$	$z_{360}(3)$	...	$z_{360}(n)$	...	$z_{360}(N)$

Notation:
$z_t(n)$ = spot rate for month t on path n
N = number of interest rate paths

where

$PV[C_T(n)]$ = present value of cash flow for month T on path n
$C_T(n)$ = cash flow for month T on path n
$z_T(n)$ = spot rate for month T on path n
K = spread

The present value for path n is the sum of the present value of the cash flow for each month on path n. That is,

$$PV[\text{Path}(n)] = PV[C_1(n)] + PV[C_2(n)] +\ldots + PV[C_{360}(n)]$$

where $PV[\text{Path}(n)]$ is the present value of interest rate path n.

Determining the Theoretical Value

The present value of a given interest rate path can be thought of as the theoretical value of a passthrough if that path was actually realized. The theoretical value of the passthrough can be determined by calculating the average of the theoretical value of all the interest rate paths. That is,

$$\text{Theoretical value} = \frac{PV[\text{Path}(1)] + PV[\text{Path}(2)] + \ldots + PV[\text{Path}(N)]}{N}$$

where N is the number of interest rate paths.

This procedure for valuing a passthrough is also followed for a CMO tranche. The cash flow for each month on each interest rate path is found according to the principal repayment and interest distribution rules of the deal. In order to do this, a CMO structuring model is needed. In any analysis of CMOs, one of the major stumbling blocks is getting a good CMO structuring model.

Option-Adjusted Spread

As explained earlier, the option-adjusted spread is a measure of the yield spread that can be used to convert dollar differences between theoretical value and market price. It represents a spread over the issuer's spot rate curve or benchmark.

In the Monte Carlo model, the OAS is the spread that, when added to all the spot rates on all interest rate paths, will make the average present value of the paths equal to the observed market price (plus accrued interest). Mathematically, OAS is the spread that will satisfy the following condition:

$$\text{Market price} = \frac{PV[\text{Path}(1)] + PV[\text{Path}(2)] + \ldots + PV[\text{Path}(N)]}{N}$$

where N is the number of interest rate paths.

Some Technical Issues

In the binomial method for valuing bonds, the interest rate tree is constructed so that it is arbitrage free. That is, if any on-the-run issue is valued, the value produced by the model is equal to the market price. This means that the tree is calibrated to the market. In contrast, in our discussion of the Monte Carlo method, there is no mechanism that we have described above that will assure the valuation model will produce a value for an on-the-run Treasury security (the benchmark in the case of agency mortgage-backed securities) equal to the market price. In practice, this is accomplished by adding a *drift term* to the short-term return generating process (Exhibit 2.16) so that the value produced by the Monte Carlo method for all on-the-run Treasury securities is their market price.[7] A technical explanation of this process is beyond the scope of this chapter.[8]

[7] This is equivalent to saying that the OAS produced by the model is zero.
[8] For an explanation of how this is done, see Lakhbir S. Hayre and Kenneth Lauterbach, "Stochastic Valuation of Debt Securities," in Frank J. Fabozzi (ed.), *Managing Institutional Assets* (New York: Harper & Row, 1990), pp. 321–364.

There is also another adjustment made to the interest rate paths. Restrictions on interest rate movements must be built into the model to prevent interest rates from reaching levels that are believed to be unreasonable (e.g., an interest rate of zero or an interest rate of 30%). This is done by incorporating *mean reversion* into the model. By this it is meant that at some point, the interest rate is forced toward some estimated average (mean) value.

The specification of the relationship between short-term rates and refinancing rates is necessary. Empirical evidence on the relationship is also necessary. More specifically, the correlation between the short-term and long-term rates must be estimated.

The number of interest rate paths determines how "good" the estimate is, not relative to the truth but relative to the valuation model used. The more paths, the more the theoretical value tends to settle down. It is a statistical sampling problem. Most Monte Carlo models employ some form of *variance reduction* to cut down on the number of sample paths necessary to get a good statistical sample. Variance reduction techniques allow us to obtain value estimates within a tick. By this we mean that if the model is used to generate more scenarios, value estimates from the model will not change by more than a tick. So, for example, if 1,024 paths are used to obtain the estimate value for a CMO tranche, there is little more information to be had from the OAS model by generating more than that number of paths. (For some very sensitive CMO tranches, more paths may be needed to estimate value within one tick.)

Distribution of Path Present Values

The Monte Carlo simulation method is a commonly used management science tool in business. It is employed when the outcome of a business decision depends on the outcome of several random variables. The product of the simulation is the average value and the probability distribution of the possible outcomes.

Unfortunately, the use of Monte Carlo simulation to value fixed-income securities has been limited to just the reporting of the average value, which is referred to as the theoretical value of the security. This means that all of the information about the distribution of the path present values is ignored. Yet, this information is quite valuable.

For example, consider a well-protected PAC bond. The distribution of the present value for the paths should be concentrated around the theoretical value. That is, the standard deviation should be small. In contrast, for a support tranche, the distribution of the present value for the paths could be wide, or equivalently, the standard deviation could be large.

Therefore, before using the theoretical value for a mortgage-backed security generated from the Monte Carlo method, a manager should ask for information about the distribution of the path's present values.

KEY POINTS

1. Valuation is the process of determining the fair value of a financial asset.

2. The fundamental principle of valuation is that the value of any financial asset is the present value of the expected cash flow, where the cash flow is the cash that is expected to be received each period from an investment.

3. For any bond in which neither the issuer nor the investor can alter the repayment of the principal before its contractual due date, the cash flow can easily be determined assuming that the issuer does not default.

4. The difficulty in determining the cash flow arises for bonds where either the issuer or the investor can alter the cash flow.

5. The base interest rate in valuing bonds is the rate on default-free securities and U.S. Treasury securities are viewed as default-free securities.

6. The traditional valuation methodology is to discount every cash flow of a bond by the same interest rate (discount rate), thereby incorrectly viewing each security as the same package of cash flows.

7. The proper approach values a bond as a package of cash flows, with each cash flow viewed as a zero-coupon instrument and each cash flow discounted at its own unique discount rate.

8. To properly value bonds, the rate on zero-coupon Treasury securities must be determined.

9. The Treasury yield curve indicates the relationship between the yield on Treasury securities and maturity. However, the securities included are a combination of zero-coupon instruments, that is, Treasury bills, and Treasury coupon securities.

10. Since the U.S. Treasury does not issue zero-coupon securities with a maturity greater than one year, a theoretical spot rate (i.e., zero-coupon rate) curve must be constructed from the yield curve.

11. One approach to constructing the spot rate curve is bootstrapping, the basic principle of which is that the value of the cash flow from an on-the-run Treasury issue when discounted at the spot rates should be equal to the observed market price.

12. From a Treasury spot rate curve, the value of any default-free security can be determined.

13. The economic force that assures that Treasury securities will be priced based on spot rates is the opportunity for government dealers to profitably strip Treasury securities or for investors to risklessly enhance portfolio returns.

14. To value a security with credit risk, it is necessary to determine a term structure of credit risk or equivalently a zero-coupon credit spread.

15. Evidence suggests that the credit spread increases with maturity and the lower the credit rating, the steeper the curve.

16. Adding the zero-coupon credit spread for a particular credit quality within a sector to the Treasury spot rate curve gives the benchmark spot rate curve that should be used to value a security.

17. The nominal spread is the difference between the yield of a non-Treasury and the yield on a comparable maturity benchmark Treasury security.

18. The zero-volatility spread is a measure of the spread that the investor would realize over the entire Treasury spot rate curve if the bond were held to maturity.

19. There are two valuation methodologies that are being used to value bonds with embedded options: the binomial method and the Monte Carlo simulation method.

20. The methodologies seek to determine the fair or theoretical value of the bond.

21. The option-adjusted spread (OAS) converts the cheapness or richness of a bond into a spread over the future possible spot rate curves.

22. The spread is option adjusted because it allows for future interest rate volatility to affect the cash flows.

23. The user of a valuation model is exposed to modeling risk and should test the sensitivity of the model to alternative assumptions.

24. The binomial method involves generating a binomial interest rate tree based on (1) an issuer's on-the-run yield curve, (2) an assumed interest rate generation process, and (3) an assumed interest rate volatility.

25. The uncertainty of interest rates is introduced into the model by introducing the volatility of interest rates.

26. In valuing a bond using the binomial interest rate tree, the cash flows at a node are modified to take into account any embedded options.

27. The option-adjusted spread is the constant spread that when added to the short rates in the binomial interest rate tree will produce a valuation for the bond equal to the market price of the bond.

28. The cash flow of mortgage-backed securities is path dependent and consequently the Monte Carlo method is commonly used to value these securities.

29. The Monte Carlo method involves randomly generating many scenarios of future interest rate paths based on some volatility assumption for interest rates.

30. The random paths of interest rates should be generated from an arbitrage-free model of the future term structure of interest rates.

31. The Monte Carlo method applied to mortgage-backed securities involves randomly generating a set of cash flows based on simulated future mortgage refinancing rates.

32. The theoretical value of a security, on any interest rate path, is the present value of the cash flow on that path where the spot rates are those on the corresponding interest rate path.

33. The theoretical value of a security is the average of the theoretical values over all the interest rate paths.

34. In the Monte Carlo method, the option-adjusted spread is the spread that, when added to all the spot rates on all interest rate paths, will make the average present value of the paths equal to the observed market price (plus accrued interest).

35. Information about the distribution of the present value for the interest rate paths provides guidance as to the degree of uncertainty associated with the theoretical value derived from the Monte Carlo method.

CHAPTER 3

Tools for Measuring Level Interest Rate Risk

A general principle of the valuation of financial assets is that the present value of an expected future cash flow changes in the opposite direction from changes in the interest rate used to discount the cash flow. This inverse relationship lies at the heart of a crucial risk faced by fixed-income investors—*interest rate risk*. Interest rate risk is the possibility that the value of a bond position or portfolio will decline in value as a result of an adverse movement in interest rates. For example, a long bond position's value will decline if interest rates rise, resulting in a loss. Conversely, for a short bond position, a loss will be realized if interest rates fall. To effectively control a portfolio's exposure to interest rate risk, it is necessary to quantify a portfolio's sensitivity to a change in interest rates. The purpose of this chapter and the next is to explain the important elements of interest rate risk for various types of fixed-income products and to illustrate the various methods used to measure it. In this chapter, we measure interest rate risk using tools that presuppose that the yield curve is flat and moves in parallel shifts. In the chapter that follows this one, we will relax this assumption and describe tools for measuring yield curve risk.

The objectives of this chapter are to:

1. Illustrate the price volatility properties of an option-free bond.
2. Provide a general formula that can be used to calculate the duration of any security.

3. Explain why the traditional duration measure, modified duration, is of limited value in determining the duration of a security with an embedded option.
4. Distinguish between modified duration, effective duration, and dollar duration.
5. Explain what is meant by negative convexity for a callable bond, a mortgage passthrough security, and asset-backed securities backed by residential mortgages.
6. Explain what the convexity measure of a bond is and the distinction between modified convexity and effective convexity.
7. Describe the relationship between Macaulay duration and modified duration.
8. Explain how the duration of a floater and inverse floater are determined.
9. Explain market-based approaches for estimating duration of a mortgage-backed security.
10. Explain price value of a basis point and yield value of a price change.
11. Explain how to control interest rate risk in active bond portfolio strategies.

PRICE VOLATILITY CHARACTERISTICS OF BONDS

There are four characteristics of a bond that affect its price volatility: (1) term to maturity, (2) coupon rate, (3) the level of yields, and (4) the presence of embedded options. In this section, we will examine each of these price volatility characteristics.

The Price/Yield Relationship

Exhibit 3.1 depicts the inverse relationship between an option-free bond's price (located on the vertical axis) and its discount rate or required yield (located on the horizontal axis).[1] There are two important ideas to be gleaned from the price/yield relationship depicted in the exhibit. First, the relationship is downward sloping. This is nothing more than the inverse relationship between present values and discount rates at work. Second, the relationship is represented as a curve rather than a straight line. In fact, the curve's shape in Exhibit 3.1 is referred to as *convex*. By convex, it simply means the curve is "bowed in" relative to the origin.[2] This second observation raises two questions about the convex or curved shape of the price/yield relationship. First, why is it curved? Second, what is the importance of the curvature?

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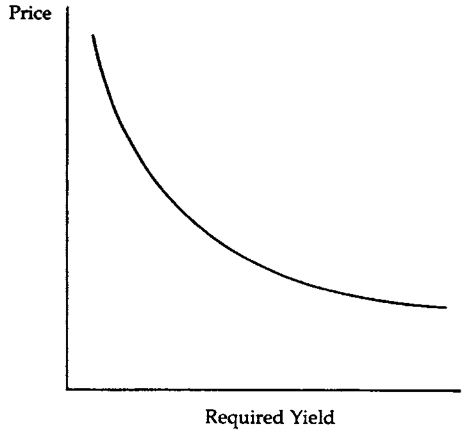
[1] In our discussion, we will use the terms "required yield," "discount rate," and "interest rate" interchangeably.
[2] Mathematically, if a relationship is convex, it implies the following: If we pick any pair of points on the curve and join them with a straight line, the line segment will lie above the curve.

EXHIBIT 3.1 Price/Yield Relationship for a Hypothetical Option-Free Bond



The answer to the first question is mathematical and becomes apparent by examining the denominator of the bond pricing formula presented in Chapter 2. Since we are raising one plus the periodic required yield to powers greater than one, it should not be surprising that the relationship between the level of the bond's price and the level of the required yield is a curve rather than a straight line.

As for the importance of the curvature to bond investors, let's consider what happens to bond prices when the required yield rises and falls. First, what happens to bond prices as the required yield falls? Bond prices rise. How about the rate at which bond prices rise as the required yield falls? If the price/yield relationship were linear, as the required yield fell, bond prices would rise at a constant rate. However, the relationship is not linear; it is curved and curved inward. See Exhibit 3.1. Accordingly, when required yields fall, bond prices increase at an increasing rate. If one has a long position in the bond, this is a benefit. Now, let's consider what happens when required yields rise. Bond prices fall. How about the rate at which bond prices fall as the required yield rises? Once again, if the price/yield relationship were linear, as required yields rose, bond price would fall as a constant rate. Since it curved inward, when required yields rise, bond prices decrease at a decreasing rate. If one has a long position in the bond, this is an appealing feature.

Recall that the slope of a curve at a particular point is equal to the slope of a straight line that just touches the curve at that point (i.e., a tangent line). What happens to the slope of the tangent line to the price/yield relationship as we move from higher required yields to lower required yields? The slope of the tangent line gets progressively steeper. This result is exactly what we have seen, namely, that bond prices increase at an increasing rate when required yields fall. Conversely, what happens to the slope of the tangent line as we move from lower to higher required yields? The slope of the tangent line gets progressively flatter. Bond prices decrease at a decreasing rate when required yields rise.

Price Volatility Characteristics of Option-Free Bonds

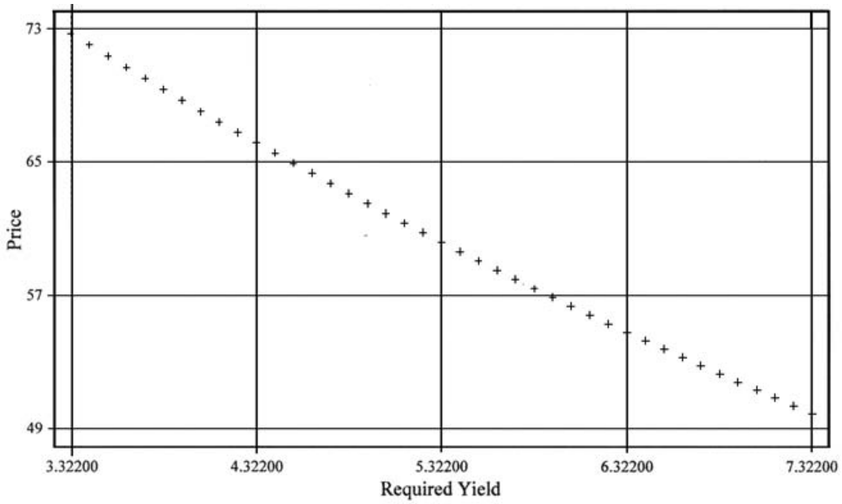
Let's begin by focusing on option-free bonds (i.e., bonds that do not have embedded options). A fundamental characteristic of an option-free bond is that the price of the bond changes in the opposite direction from a change in the bond's required yield. Exhibit 3.2 illustrates this property for four hypothetical bonds assuming a par value of $100.

When the price/yield relationship for any hypothetical option-free bond is graphed, it exhibits the basic shape depicted in Exhibit 3.1. The price/yield is for an instantaneous change in the required yield. Exhibit 3.3 shows the price/yield relationship for a U.S. Treasury principal strip that matures February 15, 2012. Using a settlement date of June 14, 2002, the yield is 5.322%. To construct the graph, the principal strip was repriced using increments and decrements of 10 basis points from 7.322% to 3.322%. Exhibit 3.4 shows the two price/yield relationships for a 3.25% coupon, 2-year Treasury note that matures on May 31, 2004 and a 4.875% coupon, 10-year note that matures on February 15, 2012. Both notes are priced with a settlement date of June 14, 2002. Note the 10-year Treasury note's price/yield relationship is more steeply sloped and more curved than the price/yield relationship for the 2-year Treasury note. The reasons for these differences will be discussed shortly.

EXHIBIT 3.2 **Price/Yield Relationship for Four Hypothetical Option-Free Bonds**

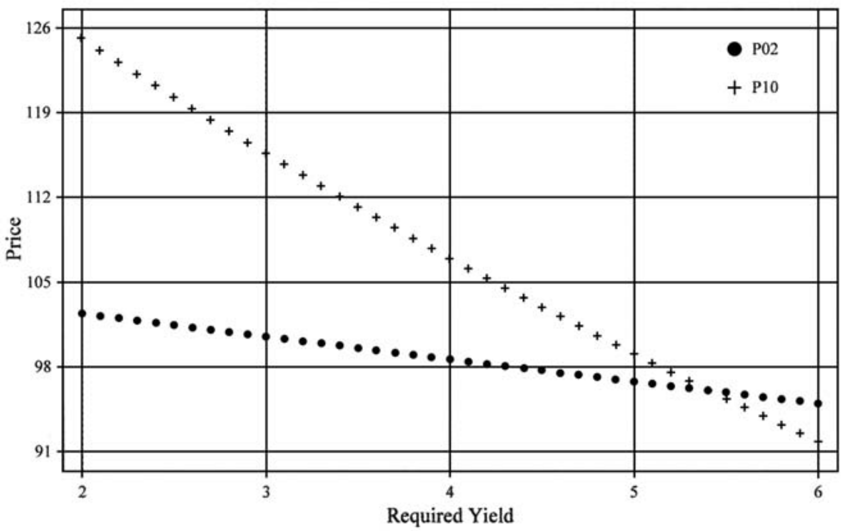
	Price ($)			
Yield (%)	7%, 10-year	7%, 30-year	9%, 10-year	9%, 30-year
5.00	115.5892	130.9087	131.1783	161.8173
6.00	107.4387	113.8378	122.3162	141.5133
6.50	103.6348	106.5634	118.1742	132.8171
6.90	100.7138	101.2599	114.9908	126.4579
6.99	100.0711	100.1248	114.2899	125.0947
7.00	100.0000	100.0000	114.2124	124.9447
7.01	99.9290	99.8754	114.1349	124.7950
7.10	99.2926	98.7652	113.4409	123.4608
7.50	96.5259	94.0655	110.4222	117.8034
8.00	93.2048	88.6883	106.7952	111.3117
9.00	86.9921	79.3620	100.0000	100.0000

EXHIBIT 3.3 Price/Yield Relationship for a 10-Year Treasury Principal Strip



Note: Priced with settlement date of 6/14/02.

EXHIBIT 3.4 Price/Yield Relationship for a 3.25% 2-Year Treasury Note and a 4.875% 10-Year Treasury Note



Note: Both notes priced with a settlement date of 6/14/02.

EXHIBIT 3.5 **Instantaneous Percentage Price Change for Four Hypothetical Bonds (Initial Yield for all Four Bonds is 7%)**

	Price ($)			
Yield (%)	7%, 10-year	7%, 30-year	9%, 10-year	9%, 30-year
5.00	15.5892	30.9087	14.8547	29.5111
6.00	7.4387	13.8378	7.0954	13.2607
6.50	3.6368	6.5634	3.4688	6.3007
6.90	0.7138	1.2599	0.6815	1.2111
6.99	0.0711	0.1248	0.0679	0.1201
7.00	0.0000	0.0000	0.0000	0.0000
7.01	–0.0710	–0.1246	–0.0679	–0.1200
7.10	–0.0707	–1.2350	–0.6750	–1.1880
7.50	–3.4740	–5.9350	–3.3190	–5.7160
8.00	–6.7950	–11.3120	–6.4940	–10.9110
9.00	–13.0080	–20.6380	–12.4440	–19.9650

The price sensitivity of a bond to changes in the required yield can be measured in terms of the dollar price change or the percentage price change. Exhibit 3.5 uses the four hypothetical bonds in Exhibit 3.2 to show the percentage change in each bond's price for various changes in yield, assuming that the initial yield for all four bonds is 7%. An examination of Exhibit 3.5 reveals the following properties concerning the price volatility of an option-free bond:

Property 1: Although the price moves in the opposite direction from the change in required yield, the percentage price change is not the same for all bonds.

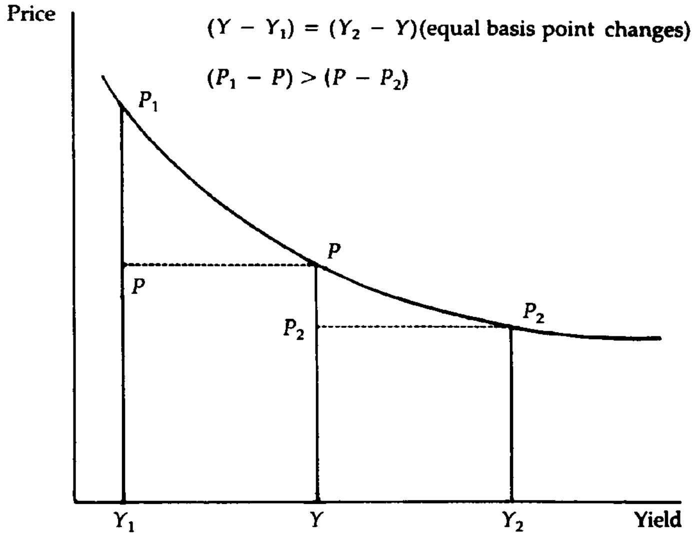
Property 2: For small changes in the required yield, the percentage price change for a given bond is roughly the same, whether the required yield increases or decreases.

Property 3: For large changes in required yield, the percentage price change is not the same for an increase in required yield as it is for a decrease in required yield.

Property 4: For a given large change in basis points in the required yield, the percentage price increase is greater than the percentage price decrease.

While the properties are expressed in terms of percentage price change, they also hold for dollar price changes.

EXHIBIT 3.6 Graphical Illustration of Properties 3 and 4 for an Option-Free Bond



An explanation for these two properties of bond price volatility lies in the convex shape of the price/yield relationship. Exhibit 3.6 illustrates this. The following notation is used in the exhibit

Y = initial yield
Y_1 = lower yield
Y_2 = higher yield
P = initial price
P_1 = price at lower yield Y_1
P_2 = price at higher yield Y_2

What was done in the exhibit was to change the initial yield (Y) up and down by the same number of basis points. That is, in Exhibit 3.6, the yield is decreased from Y to Y_1 and increased from Y to Y_2 such that the magnitude of the change is the same:

$$Y - Y_1 = Y_2 - Y$$

Also, the amount of the change in yield is a large number of basis points.

The vertical distance from the horizontal axis (the yield) to the intercept on the graph shows the price. The change in the initial price (P) when the yield declines from Y to Y_1 is equal to the difference between the new price (P_1) and the initial price. That is,

$$\text{Change in price when yield decreases} = P_1 - P$$

The change in the initial price (P) when the yield increases from Y to Y_2 is equal to the difference between the new price (P_2) and the initial price. That is,

$$\text{Change in price when yield increases} = P - P_2$$

As can be seen in the exhibit, the change in price when yield decreases is not equal to the change in price when yield increases by the same number of basis points. That is,

$$P_1 - P \neq P - P_2$$

This is what Property 3 states. Moreover, a comparison of the price change shows that the change in price when yield decreases is greater than the change in price when yield increases. That is,

$$P_1 - P > P - P_2$$

This is Property 4.

The implication of Property 4 is that if an investor is long a bond, the price appreciation that will be realized, if the required yield decreases, is greater than the capital loss that will be realized if the required yield increases by the same number of basis points. For an investor who is short a bond, the reverse is true: The potential capital loss is greater than the potential capital gain if the yield changes by a given number of basis points.

To see how the convexity of the price/yield relationship impacts Property 4, look at Exhibits 3.7 and 3.8. Exhibit 3.7 shows a less convex price/yield relationship than Exhibit 3.6. That is, the price/yield relationship in Exhibit 3.7 is less bowed than the price/yield relationship in Exhibit 3.6. Because of the difference in the convexities, look at what happens when the yield increases and decreases by the same number of basis points and the yield change is a large number of basis points. We use the same notation in Exhibits 3.7 and 3.8 as in Exhibit 3.6. Notice that while the price gain, when the required yield decreases, is greater than the price decline, when the required yield increases, the gain is not much greater than the loss. In contrast, Exhibit 3.8 has much greater convexity than the bonds in Exhibits 3.6 and 3.7 and the price gain is significantly greater than the loss for the bonds depicted in Exhibits 3.6 and 3.7.

EXHIBIT 3.7 Impact of Convexity on Property 4: Less Convex Bond

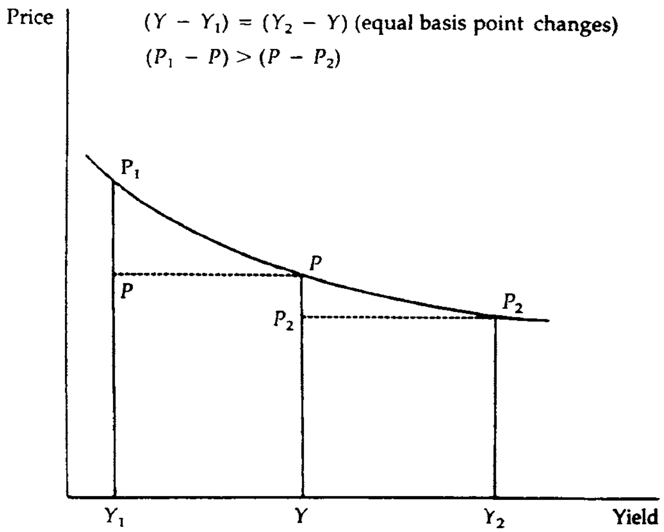
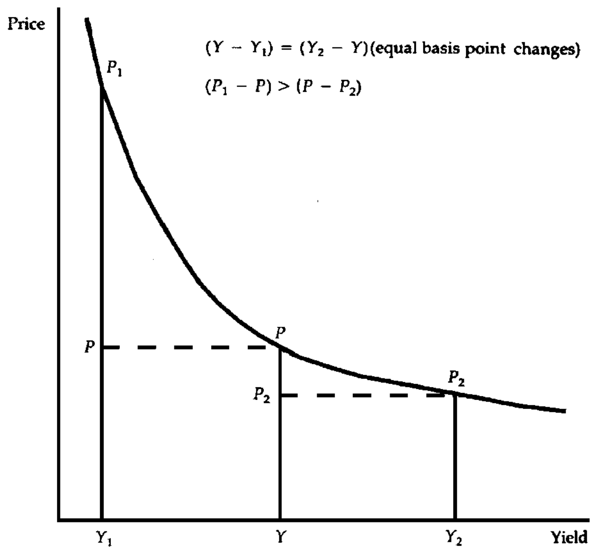


EXHIBIT 3.8 Impact of Convexity on Property 4: Highly Convex Bond



Price Volatility Characteristics of Bonds with Embedded Options

Now let's turn to the price volatility characteristics of bonds with embedded options. As explained in previous chapters, the price of a bond with an embedded option is comprised of two components. The first is the value of the same bond if it had no embedded option. That is, the price if the bond is option free. The second component is the value of the embedded option.

The two most common types of embedded options are *call* (or prepay) *options* and *put options*. As interest rates in the market decline, the issuer may call or prepay the debt obligation prior to the scheduled principal repayment date. The other type of option is a put option. This option gives the investor the right to require the issuer to purchase the bond at a specified price. Below we will examine the price/yield relationship for bonds with both types of embedded options (calls and puts) and implications for price volatility.

Bonds with Call and Prepay Options

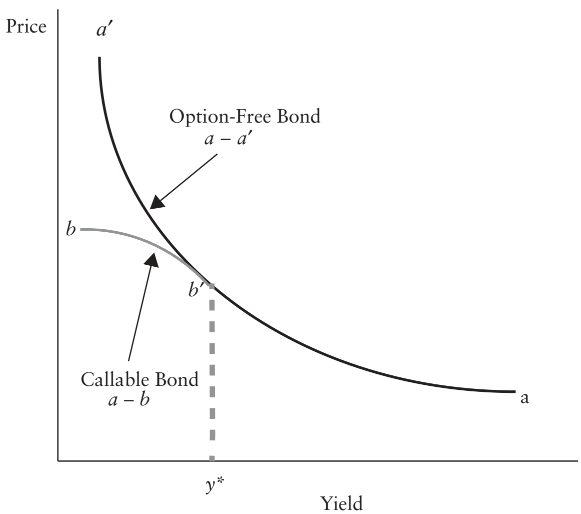
In the discussion below, we will refer to a bond that may be called or is prepayable as a callable bond. Exhibit 3.9 shows the price/yield relationship for an option-free bond and a callable bond. The convex curve given by *a-a´* is the price/yield relationship for an option-free bond. The unusual shaped curve denoted by *a-b* in the exhibit is the price/yield relationship for the callable bond.

The reason for the price/yield relationship for a callable bond is as follows. When the prevailing market yield for comparable bonds is higher than the coupon rate on the callable bond, it is unlikely that the issuer will call the issue. For example, if the coupon rate on a bond is 7% and the prevailing market yield on comparable bonds is 12%, it is highly unlikely that the issuer will call a 7% coupon bond so that it can issue a 12% coupon bond. Since the bond is unlikely to be called, the callable bond will have a similar price/yield relationship as an otherwise comparable option-free bond. Consequently, the callable bond is going to be valued as if it is an option-free bond. However, since there is still some value to the call option, the bond won't trade exactly like an option-free bond.

As yields in the market decline, the concern is that the issuer will call the bond. The issuer won't necessarily exercise the call option as soon as the market yield drops below the coupon rate. Yet, the value of the embedded call option increases as yields approach the coupon rate from higher yield levels. For example, if the coupon rate on a bond is 7% and the market yield declines to 7.5%, the issuer will most likely not call the issue. However, market yields are at a level at which the investor is concerned that the issue may eventually be called if market yields decline further. Cast in terms of the value of the embedded call option, that option becomes more valuable to the issuer and therefore it reduces the price relative to an otherwise comparable option-free bond.[3] In Exhibit 3.9, the value of the embedded call option at a given yield can be measured by the difference between the price of an option-free bond (the price shown on the curve *a-a′*) and the price on the curve *a-b*. Notice that at low yield levels (below y^* on the horizontal axis), the value of the embedded call option is high.

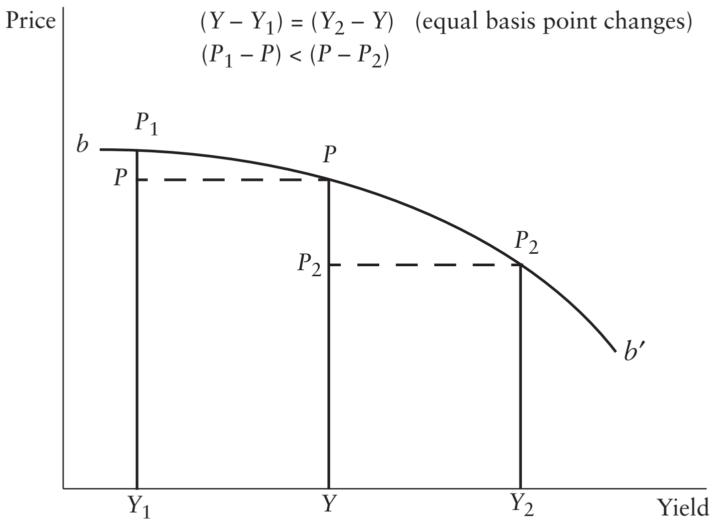
Let's look at the difference in the price volatility properties relative to an option-free bond given the price/yield relationship for a callable bond shown in Exhibit 3.9. Exhibit 3.10 blows up the portion of the price/yield relationship for the callable bond where the two curves in Exhibit 3.9 depart (segment *b-b′* in Exhibit 3.9). We know from our discussion of the price/yield relationship that for a large change in yield of a given number of basis points, the price of an option-free bond increases by more than it decreases (Property 4 above). Is that what happens for a callable bond in the region of the price/yield relationship shown in Exhibit 3.10? No, it is not. In fact, as can be seen in the exhibit, the opposite is true! That is, for a given large change in yield, the price appreciation is less than the price decline.

EXHIBIT 3.9 **Price/Yield Relationship for a Callable Bond and an Option-Free Bond**



[3] For readers who are already familiar with option theory, this characteristic can be restated as follows: When the coupon rate for the issue is below the market yield, the embedded call option is said to be "out-of-the-money." When the coupon rate for the issue is above the market yield, the embedded call option is said to be "in-the-money."

EXHIBIT 3.10 **Negative Convexity Region of the Price/Yield Relationship for a Callable Bond**

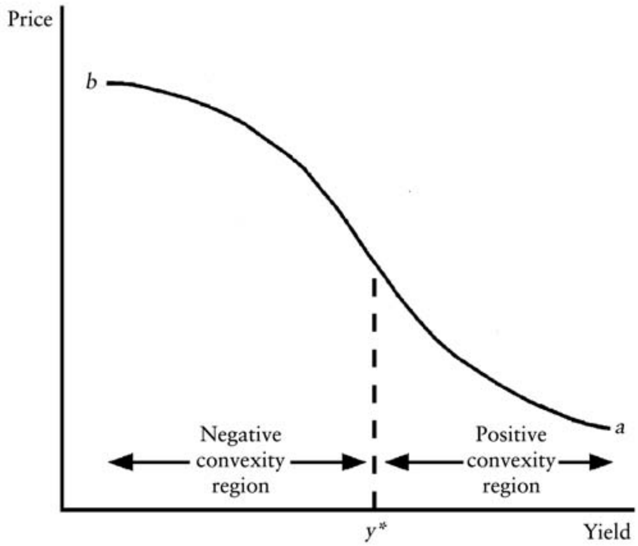


The price volatility characteristic of a callable bond is important to understand. The characteristic of a callable bond—that its price appreciation is less than its price decline when rates change by a large number of basis points—is referred to as *negative convexity*.[4] But notice from Exhibit 3.9 that callable bonds do not exhibit this characteristic at every yield level. When yields are high (relative to the issue's coupon rate), the bond exhibits the same price/yield relationship as an option-free bond and therefore at high yield levels it also has the characteristic that the gain is greater than the loss. Because market participants have referred to the shape of the price/yield relationship shown in Exhibit 3.10 as negative convexity, market participants refer to the relationship for an option-free bond as *positive convexity*. Consequently, a callable bond exhibits negative convexity at low yield levels and positive convexity at high yield levels. This is depicted in Exhibit 3.11.

As can be seen from the exhibits, when a bond exhibits negative convexity, the bond compresses in price as rates decline. That is, at a certain yield level there is very little price appreciation when rates decline. When a bond enters this region, the bond is said to exhibit "price compression."

[4] Mathematicians refer to this shape as being "concave."

EXHIBIT 3.11 Negative and Positive Convexity Exhibited by a Callable Bond



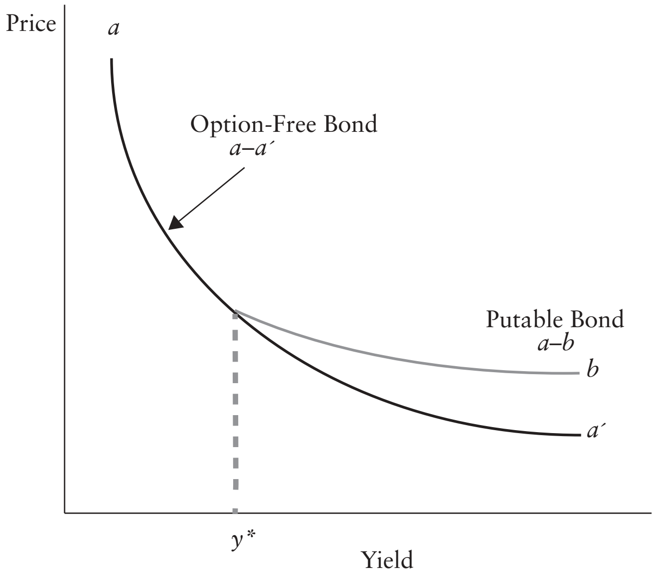
Bonds with Embedded Put Options

Putable bonds may be redeemed by the bondholder on the dates and at the put price specified in the indenture. Typically, the put price is par value. The advantage to the investor is that if yields rise such that the bond's value falls below the put price, the investor will exercise the put option. If the put price is par value, this means that if market yields rise above the coupon rate, the bond's value will fall below par and the investor will then exercise the put option.

The value of a putable bond is equal to the value of an option-free bond plus the value of the put option. Thus, the difference between the value of a putable bond and the value of an otherwise comparable option-free bond is the value of the embedded put option. This can be seen in Exhibit 3.12 which shows the price/yield relationship for a putable bond (the curve *a–b*) and an option-free bond (the curve *a–a´*).

At low yield levels (low relative to the issue's coupon rate), the price of the putable bond is basically the same as the price of the option-free bond because the value of the put option is small. As rates rise, the price of the putable bond declines, but the price decline is less than that for an option-free bond. The divergence in the price of the putable bond and an otherwise comparable option-free bond at a given yield level is the value of the put option. When yields rise to a level where the bond's price would fall below the put price, the price at these levels is the put price.

EXHIBIT 3.12 **Price/Yield Relationship for a Putable Bond and an Option-Free Bond**



DURATION

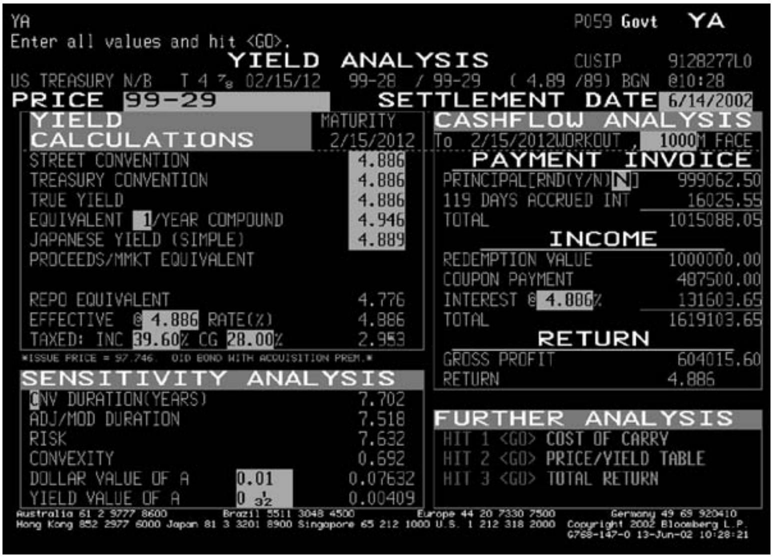
Given the background about a bond's price volatility characteristics, we can now turn our attention to an alternate approach to full valuation discussed in Chapter 1: the duration/convexity approach. Simply put, *duration* is a measure of the approximate sensitivity of a bond's value to rate changes. More specifically, *duration is the approximate percentage change in value for a 100-basis-point change in rates*. We will see in this section that duration is the first approximation (i.e., linear) of the percentage price change. To improve the estimate obtained using duration, a measure called "convexity" can be used. Hence, using duration and convexity together to estimate a bond's percentage price change resulting from interest rate changes is called the *duration/convexity approach*.

Calculating Duration

The duration of a bond is estimated as follows:

$$\frac{\text{Price if yields decline} - \text{Price if yields rise}}{2(\text{Initial price})(\text{Change in yield in decimal})}$$

EXHIBIT 3.13 Bloomberg Yield Analysis for 4.875% 10-Year Treasury Note



Source: Bloomberg Finanical Markets

If we let

Δy = change in yield in decimal
V_0 = initial price
V_- = price if yields decline by Δy
V_+ = price if yields increase by Δy

then duration can be expressed as

$$\text{Duration} = \frac{V_- - V_+}{2(V_0)(\Delta y)} \tag{3.1}$$

For example, consider the 4.875% coupon, 10-year note discussed earlier that matures on February 15, 2012 and on a settlement date of June 14, 2002 is priced to yield 4.886% with a full price of 101.5119 since it is between coupon payment dates. Exhibit 3.13 presents Bloomberg's Yield Analysis (YA) screen for this security. Let's change (i.e., shock) the note's required yield up and down by 20 basis points and determine what the new prices will be in the numerator of equation (3.1). If the required yield were decreased by 20 basis points from 4.886% to 4.686%, the note's full price would increase to 103.0525. Conversely, if the yield increases by 20 basis points, the full price would decrease to 99.9995. Thus,

$$\Delta y = 0.002$$
$$V_0 = 101.5119$$
$$V_- = 103.0525$$
$$V_+ = 99.9995$$

Then

$$\text{Duration} = \frac{103.0525 - 99.9995}{2 \times (101.5119) \times (0.002)} = 7.519$$

Note that our calculation for duration of 7.519 agrees (within rounding error) with Bloomberg's calculation in Exhibit 3.13. Bloomberg's interest rate risk measures are located in a box titled "Sensitivity Analysis" in the lower left-hand corner of the screen. The duration measure we just calculated is labeled "Adj/Mod Duration" which stands for adjusted/modified duration. We'll discuss this further later in this chapter.

Duration is interpreted as the approximate percentage change in price for a 100-basis-point change in the required yield. Consequently, a duration of 7.519 means that the approximate percentage change in the bond's price will be 7.519% for a 100-basis-point change in the required yield. Moreover, since duration is a linear approximation, the approximate percentage price change for a 50-basis-point change in required yield is one-half the duration or in the case 3.7595%. This result generalizes.

It is paramount to keep in mind when interpreting duration that the change in yield referred to above is the same change in yield for all maturities. This assumption is commonly referred to as the parallel yield curve shift assumption. Thus, the foregoing discussion about the price sensitivity of a security to interest rate changes is limited to parallel shifts in the yield curve. In the next chapter, we will address the case where the yield curve shifts in a nonparallel manner.

A common question often raised at this juncture is the consistency between the yield change that is used to compute duration (Δy) using equation (3.1) and the interpretation of duration. For example, recall that in computing the duration of the 10-year Treasury note, we used a 20-basis-point yield change to obtain the two prices used in the numerator in equation (3.1). Yet, we interpret the duration measure computed using equation (3.1) as the approximate percentage price change for a 100-basis-point change in yield. The reason is that regardless of the yield change used to estimate duration in equation (3.1), the interpretation is unchanged. If we used a 30-basis-point change in yield to compute the prices used in the numerator of equation, the resulting duration measure is interpreted as the approximate percentage price change for a 100-basis-point change in yield. Simply put, the choice of Δy in equation (3.1) is arbitrary. Shortly, we will use different changes in yield to illustrate the sensitivity (or lack thereof) of the computed duration using equation (3.1).

Approximating the Percentage Price Change Using Duration

In order to approximate the percentage price change for a given change in yield and a given duration, we employ the following formula:

$$\text{Approximate percentage price change} = -\text{Duration} \times \Delta y \times 100 \qquad (3.2)$$

The reason for the negative sign on the right-hand side of equation (3.2) is due to the inverse relationship between price change and yield change.

For example, consider the 4.875% coupon, 10-year U.S. Treasury note trading at a full price of 101.5119 whose duration we just computed is 7.519. The approximate percentage price change for a 10-basis-point increase in the required yield (i.e., $\Delta y = +0.001$) is

$$\text{Approximate percentage price change} = -7.519 \times (+0.001) \times 100 = -0.7519$$

How good is this approximation? The actual percentage price change is –0.7484 (= (100.7522 – 101.5119)/101.5119). Duration, in this instance, did an accurate job of estimating the percentage price change. We would reach the same conclusion if we used duration to estimate the percentage price change if the yield declined by 10 basis points (i.e., $\Delta y = -0.001$). In this case, the approximate percentage price change would be +0.7519 (i.e., the direction of the estimated price change is the reverse but the magnitude of the change is the same because it is a linear approximation.)

In terms of estimating the new price, let's see how duration performs. The initial full price is 101.5119. For a 10-basis-point increase in yield, duration estimates that the price will decline by –0.7519%. Thus, the full price will decline to 100.7486 (found by multiplying 101.5119 by one minus 0.007519). The actual full price if the yield increases by 10 basis points is 100.7522. Thus, the price estimate using duration is very close to the actual price. For a 10-basis-point decrease in yield, the actual full price is 102.2787 and the estimated price using duration is 102.2752 (a price increase of 0.7519%).

Now let us examine how well duration does in estimating the percentage price change when the yield increases by 200 basis points rather than a 10 basis points. In this case, Δy is equal to +0.02. Substituting into equation (3.2) we have

$$\begin{aligned}\text{Approximate percentage price change} &= -7.519 \times (+0.02) \times 100 \\ &= -15.038\%\end{aligned}$$

EXHIBIT 3.14 Application of Duration to Approximate the Percentage Price Change

		New Price		Percent Price Change		
Yield Change (bp)	Initial Price	Based on Duration	Actual	Based on Duration	Actual	Comment
+10	101.5119	100.7486	100.7522	–0.7519	–0.7484	Estimated price close to new price.
–10	101.5119	102.2787	102.2787	+0.7519	+0.7554	Estimated price close to new price.
+200	101.5119	86.2470	87.5627	–15.038	–13.740	Underestimates new price.
–200	101.5119	116.7772	118.2794	+15.038	+16.520	Underestimates new price.

How accurate is this estimate? The actual percentage price change when the yield increases by 200 basis points (4.886% to 6.886%) is –13.74%. Thus, the estimate is considerably less accurate than when we used duration to approximate the percentage price change for a change in yield of only 10 basis points. If we use duration to approximate the percentage price change when the yield decreases by 200 basis points, the approximate percentage price change in this scenario is +15.038% (remember only the sign changes). The actual percentage price change is +16.52%.

As before, let's examine the use of duration in terms of estimating the new price. Since the initial full price is 101.5119 and a 200-basis-point increase in yield will decrease the price by –13.74%, the estimated new price using duration is 86.247 (found by multiplying 101.5119 by one minus 0.15038). The actual full price if the yield rises by 200 basis points (4.886% to 6.886%) is 87.5627. Consequently, the estimate is not as accurate as the estimate for a 10-basis-point change in yield. The estimated new price using duration for a 200-basis-point decrease in yield (4.886% to 2.886%) is 116.7772 compared to the actual price of 118.2794. Once again, the estimation of the price using duration is not as accurate as for a 10-basis-point change. Notice that whether the yield is increased or decreased by 200 basis points, duration underestimates what the new price will be. We will discover why shortly. Exhibit 3.14 summarizes what we found in our application to approximate the 10-year U.S. Treasury note's percentage price change.

This result should come as no surprise to careful readers of the last section on price volatility characteristics of bonds. Specifically equation (3.2) is somewhat at odds with the properties of the price/yield relationship. We are using a linear approximation for a price/yield relationship that is convex.

Graphical Depiction of Using Duration to Estimate Price Changes

Earlier we used the graph of the price/yield relationship to demonstrate the price volatility properties of bonds. We can use graphs to illustrate what we observed in our examples about how duration estimates the percentage price change, as well as some other noteworthy points.

The shape of the price/yield relationship for an option-free bond is convex. Exhibit 3.15 shows this relationship. In the exhibit a tangent line is drawn to the price/yield relationship at yield y^*. (For those unfamiliar with the concept of a tangent line, it is a straight line that just touches a curve at one point within a relevant (local) range. In Exhibit 3.15, the tangent line touches the curve at the point where the yield is equal to y^* and the price is equal to p^*.) The tangent line is used to *estimate* the new price if the yield changes. If we draw a vertical line from any yield (on the horizontal axis), as in Exhibit 3.15, the distance between the horizontal axis and the tangent line represents the price approximated by using duration starting with the initial yield y^*.

EXHIBIT 3.15 Price/Yield Relationship for an Option-Free Bond with a Tangent Line

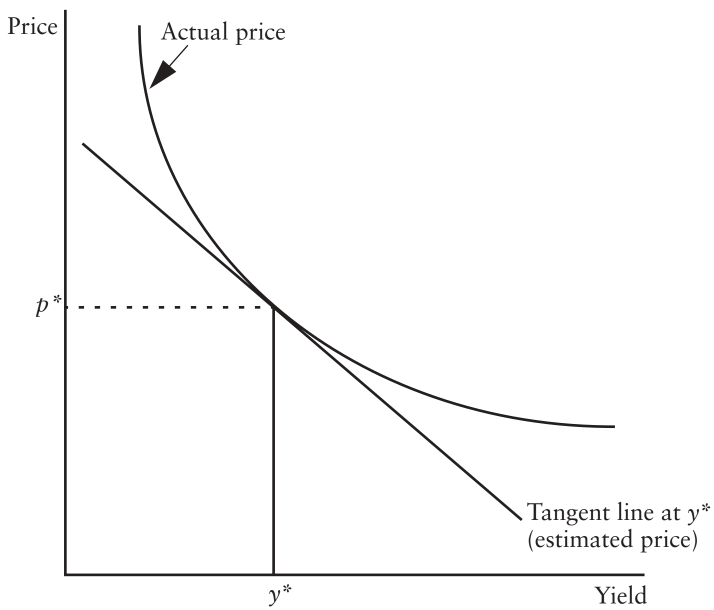
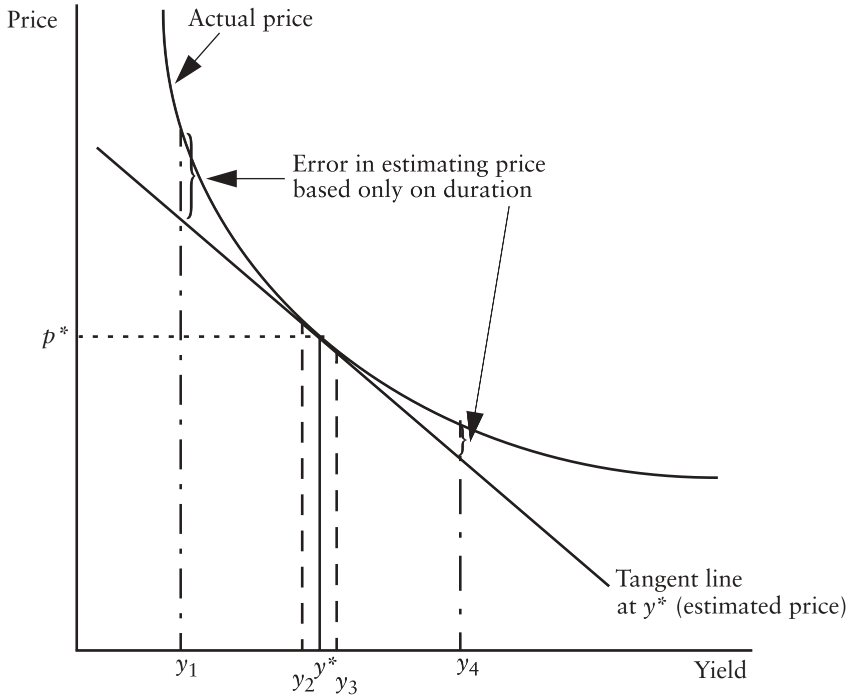


EXHIBIT 3.16 Estimating the New Price Using a Tangent Line



Now how is the tangent line, used to approximate what the new price will be if yields change, related to duration? The tangent line tells us the approximate new price of a bond if the yield changes. Given (1) the initial price and (2) the new price of a bond if the yield changes using the tangent line, the approximate percentage price change can be computed for a given change in yield. But this is precisely what duration, using equation (3.2), gives us: the approximate percentage change for a given change in yield. Thus, using the tangent line one obtains the same approximate percentage price change as using equation (3.2).

This helps us understand why duration did an effective job of estimating the percentage price change, or equivalently the new price, when the yield changes by a small number of basis points. Look at Exhibit 3.16. Notice that for a small change in yield, the tangent line does not depart much from the price/yield relationship. Hence, when the yield changes up or down by 10 basis points, the tangent line does a good job of estimating the new price, as we found in our earlier numerical illustration.

EXHIBIT 3.17 **Estimating the New Price for a Large Yield Change for Bonds with Different Convexities**

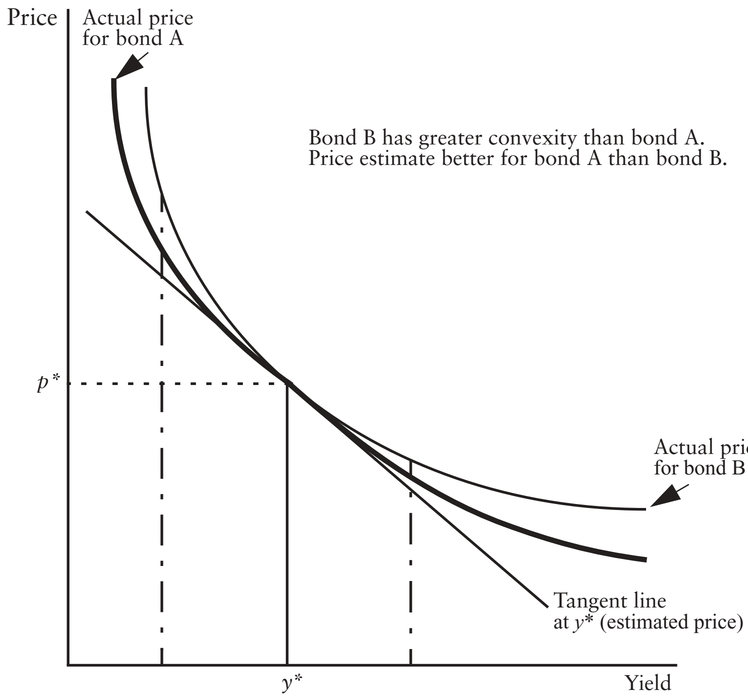


Exhibit 3.16 also shows what happens to the estimate using the tangent line when the yield changes by a large number of basis points. Notice that the error in the estimate gets larger the further one moves from the initial yield. The estimate is less accurate the more convex the bond. This is illustrated in Exhibit 3.17.

Also note that regardless of the magnitude of the yield change, the tangent line always underestimates what the new price will be for an option-free bond because the tangent line is below the price/yield relationship. This explains why we found in our illustration that when using duration we underestimated what the actual price will be.

Rate Shocks and Duration Estimate

In calculating duration using equation (3.1), it is necessary to shock interest rates (yields) up and down by the same number of basis points to obtain the values for V_- and V_+. In our illustration, 20 basis points was arbitrarily selected. But how large should the shock be? That is, how many basis points should be used to shock the rate? Looking at equation (3.1) it is relatively easy to discern why the size of the interest rate shock should not matter too much. Specifically, the choice of Δy has two effects on equation (3.1). In the numerator, the choice of Δy affects the spread between V_- and V_+ in that the larger the interest rate shock, the larger the spread between the two prices. In the denominator, the choice of Δy appears directly and the denominator is larger for larger values of Δy. The two effects should largely neutralize each other, unless the price/yield relationship is highly convex (i.e., curved).

In Exhibit 3.18, the duration estimate for our three U.S. Treasury securities from Exhibits 3.3 and 3.4 using equation (3.1) for rate shocks of 1 basis point to 100 basis points is reported. The duration estimates for the 2-year note are unaffected by the size of the shock. The duration estimates for the 10-year note are affected only slightly even though a 10-year note will have higher positive convexity (i.e., a price/yield relationship that is more curved) than a 2-year note. Lastly, if the duration estimates are ever going to be affected by the size of the interest rate shock, this should be evident when this exercise is performed on a 10-year principal strip, which has large positive convexity relative to the other two securities (i.e., a price/yield relationship that is very curved). However, even in this case, the duration estimates are affected only marginally. It would appear that the size of the interest rate shock is unimportant for approximating the duration of option-free bonds using equation (3.1).[5]

EXHIBIT 3.18 Duration Estimates for Different Rate Shocks
Assumptions: All of these bonds are priced with a settlement date of 6/14/02. The initial yields for the 2-year note, 10-year note and 10-year principal strip are 2.969%, 4.886%, and 5.322% respectively.

Bond	1 bp	10 bps	20 bps	50 bps	100 bps
2-year, 3.25% coupon U.S. Treasury note maturing 5/31/04	1.886	1.886	1.886	1.886	1.887
10-year, 4.875% coupon U.S. Treasury note maturing 2/15/12	7.518	7.519	7.519	7.521	7.530
10-year U.S. Treasury principal strip maturing 2/15/12	9.421	9.421	9.421	9.425	9.439

[5] When calculating modified duration, Bloomberg uses an interest rate shock of 100 basis points to calculate V_- and V_+.

When we deal with more complicated securities, small rate shocks that do not reflect the types of rate changes that may occur in the market do not permit the determination of how prices can change because expected cash flows may change when dealing with bonds with embedded options. In comparison, if large rate shocks are used, we encounter the asymmetry caused by convexity. Moreover, large rate shocks may cause dramatic changes in the expected cash flows for bonds with embedded options that may be far different from how the expected cash flows will change for smaller rate shocks.

There is another potential problem with using small rate shocks for complicated securities. The prices that are inserted into the duration formula as given by equation (3.1) are derived from a valuation model. The duration measure depends crucially on a valuation model. If the rate shock is small and the valuation model used to obtain the prices for equation (3.1) is poor, dividing poor price estimates by a small shock in rates in the denominator will have a significant affect on the duration estimate.

What is done in practice by dealers and vendors of analytical systems? Each system developer uses rate shocks that they have found to be realistic based on historical rate changes.

Dollar Duration

Duration is related to percentage price change. However, for two bonds with the same duration, the dollar price change will not be the same. For example, consider two bonds, W and X. Suppose that both bonds have a duration of 5, but that W is trading at par while X is trading at 90. A 100-basis-point change for both bonds will change the price by approximately 5%. This means a price change of \$5 (5% times \$100) for W and a price change of \$4.5 (5% times \$90) for X.

The dollar price volatility of a bond can be measured by multiplying duration by the full dollar price and the number of basis points (in decimal form). That is,

$$\begin{aligned}\text{Dollar price change} = {} & \text{Duration} \times \text{Dollar price} \\ & \times \text{Yield change (in decimal)}\end{aligned}$$

The dollar price volatility for a 100-basis-point change in yield is

$$\text{Dollar price change} = \text{Duration} \times \text{Dollar price} \times 0.01$$

or equivalently,

$$\text{Dollar price change} = \text{Duration} \times \text{Dollar price}/100$$

The dollar price change calculated using the above formula is called *dollar duration*. In some contexts, dollar duration refers to the price change for a 100-basis-point change in yield. The dollar duration for any number of basis points can be computed by scaling the dollar price change accordingly. For example, for a 50-basis-point change in yields, the dollar price change or dollar duration is

$$\text{Dollar price change} = \text{Duration} \times \text{Dollar price}/200$$

For a one basis point change in yield, the dollar price change will give the same result as the price value of a basis point.

The dollar duration for a 100-basis-point change in yield for bonds W and X is

$$\text{For bond W: Dollar duration} = 5 \times 100/100 = 5.0$$
$$\text{For bond X: Dollar duration} = 5 \times 90/100 = 4.5$$

Modified Duration versus Effective Duration

One form of duration that is cited by practitioners is *modified duration*. Modified duration is the approximate percentage change in a bond's price for a 100-basis-point change in yield *assuming that the bond's expected cash flows do not change when the yield changes*. What this means is that in calculating the values of V_- and V_+ in equation (3.1), the same cash flows used to calculate V_0 are used. Therefore, the change in the bond's price when the yield is changed is due solely to discounting cash flows at the new yield level.

The assumption that the cash flows will not change when the yield is changed makes sense for option-free bonds such as noncallable Treasury securities. This is because the payments made by the U.S. Department of the Treasury to holders of its obligations do not change when interest rates change. However, the same cannot be said for bonds with embedded options (i.e., callable and putable bonds, mortgage-backed securities, and certain asset-backed securities). For these securities, a change in yield may significantly alter the expected cash flows.

Earlier in the chapter, we presented the price/yield relationship for callable and prepayable bonds. Failure to recognize how changes in yield can alter the expected cash flows will produce two values used in the numerator of equation (3.1) that are not good estimates of how the price will actually change. The duration is then not a good number to use to estimate how the price will change.

When we discussed valuation models for bonds with embedded options, we learned how these models (lattice models and Monte Carlo simulation) take into account how changes in yield will affect the expected cash flows. Thus, when V_- and V_+ are the values produced from these valuation models, the resulting duration takes into account both the discounting at different interest rates and how the expected cash flows may change. When duration is calculated in this manner, it is referred to as *effective duration* or *option-adjusted duration* or *OAS duration*. Below we explain how effective duration is calculated based on the lattice model and the Monte Carlo model.

Calculating the Effective Duration Using the Lattice Model

In Chapter 2, we explained how the lattice model is used to value bonds with embedded options. In our illustrations we used one form of the lattice model, the binomial model. The procedure for calculating the values to be substituted into the duration formula, equation (3.1), using the binomial model is now described. V_+ is determined as follows:

Step 1: Calculate the option-adjusted spread (OAS) for the issue.

Step 2: Shift the on-the-run yield curve up by a small number of basis points.

Step 3: Construct a binomial interest rate tree based on the new yield curve in Step 2.

Step 4: To each of the short rates in the binomial interest rate tree, add the OAS to obtain an "adjusted tree."

Step 5: Use the adjusted tree found in Step 4 to determine the value of the bond, which is V_+.

To determine the value of V_-, the same five steps are followed except that in Step 2, the on-the-run yield curve is shifted down by a small number of basis points.

Notice that in the calculation of V_+ and V_- the yield for each maturity is changed by the same number of basis points. This assumption is called the parallel yield curve shift assumption that we referred to earlier.

To illustrate how V_+ and V_- are determined in order to calculate effective duration, we will use the same on-the-run yield curve that we used in Chapter 2 assuming a volatility of 10%. The 4-year callable bond with a coupon rate of 6.5% and callable at par selling at 102.218

will be used in this illustration. We showed that the OAS for this issue is 35 basis points.

Exhibit 3.19a shows the adjusted tree by shifting the yield curve up by an arbitrarily small number of basis points, 25 basis points, and then adding 35 basis points (the OAS) to each 1-year rate. The adjusted tree is then used to value the bond. The resulting value, V_+, is 101.621. Exhibit 3.19b shows the adjusted tree by shifting the yield curve down by 25 basis points and then adding 35 basis points to each 1-year rate. The resulting value, V_-, is 102.765.

The results are summarized below:

$$\Delta y = 0.0025 \quad V_+ = 101.621 \quad V_- = 102.765 \quad V_0 = 102.218$$

Therefore,

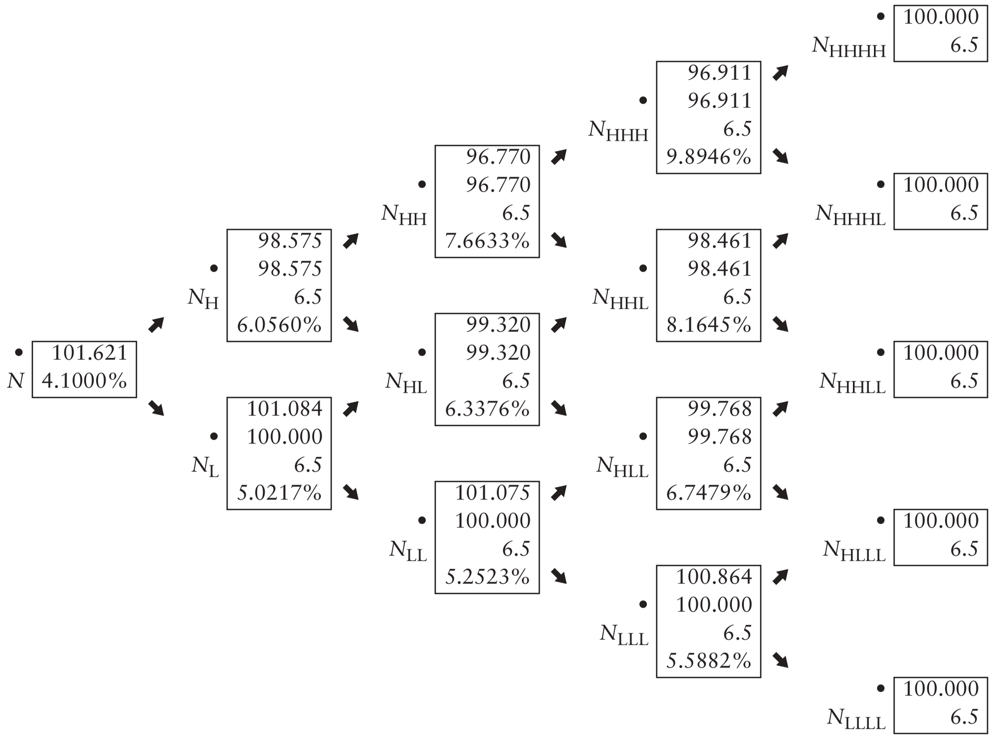
$$\text{Effective duration} = \frac{102.765 - 101.621}{2(102.218)0.0025} = 2.24$$

This procedure is the one used by Bloomberg to calculate effective duration.

As an illustration of the difference between modified and effective duration, let's consider a callable bond issued by Fannie Mae. Bloomberg's Security Description screen (DES) for this issue is presented in Exhibit 3.20. Note that this 6% coupon bond matures on January 18, 2012 and is callable at par only on January 18, 2005. This type of call structure is known as a "European call." Bloomberg's Option-Adjusted Spread Analysis (YAS) screen shown in Exhibit 3.21 gives three duration measures in center of the screen. Based on a settlement date of June 18, 2002, the modified duration is 7.07. This duration measure treats the bond as if it is option-free and values the bond as if changes in interest rates have no impact on the bond's expected cash flows. This number is located in the row labeled "M Dur" and the column labeled "To Mty." Next, Bloomberg calculates a duration measure assuming the bond will be called on the first call date (and in the case the only call date) of January 18, 2005. Essentially, the bond is valued as if it is straight bond that matures on the first call date and the investor receives the call price at this time (which for this issue is par). It is important to recognize that the call option impact's on the bond's expected cash flows is not considered explicitly. This duration measure is located in the column labeled "To Call on 1/18/2005" and is 2.31. It is lower, of course, because the bond's maturity is assumed to be seven years shorter namely, January 18, 2005 as opposed to January 18, 2012.

EXHIBIT 3.19 Calculating Effective Duration and Convexity Using the Binomial Model

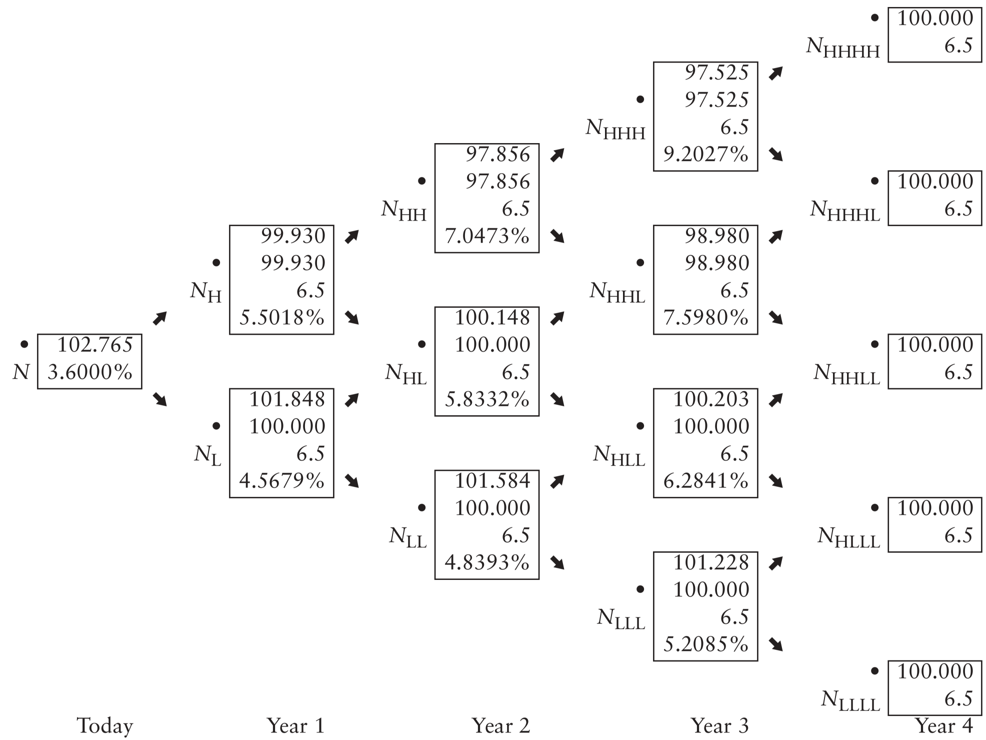
a. Determination of V_+[a]



[a] +25 basis point shift in on-the-run yield curve.

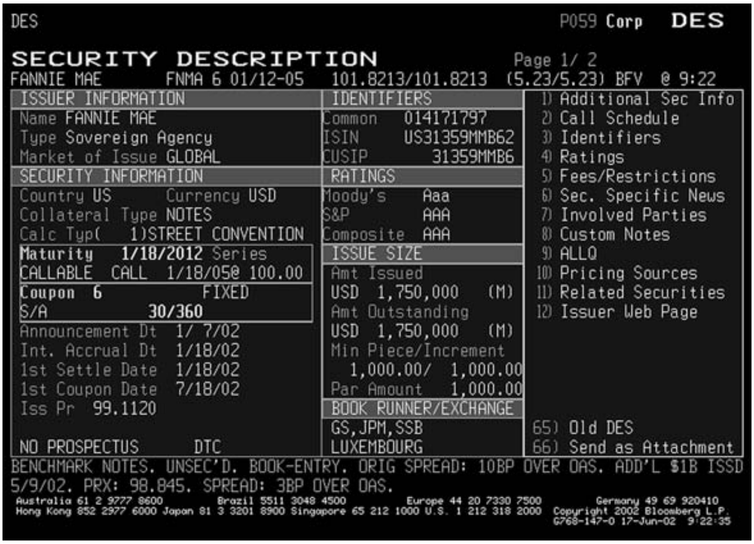
EXHIBIT 3.19 (Continued)

b. Determination of V_- [a]



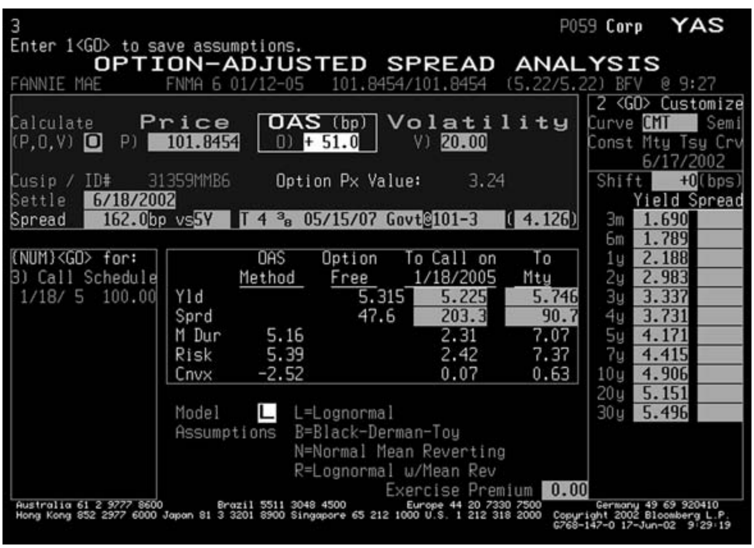
[a] –25 basis point shift in on-the-run yield curve.

EXHIBIT 3.20 Bloomberg Security Description Screen for a 6% Callable Fannie Mae Bond



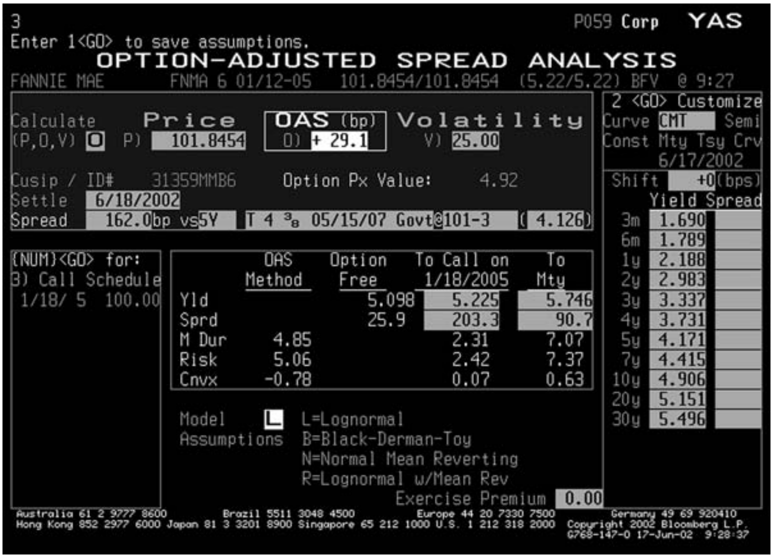
Source: Bloomberg Financial Markets

EXHIBIT 3.21 Bloomberg Option-Adjusted Spread Analysis Screen for a 6% Callable Fannie Mae Bond
Assumption: Volatility is 20%



Source: Bloomberg Financial Markets

EXHIBIT 3.22 **Bloomberg Option-Adjusted Spread Analysis Screen for a 6% Callable Fannie Mae Bond**
Assumption: Volatility is 25%



Source: Bloomberg Financial Markets

The third duration measure explicitly considers the embedded call option's impact on the bond's expected future cash flows. In this example, the valuation model used to compute the effective duration is the lognormal binomial interest rate tree just as in the illustration in Exhibit 3.19 earlier in this section. Bloomberg allows the user to select the valuation model used in the calculation and these are located in bottom-center of the screen.[6] Moreover, the benchmark yield curve used in the calculation is the Constant Maturity Treasury curve and this is located on the right-hand side of the screen. Given an interest rate volatility assumption of 20%, the OAS is 51 basis points and the value of the embedded call option is 3.24 (per $100 of the par value). The effective duration is 5.16 and this is located in the column labeled "OAS Method."

Since effective duration explicitly considers the impact of the embedded call option on the bond's expected cash flows using a valuation model, if the call option's value changes, then the bond price's sensitivity to changes in the required yield should change as well. To see this, consider the Bloomberg Option-Adjusted Spread Analysis screen shown in Exhibit 3.22 for the same Fannie Mae issue. The important

[6] Once again, the user has a choice among several different benchmark yield curves.

difference between this exhibit and the previous one is that the callable bond is valued assuming an interest rate volatility of 25% rather than 20%, all else is the same. As a result of the higher assumed interest rate volatility, the value of the embedded call option increases to 4.92 (per \$100 of par value) and the OAS declines to 29.1 basis points just as we demonstrated in Chapter 2. Let's examine the impact of the higher interest rate volatility assumption on our three duration measures.

The "To Mty" duration (i.e., modified duration) and the "To Call on 1/18/2005" duration are the same as before 7.07 and 2.31, respectively. The reason being is that these two measures ignore the impact of the embedded call option on the bond's expected cash flows so they are unaffected by a change in the interest rate volatility assumption. Note, however, that the effective duration (in the column labeled "OAS Method") is lower, 4.85, as opposed to 5.16. Specifically, as the assumed interest rate volatility increases from 20% to 25%, the effective duration decreases from 5.16 to 4.85. The reason for this result is straightforward. As the interest rate volatility increases, the likelihood the call option will be exercised increases. In other words, the probability that the bond will called on January 18, 2005 increases, so the bond's expected maturity shortens. Decreasing the expected maturity, all else equal, will decrease the bond price's sensitivity to changes in the required yield so the effective duration decreases.

Calculating the Effective Duration Using the Monte Carlo Model

The same procedure is used to calculate the effective duration for a security valued using the Monte Carlo model. The short-term rates are used to value the cash flow on each interest rate path. To obtain the two values to substitute into the duration formula, the OAS is calculated first. The short-term rates are then shifted up a small number of basis points, obtaining new refinancing rates and cash flows. V_+ is then calculated by discounting the cash flow on an interest rate path using the new short-term rates plus the OAS. V_- is then calculated in the same manner by shifting the short-term rates down by a small number of basis points. Again, since all rates are shifted by the same number of basis points, the resulting duration assumes a parallel shift in the yield curve.

Macaulay Duration and Modified Duration

It is worth comparing the relationship between modified duration to another duration measure. Modified duration can also be written as[7]:

[7] More specifically, this is the formula for the modified duration of a bond on a coupon anniversary date.

$$\frac{1}{(1+\text{yield}/k)}\left[\frac{1 \times PVCF_1 + 2 \times PVCF_2 + \ldots + n \times PVCF_n}{k \times \text{Price}}\right] \tag{3.3}$$

where

k = number of periods, or payments, per year (e.g., $k = 2$ for semiannual-pay bonds and $k = 12$ for monthly-pay bonds)
n = number of periods until maturity (i.e., number of years to maturity times k)
yield = yield to maturity of the bond
$PVCF_t$ = present value of the cash flow in period t discounted at the yield to maturity where $t = 1, 2, \ldots, n$

The expression in the brackets of the modified duration formula given by equation (3.3) is a measure formulated in 1938 by Frederick Macaulay.[8] This measure is popularly referred to as *Macaulay duration*. Thus, modified duration is commonly expressed as

$$\text{Modified duration} = \frac{\text{Macaulay duration}}{(1 + \text{yield}/k)}$$

Bloomberg reports Macaulay duration on its YA (yield analysis) screen in the Sensitivity Analysis box in the lower left-hand corner of Exhibit 3.13. Macaulay duration is labeled "CNV DURATION (YEARS)" where the CNV stands for "conventional."

The general formulation for duration as given by equation (3.1) provides a short-cut procedure for determining a bond's modified duration. Because it is easier to calculate the modified duration using the short-cut procedure, almost all vendors of analytical software will use equation (3.1) rather than equation (3.3) to reduce computation time.

However, it must be clearly understood that modified duration is a flawed measure of a bond's price sensitivity to interest rate changes for a bond with an embedded option and therefore so is Macaulay duration. The use of the formula for duration given by equation (3.3) *misleads* the user because it masks the fact that changes in the expected cash flows must be recognized for bonds with embedded options. Although equation (3.3) will give the same estimate of percentage price change for an option-free bond as equation (3.1), equation (3.1) is still better because it acknowledges that cash flows and thus value can change due to yield changes.

[8] Frederick Macaulay, *Some Theoretical Problems Suggested by the Movement of Interest Rates, Bond Yields, and Stock Prices in the U.S. Since 1856* (New York: National Bureau of Economic Research, 1938).

EXHIBIT 3.23 Summary of a 3-Treasury Bond Portfolio

Bond	Full Price ($)	Yield (%)	Par Amount Owned($)	Market Value ($)	Duration
2-year, 3.25% coupon U.S. Treasury note maturing 5/31/04	100.6555	2.969	5,000,000	5,032,778.35	1.886
10-year, 4.875% coupon U.S. Treasury note maturing 2/15/12	101.5119	4.886	4,000,000	4,060,352.20	7.518
10-year U.S. Treasury principal strip maturing 2/15/02	60.1730	5.322	2,000,000	1,203,461.86	9.421

Portfolio Duration

A portfolio's duration can be obtained by calculating the weighted average of the duration of the bonds in the portfolio. The weight is the proportion of the portfolio that a security comprises. Mathematically, a portfolio's duration can be calculated as follows:

$$w_1 D_1 + w_2 D_2 + w_3 D_3 + \ldots + w_K D_K$$

where

w_i = market value of bond *i*/market value of the portfolio
D_i = duration of bond *i*
K = number of bonds in the portfolio

To illustrate the calculation, consider the following 3-bond portfolio in which all three bonds are U.S. Treasuries from Exhibits 3.3 and 3.4. Exhibit 3.23 presents the full price per \$100 of par value for each bond, its yield, the par amount owned, the market value and its duration assuming a settlement date of June 14, 2002.

In this illustration, the 2-year note and the 10-year note are priced with a settlement date between coupon payments dates so the market prices reported are full prices. The market value for the portfolio is \$10,296,592.21. Since each bond is option-free, modified duration can be used.

In this illustration, K is equal to 3 and

w_1 = \$5,032,778.35/\$10,296,592.21 = 0.489 D_1 = 1.886
w_2 = \$4,060,352.20/\$10,296,592.21 = 0.394 D_2 = 7.518
w_3 = \$1,203,461.86/\$10,296,592.21 = 0.117 D_3 = 9.421

The portfolio's duration is

$$0.489(1.886) + 0.394(7.518) + 0.117(9.421) = 4.987$$

A portfolio duration of 4.987 means that for a 100-basis-point change in the yield for each of the three bonds, the portfolio's market value will change by approximately 4.987%. It is paramount to keep in mind; the yield for each of the three bonds must change by 100 basis points for the duration measure to be useful. This is a critical assumption and its importance cannot be overemphasized. Portfolio managers will find it necessary to be able to measure a portfolio's exposure to a reshaping of the yield curve. We will examine methods for doing so in the next chapter.

An alternative procedure for calculating a portfolio's duration is to calculate the dollar price change for a given number of basis points for each security in the portfolio and then sum up all the changes in market value. Dividing the total of the changes in market value by the portfolio's initial market value produces a percentage change in market value that can be adjusted to obtain the portfolio's duration.

For example, consider the 3-bond portfolio given in Exhibit 3.23. Suppose that we calculate the dollar change in market value for each bond in the portfolio based on its respective duration for a 50-basis-point change in yield. We would then have

Bond	Market Value ($)	Duration	Change in Value for 50 bp Yield Change ($)
2-year, 3.25% coupon U.S. Treasury note maturing 5/31/04	5,032,778.35	1.886	47,459.10
10-year, 4.875% coupon U.S. Treasury note maturing 2/15/12	4,060,352.20	7.518	152,628.64
10-year U.S. Treasury principal strip maturing 2/15/12	1,203,461.86	9.421	55,689.07
		Total	255,776.81

Thus, a 50-basis-point change in all rates will change the market value of the 3-bond portfolio by $255,776.81. Since the market value of the portfolio is $10,296,592.21, a 50-basis-point change produced a change in value of 2.484% ($255,776.81 divided by $10,296,592.21). Since duration is the approximate percentage change for a 100-basis-point change in rates, this means that the portfolio is 4.968 (found by doubling 2.484). This is virtually the same value for the portfolio's duration as found earlier.

Contribution to Portfolio Duration

Some portfolio managers view their exposure to a particular issue or to a sector in terms of the percentage of that issue or sector in the portfolio. A better measure of exposure of an individual issue or sector to changes in interest rates is in terms of its *contribution to the portfolio duration*. Contribution to portfolio duration is computed by multiplying the percentage that the individual issues comprises of the portfolio by the duration of the individual issue or sector. Specifically,

$$\text{Contribution to portfolio duration} = \frac{\text{Market value of issue or sector}}{\text{Market value of portfolio}} \times \text{Duration of issue or sector}$$

This exposure can also be cast in terms of dollar exposure. To accomplish this, the dollar duration of the issue or sector is used instead of the duration of the issue or sector.

A portfolio manager who desires to determine the contribution to a portfolio of a sector relative to the contribution of the same sector in a broad-based market index can compute the difference between these two contributions.

OTHER DURATION MEASURES

Numerous duration measures are routinely employed by fixed-income practitioners and risk managers that relate to both fixed-rate and floating-rate securities. We discuss these measures in this section.

Spread Duration for Fixed-Rate Bonds

As we have seen, duration is a measure of the change in a bond's value when interest rates change. The interest rate that is assumed to shift is the Treasury rate, which serves as the benchmark interest rate. However, for non-Treasury instruments, the yield is equal to the Treasury yield plus a spread to the Treasury yield curve. This is why non-Treasury securities are often called "spread products." Of course, the price of a bond exposed to credit risk can change even though Treasury yields are unchanged because the spread required by the market changes. A measure of how a non-Treasury security's price will change if the spread sought by the market changes is called *spread duration*.

The problem is, what spread is assumed to change? There are three measures that are commonly used for fixed-rate bonds: nominal spread, zero-volatility spread, and option-adjusted spread. Each of these spread measures were defined earlier in the book.

The nominal spread is the traditional spread measure. The nominal spread is simply the difference between the yield on a non-Treasury issue and the yield on a comparable maturity Treasury. When the spread is taken to be the nominal spread, spread duration indicates the approximate percentage change in price for a 100-basis-point change in the nominal spread holding the Treasury yield constant.

The zero-volatility or static spread is the spread that when added to the Treasury spot rate curve will make the present value of the cash flows equal to the bond's price plus accrued interest. When spread is defined in this way, spread duration is the approximate percentage change in price for a 100-basis-point change in the zero-volatility spread holding the Treasury spot rate curve constant.

Finally, the option-adjusted spread (OAS) is the constant spread that, when added to all the rates on the interest rate tree, will make the theoretical value equal to the market price. Spread duration based on OAS can be interpreted as the approximate percentage change in price of a non-Treasury for a 100-basis-point change in the OAS, holding the Treasury rate constant.

A sensible question arises: How do you know whether a spread duration for a fixed-rate bond is a spread based on the nominal spread, zero-volatility spread or the OAS? The simple answer is you do not know! You must ask the broker/dealer or vendor of the analytical system. To add further to the confusion surrounding spread duration, consider the term "OAS duration" that is referred to by some market participants. What does it mean? On the one hand, it could mean simply the spread duration that we just described. On the other hand, many market participants use the term "OAS duration" interchangeably with the term "effective duration." Once again, the only way to know what OAS is measuring is to ask the broker/dealer or vendor.

Spread Duration for Floaters

Two measures have been developed to estimate the sensitivity of a floater to each component of the coupon reset formula: the index (i.e., reference rate) and the spread (i.e., quoted margin). *Index duration* is a measure of the price sensitivity of a floater to changes in the reference rate holding the spread constant. *Spread duration* measures a floater's price sensitivity to a change in the spread assuming that the reference rate is unchanged.

Duration of an Inverse Floater

An inverse floater possesses a coupon rate that changes in the direction opposite to that of some reference rate or market rate. Inverse floaters exist in the corporate and municipal bond markets. However, the largest issuance of inverse floaters has been in the collateralized mortgage obligations (CMO) market. There are several methods that can be employed to create to create an inverse floater. For example, a dealer buys a fixed-rate bond in the secondary market and places the bond in a trust. Subsequently, the trust issues a floating-rate security and an inverse floating-rate security.

As noted throughout this chapter, duration is measure of a security's price sensitivity to a change in required yield. Because valuations are additive (i.e., the value of the underlying collateral is the sum of the floater and inverse floater values), durations (properly weighted) as we have seen are additive as well. Accordingly, the duration of an inverse floater is related in a particular fashion to the duration of the collateral and the duration of the floater.

The duration of an inverse floater will be a multiple of the duration of the collateral from which it is created. To understand this, suppose that a 30-year fixed-rate bond with a market value of \$100 million is split into a floater and an inverse floater with market values of \$80 million and \$20 million, respectively. Assume that the duration of the collateral (i.e., the 30-year fixed-rate bond) is 8. Given this information, we know that for a 100-basis-point change in required yield that the collateral's value will change by approximately 8% or \$8 million (8% times \$100 million). Since the floater and inverse floater are created from the same collateral, the combined change in value of the floater and the inverse floater must also be \$8 million given a 100-basis-point change in required yield. The question becomes how one allocates the change in value between the floater and inverse floater. The duration of the floater will be small because on each coupon reset date, any change in interest rates (via the reference rate) is also reflected in the size of the floater's coupon payment. Accordingly if the duration of the floater is small, then the inverse floater must experience the full force of the \$8 million change in value. For this to occur, the duration of the inverse floater must be approximately 40. A duration of 40 will mean a 40% change in the inverse floater's value for a 100-basis-point change in required yield and a change in value of approximately \$8 million (40% times \$20 million).

Notice from our illustration that the duration of an inverse floater is greater than the collateral's term to maturity. For those individuals who interpret duration in terms of years (i.e., Macaulay duration presented earlier in the chapter) this presents something of a puzzle. After all, how can a security have a duration greater than the underlying collateral from which it is created? Of course, there is no puzzle. The confusion is the lingering residue from continuing to think about duration in the context in which it was developed by Frederick Macaulay in 1938—as a measure of the average time taken by a security, on a discounted basis, to return the original investment. The significance and interpretation of Macaulay duration lie in its link to bond price volatility.

In general, assuming that the duration of the floater is close to zero, it can be shown that the duration of an inverse floater is:

$$\begin{aligned}&\text{Duration of inverse floater}\\&= (1 + L)(\text{Duration of collateral}) \times \frac{\text{Collateral price}}{\text{Inverse price}}\end{aligned}$$

where L = leverage of inverse floater.

Empirical Duration for an MBS

Empirical duration, sometimes referred to as *implied duration*, is the sensitivity of a mortgage-backed security (MBS) as estimated empirically from historical prices and yields.[9] Regression analysis, a statistical technique described in Chapter 6, is used to estimate the relationship. More specifically, the relationship estimated is the percentage change in the price of the MBS of interest to the change in the general level of Treasury yields.

To obtain the empirical duration, Paul DeRossa, Laurie Goodman, and Mike Zazzarino suggest the following relationship be estimated using multiple regression analysis:[10]

$$\begin{aligned}&\text{Percentage change in price } = c + b_1(\Delta y) + b_2(P - 100)(\Delta y)\\&+ b_3[(P - 100)^2(\Delta y) \text{ if P } > 100\text{, otherwise } 0] + \text{error term}\end{aligned}$$

where

P = price (with par equal to 100)
Δy = change in yield

and c, b_1, b_2, and b_3 are the parameters to be estimated.

The inclusion of the second and third terms in the relationship is to allow for the price sensitivity to vary depending on the price level of the mortgages. The reason for the inclusion of the error term is explained in Chapter 6.

[9] The first attempt to calculate empirical duration was by Scott M. Pinkus and Marie A. Chandoha, "The Relative Price Volatility of Mortgage Securities," *Journal of Portfolio Management* (Summer 1986), pp. 9–22.

[10] Paul DeRossa, Laurie Goodman, and Mike Zazzarino, "Duration Estimates on Mortgage-Backed Securities," *Journal of Portfolio Management* (Winter 1993), pp. 32–37.

The expectation is that the parameter c would be equal to zero when the relationship is estimated. The expected sign of b_2 is negative. That is, there is an inverse relationship between yield changes and price changes. Finally, the terms b_2 and b_3 are expected to have a positive sign.

DeRossa, Goodman, and Zazzarino estimated the relationship using daily data for the 5-year period (11/19/86 to 11/18/91) for Ginnie Mae and Fannie Mae 8s, 9s, 10s, and 11s. The yield used was the 10-year Treasury, although they indicate that nearly identical results were realized if they used the 7-year Treasury. In all of their estimated regressions, all of the parameters had the expected sign.

Given the estimated relationship, the empirical duration for different coupons at different price levels can be found by dividing the estimated relationship by the change in yield. That is,

$$\text{Duration} = \frac{\text{Percentage change in price}}{\Delta y}$$
$$= c + b_1 + b_2(P - 100) + b_3[(P - 100)^2(\Delta y) \text{ if } P > 100, \text{ otherwise } 0]$$

For an MBS trading at par, P is 100, and the empirical duration is therefore b_1.

There are three advantages to the empirical duration approach.[11] First, the duration estimate does not rely on any theoretical formulas or analytical assumptions. Second, the estimation of the required parameters are easy to compute using regression analysis. Finally, the only inputs that are needed are a reliable price series and Treasury yield series.

There are disadvantages.[12] First, a reliable price series for the data may not be available. For example, there may be no price series available for a thinly traded mortgage derivative security or the prices may be matrix priced or model priced rather than actual transaction prices. Second, an empirical relationship does not impose a structure for the options embedded in an MBS and this can distort the empirical duration. Third, the price history may lag current market conditions. This may occur after a sharp and sustained shock to interest rates has been realized. Finally, the volatility of the spread to Treasury yields can distort how the price of an MBS reacts to yield changes.

[11] See Bennett W. Golub, "Towards a New Approach to Measuring Mortgage Duration," Chapter 32 in Frank J. Fabozzi (ed.), *The Handbook of Mortgage-Backed Securities* (Chicago: Probus Publishing, 1995), p. 672.

[12] Golub, "Towards a New Approach to Measuring Mortgage Duration."

CONVEXITY

The duration measure indicates that regardless of whether interest rates increase or decrease, the approximate percentage price change is the same. However, as we noted earlier, this is not consistent with Property 3 of a bond's price volatility. Specifically, while for small changes in yield the percentage price change will be the same for an increase or decrease in yield, for large changes in yield this is not true. This suggests that duration is only a good approximation of the percentage price change for small changes in yield.

We demonstrated this property earlier using a 4.875% coupon, 10-year Treasury note priced to yield 4.886% with a duration of 7.518. For a 10-basis-point change in yield, the estimate was accurate for both an increase and decrease in yield. However, for a 200-basis-point change in yield the approximate percentage price change was off considerably.

The reason for this result is that duration is in fact a first (linear) approximation for a small change in yield.[13] The approximation can be improved by using a second approximation. This approximation is referred to as "convexity." *The use of this term in the industry is unfortunate since the term convexity is also used to describe the shape or curvature of the price/yield relationship.* The *convexity measure* of a security can be used to approximate the change in price that is not explained by duration.[14]

Convexity Measure

The convexity measure of a bond is approximated using the following formula:

$$\text{Convexity measure} = \frac{V_+ + V_- - 2V_0}{2V_0(\Delta y)^2} \tag{3.4}$$

[13] The reason it is a linear approximation can be seen in Exhibit 3.16, where the tangent line is used to estimate the new price. That is, a straight line is being used to approximate a nonlinear (i.e., convex) relationship.

[14] Mathematically, any function can be estimated by a series of approximations referred to a Taylor series expansion. Each approximation or term of the Taylor series is based on a corresponding derivative. For a bond, duration is the first term approximation of the price change and is related to the first derivative of the bond's price with respect to a change in the required yield. The convexity measure is the second approximation and related to the second derivative of the bond's price. We will see the technique used again when we discuss the delta and gamma of an option in Chapter 12.

where the notation is the same as used earlier for duration as given by equation (3.4).

For the 4.875%, 10-year Treasury note priced to yield 4.886% with a settlement date of June 14, 2002, we know that for a 20-basis-point change in yield ($\Delta y = 0.002$):

$$V_0 = 101.5119,\ V_{-} = 103.0525,\ V_{+} = 99.9995$$

Note once again, because the settlement date is not a coupon payment date (see Exhibit 3.4), that we use full prices in the calculation. Substituting these values into the convexity measure given by equation (3.4):

$$\text{Convexity measure} = \frac{99.9995 + 103.0525 - 2(101.5119)}{2(101.5088)(0.002)^2} = 34.58$$

We'll see how to use this convexity measure shortly. Before doing so, there are three points that should be noted. First, there is no simple interpretation of the convexity measure as there is for duration. Second, it is more common for market participants to refer to the value computed in equation (3.4) as the "convexity of a bond" rather than the "convexity measure of a bond." Finally, the convexity measure reported by dealers and vendors will differ for an option-free bond. The reason is that the value obtained from equation (3.4) is often scaled for the reason explained after we demonstrate how to use the convexity measure.

Convexity Adjustment to Percentage Price Change

Given the convexity measure, the approximate percentage price change adjustment due to the bond's convexity (i.e., the percentage price change not explained by duration) is

$$\begin{aligned}&\text{Convexity adjustment to percentage price change}\\&= \text{Convexity measure} \times (\Delta y)^2 \times 100\end{aligned} \tag{3.5}$$

For example, for the 4.875% 10-year Treasury note, the convexity adjustment to the percentage price change based on duration if the yield increases from 4.886% to 6.886% is

$$34.58 \times (0.02)^2 \times 100 = 1.383\%$$

If the yield decreases from 4.886% to 6.886%, the convexity adjustment to the approximate percentage price change based on duration would also be 1.383%.

The approximate percentage price change based on duration and the convexity adjustment is found by summing the two estimates. So, for example, if yields change from 4.886% to 6.886%, the estimated percentage price change would be as follows:

Estimated change using duration alone	=	−15.038
Convexity adjustment	=	+1.383
Total estimated percentage price change	=	−13.655%

The actual percentage price change is −13.741%.

For a decrease of 200 basis points, from 4.886% to 2.886%, the approximate percentage price change would be as follows:

Estimated change using duration alone	=	+15.038
Convexity adjustment	=	+1.383
Total estimated percentage price change	=	+16.42%

The actual percentage price change is +16.518%. Thus, duration combined with the convexity adjustment does a much better job of estimating the sensitivity of a bond's price change to large changes in yield. Accordingly, for large changes in required yield, duration and convexity used together deliver a more accurate estimate of how much a bond's price will change for a given change in required yield than duration used alone.

Notice that when the convexity measure is positive, we have the situation described earlier that the gain is greater than the loss for a given large change in rates. That is, the bond exhibits positive convexity. We can see this in the example above. However, if the convexity measure is negative, we have the situation where the loss will be greater than the gain. For example, suppose that a callable bond has an effective duration of 4 and a convexity measure of −30. This means that the approximate percentage price change for a 200-basis-point change is 8%. The convexity adjustment for a 200-basis-point change in rates is then

$$-30 \times (0.02)^2 \times 100 = -1.2$$

The convexity adjustment is −1.2%, and therefore the bond exhibits the negative convexity property illustrated in Exhibit 3.17. The approximate percentage price change after adjusting for convexity is as follows:

Estimated change using duration	=	−8.0%
Convexity adjustment	=	−1.2%
Total estimated percentage price change	=	−9.2%

For a decrease of 200 basis points, the approximate percentage price change would be as follows:

Estimated change using duration	=	+8.0%
Convexity adjustment	=	–1.2%
Total estimated percentage price change	=	+6.8%

Notice that the loss is greater than the gain—a property called *negative convexity* that we discussed earlier and illustrated in Exhibit 3.11.

Scaling the Convexity Measure

The convexity measure as given by equation (3.4) means nothing in isolation. It is the substitution of the computed convexity measure into equation (3.5) that provides the estimated adjustment for convexity that is meaningful. Therefore, it is possible to scale the convexity measure in any way as long as the same convexity adjustment is obtained.

For example, in some books the convexity measure is defined as follows:

$$\text{Convexity measure} = \frac{V_+ + V_- - 2V_0}{V_0(\Delta y)^2} \tag{3.6}$$

Equation (3.6) differs from equation (3.4) since it does not include 2 in the denominator. Thus, the convexity measure computed using equation (3.6) will be double the convexity measure using equation (3.4). So, for our earlier illustration, since the convexity measure using equation (3.4) is 34.58, the convexity measure using equation (3.6) would be 69.16.

Which is correct, 34.58 or 69.16? The answer is both. The reason is that the corresponding equation for computing the convexity adjustment would not be given by equation (3.5) if the convexity measure is obtained from equation (3.6). Instead, the corresponding convexity adjustment formula would be as follows:

$$\begin{aligned}&\text{Convexity adjustment to percentage price change}\\&= (\text{Convexity measure}/2) \times (\Delta y)^2 \times 100\end{aligned} \tag{3.7}$$

Equation (3.7) differs from equation (3.5) in that the convexity measure is divided by 2. Thus, the convexity adjustment will be the same whether one uses equation (3.4) to get the convexity measure and equation (3.5) to get the convexity adjustment or one uses equation (3.6) to compute the convexity measure and equation (3.7) to determine the convexity adjustment.

Some dealers and vendors scale convexity in a different way. One can also compute the convexity measure as follows:

$$\text{Convexity measure} = \frac{V_+ + V_- - 2V_0}{2V_0(\Delta y)^2(100)} \tag{3.8}$$

Equation (3.8) differs from equation (3.4) by the inclusion of 100 in the denominator. In our illustration, the convexity measure would be 0.3458 rather than 34.58 using equation (3.4). The convexity adjustment formula corresponding to the convexity measure given by equation (3.8) is then

$$\begin{aligned}&\text{Convexity adjustment to percentage price change}\\&= \text{Convexity measure} \times (\Delta y)^2 \times 10{,}000\end{aligned} \tag{3.9}$$

Similarly, one can express the convexity measure as shown in equation (3.10):

$$\text{Convexity measure} = \frac{V_+ + V_- - 2V_0}{V_0(\Delta y)^2(100)} \tag{3.10}$$

For the 10-year Treasury note we have been using in our illustrations, the convexity measure is 0.692. The corresponding convexity adjustment is

$$\begin{aligned}&\text{Convexity adjustment to percentage price change}\\&= (\text{Convexity measure}/2) \times (\Delta y)^2 \times 10{,}000\end{aligned} \tag{3.11}$$

Consequently, the convexity measure (or just simply "convexity" as it is referred to by some market participants) that could be reported for this option-free bond are 34.58, 69.16, 0.3458, or 0.6916. All of these values are correct, but they mean nothing in isolation. To use them to obtain the convexity adjustment to the price change estimated by duration requires knowing how they are computed so that the correct convexity adjustment formula is used. *It is the convexity adjustment that is important—not the convexity measure in isolation.*

It is also important to understand this when comparing the convexity measures reported by dealers and vendors. For example, if one dealer shows a manager Bond A with a duration of 4 and a convexity measure of 50, and a second dealer shows the manager Bond B with a duration of 4 and a convexity measure of 80, which bond has the greater percentage price change response to changes in interest rates? Since the duration of the two bonds is identical, the bond with the larger convexity measure will change more when rates decline. However, not knowing how the two dealers computed the convexity measure means that the manager does not know which bond will have the greater convexity adjustment. If the first dealer used equation (3.4) while the second dealer used equation (3.6), then the convexity measures must be adjusted in terms of either equation. For example, the convexity measure of 80 computed using equation (3.6) is equal to a convexity measure of 40 based on equation (3.4).

Let's return to Exhibit 3.13 which is the Bloomberg Yield Analysis screen for the 10-year Treasury note in our illustration. Bloomberg's convexity measure is displayed in the Sensitivity Analysis box in lower left-hand corner of the screen. Specifically, the convexity measure reported is 0.6916 which is the same number we calculated using equation (3.10). This means that equation (3.11) should be employed to obtain the convexity adjustment when using the convexity measure reported by Bloomberg.

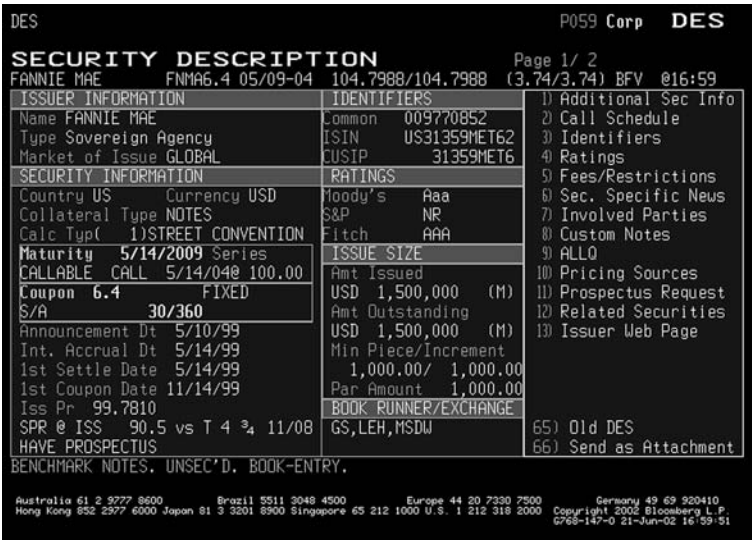
Modified Convexity and Effective Convexity

The prices used in equation (3.4) to calculate convexity can be obtained by either assuming that when the yield changes the expected cash flows either do not change or they do change. In the former case, the resulting convexity is referred to as *modified convexity.* (Actually, in the industry, convexity is not qualified by the adjective "modified.") In contrast, *effective convexity* assumes that the cash flows do change when yields change. This is the same distinction made for duration.

As with duration, there is little difference between modified convexity and effective convexity for option-free bonds. However, for bonds with embedded options there can be quite a difference between the calculated modified convexity and effective convexity measures. In fact, for all option-free bonds, either convexity measure will have a positive value. For bonds with embedded options, the calculated effective convexity measure can be negative when the calculated modified convexity measure is positive.

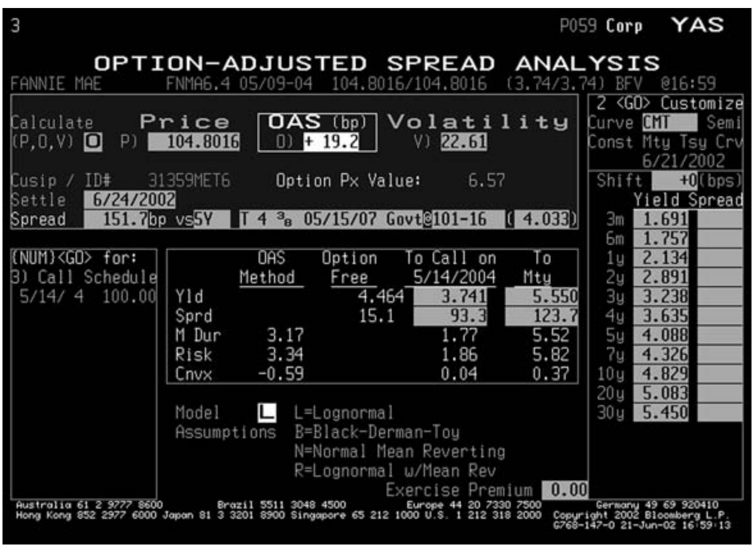
As an illustration, consider a 6.4% coupon bond issued by Fannie Mae that matures on May 14, 2009. Exhibit 3.24 presents the Bloomberg Security Description screen for this issue. The bond is callable on or after May 14, 2004 with at a minimum of 10 business days notice. Using a settlement date of June 24, 2002, let's compare the modified and effective convexities. Exhibit 3.25 shows the Bloomberg Option-Adjusted Spread Analysis screen. The modified convexity is 0.37 and is located in the center-right portion of the screen in the column labeled "To Mty." The effective convexity is –0.59 and is located in the column labeled "OAS Method."

EXHIBIT 3.24 Bloomberg Security Description Screen for a 6.4% Callable Fannie Mae Bond



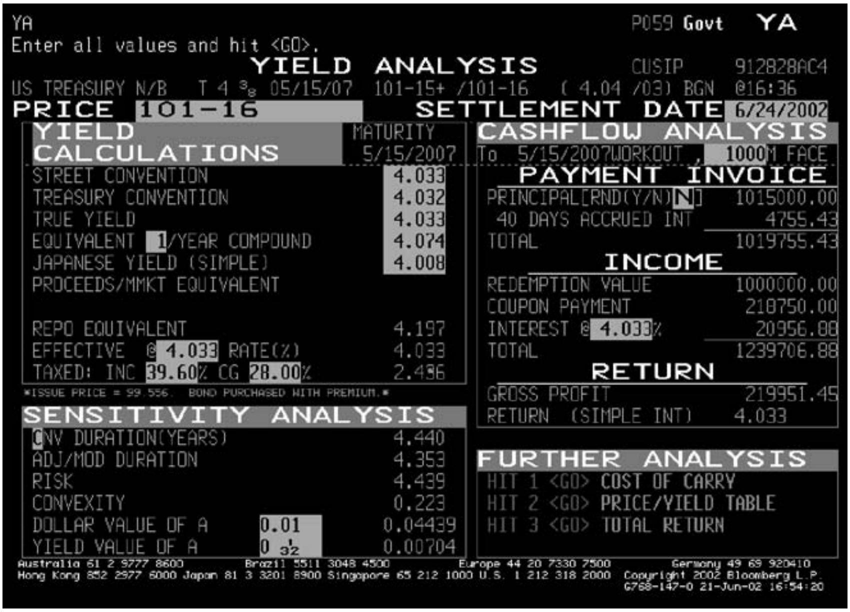
Source: Bloomberg Financial Markets

EXHIBIT 3.25 Bloomberg Option-Adjusted Spread Analysis Screen for a 6.4% Callable Fannie Mae Bond



Source: Bloomberg Financial Markets

EXHIBIT 3.26 Bloomberg Yield Analysis for a 4.375% 5-Year Treasury Note



Source: Bloomberg Financial Markets

PRICE VALUE OF A BASIS POINT

Some managers use another measure of the price volatility of a bond to quantify interest rate risk—the *price value of a basis point* (PVBP). This measure, also called the *dollar value of an 01* (DV01), is the absolute value of the change in the price of a bond for a 1-basis-point change in yield. That is,

$$\text{PVBP} = |\text{ initial price} - \text{price if yield is changed by 1 basis point }|$$

Does it make a difference if the yield is increased or decreased by 1 basis point? It does not because of Property 2—the change will be about the same for a small change in basis points.

To illustrate the computation, let examine a 4.375% coupon, 5-year U.S. Treasury note that matures on May 15, 2012. Bloomberg's YA (Yield Analysis) Screen is presented in Exhibit 3.26. If the bond is priced to yield 4.033% on a settlement date of June 24, 2002, we can compute the PVBP by using the prices for either the yield at 4.043 or 4.023. The bond's initial full price at 4.033% is 101.9763. If the yield is decreased by 1 basis point to 4.023%, the PVBP is 0.0444 (|102.0207 – 101.9763|). If the yield is increased by 1 basis point to 4.043%, the PVBP is 0.0444 (|101.9319 - 101.9763|). Note that our PVBP calculation agrees with Bloomberg's calculation labeled "DOLLAR VALUE OF A 0.01" that is presented in the Sensitivity Analysis box located in the lower left-hand corner of the screen.

The PVBP is related to duration. In fact, PVBP is simply a special case of a measure called *dollar duration*. Dollar duration is the approximate dollar price change for a 100-basis-point change in yield. We know that a bond's duration is the approximate percentage price change for a 100-basis-point change in interest rates. We also know how to compute the approximate percentage price change for any number of basis points given a bond's duration using equation (3.2). Given the initial price and the approximate percentage price change for 1 basis point, we can compute the change in price for a 1-basis-point change in rates.

For example, consider once again the 4.375% coupon, 5-year Treasury note. From Exhibit 3.26, the duration is 4.353. Using equation (3.2), the approximate percentage price change for a 1 basis point increase in interest rates (i.e., $\Delta y = 0.0001$) ignoring the negative sign in equation (3.2) is

$$4.353 \times (0.0001) \times 100 = 0.04353\%$$

Given the initial full price of 101.9763, the dollar price change estimated using duration is

$$0.04353\% \times 101.9763 = \$0.0444$$

This is the same price change as shown above for a PVBP for this bond.[15]

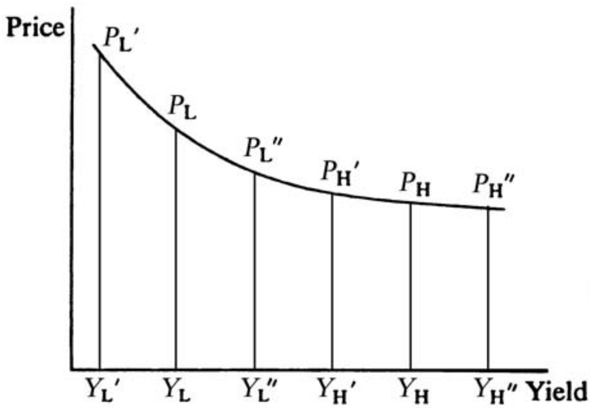
Yield Value of Price Change

Another common measure of interest rate risk is called the *yield value of a price change*. The price change is the tick (e.g., 1⁄32 for Treasuries or 1⁄8 for corporates) for the particular bond being examined. Suppose we are examining a Treasury so a tick is 1⁄32. The yield value of a price change for a Treasury is the change in yield for a 1⁄32 change in price. The yield value of a price change is determined by calculating the difference between the yield to maturity at the current price and the yield to maturity if the bond price's was increased/decreased by 1⁄32. In other words, how much does the current yield to maturity have to change to either increase or decrease the current price by 1⁄32 (i.e., 1 tick)? The smaller the yield value of a price change, the greater the dollar price volatility.

[15] Bloomberg's "Risk" measure is simply the PVBP × 100. For bonds that are trading close to par, Risk should be close to modified duration.

EXHIBIT 3.27 The Effect of Yield Level on Price Volatility

$(Y_H' - Y_H) = (Y_H - Y_H'') = (Y_L' - Y_L) = (Y_L - Y_L'')$
$(P_H - P_H') < (P_L - P_L')$ and
$(P_H - P_H'') < (P_L - P_L'')$



To illustrate, let us return to the 4.375% coupon, 5-year Treasury note in Exhibit 3.26. On a settlement date of June 24, 2002, the bond is yielding 4.033% with a full price of 101.9763. The yield value of a 1⁄32, reported by Bloomberg in the Sensitivity Analysis box in the lower left-hand corner of the screen, is 0.00704. This number tells us how the yield must fall/rise to increase/decrease the bond's price by one tick (i.e., 1⁄32). If we reprice the bond at 4.02596% (4.033% – 0.00704%), the full price is 102.0076. The difference between these two prices is 0.03125 (102.0076 – 101.9763) which is the dollar value of 1/32 when par is $100.

THE IMPORTANCE OF YIELD VOLATILITY

What we have not considered thus far is the volatility of interest rates. For example, as we explained earlier, all other factors equal, the higher the coupon rate, the lower the price volatility of a bond to changes in interest rates. In addition, the higher the level of yields, the lower the price volatility of a bond to changes in interest rates. This is illustrated in Exhibit 3.27, which shows the price/yield relationship for an option-free bond. When the yield level is high (Y_H in the exhibit) a change in interest rates does not produce a large change in the initial price (P_H in the exhibit). However, when the yield level is low (Y_L in the exhibit) a change in interest rates of the same number of basis points as shown

when the yield is high does produce a large change in the initial price (P_L in the exhibit).

This can also be cast in terms of duration properties: the higher the coupon, the lower the duration; and the higher the yield level the lower the duration. Given these two properties, a 10-year noninvestment grade bond has a lower duration than a current coupon 10-year Treasury note since the former has a higher coupon rate and trades at a higher yield level. Does this mean that a 10-year noninvestment grade bond has less interest rate risk than a current coupon 10-year Treasury note? Consider also that a 10-year Swiss government bond has a lower coupon rate than a current coupon 10-year U.S. Treasury note and trades at a lower yield level. Therefore, a 10-year Swiss government bond will have a higher duration than a current coupon 10-year Treasury note. Does this mean that a 10-year Swiss government bond has greater interest rate risk than a current coupon 10-year U.S. Treasury note? The missing link is the relative volatility of rates, which we shall refer to as *yield volatility* or *interest rate volatility.*

The greater the expected yield volatility, the greater the interest rate risk for a given duration and current value of a position. In the case of noninvestment grade bonds, while their durations are less than current coupon Treasuries of the same maturity, the yield volatility is greater than that of current coupon Treasuries. For the 10-year Swiss government bond, while the duration is greater than for a current coupon 10-year U.S. Treasury note, the yield volatility is considerably less than that of 10-year U.S. Treasury notes.

Consequently, to measure the exposure of a portfolio or position to rate changes, it is necessary to measure yield volatility. This requires an understanding of the fundamental principles of probability distributions. The measure of yield volatility is the standard deviation of yield changes. In Chapter 7, we show how to estimate yield volatility. As we will see, depending on the underlying assumptions, there could be a wide range for the yield volatility estimate.

A framework that ties together the price sensitivity of a bond position to rate changes and yield volatility is the *value-at-risk (VaR) framework*, discussed in Chapter 8.

Controlling Interest Rate Risk in Active Bond Portfolio Strategies

Bond portfolio strategies can be classified as either active portfolio strategies or structured portfolio strategies. Essential to all active strategies is specification of expectations about the factors that influence the performance of bonds. Structured portfolio strategies involve minimal expectational input. The goal is to design a portfolio so as to replicate the performance of a bond index or to satisfy predetermined liabilities.

There are four types of active strategies: rate expectations strategies, yield curve strategies, yield spread strategies, and individual bond selection strategies. Here we will explain how to control the interest rate risk for all but yield curve strategies. Measuring and controlling risk in yield curve strategies is the subject of later chapters.

Rate expectations strategies seek to capitalize on expectations about interest rate movements. A manager who believes that he or she can accurately forecast the future level of interest rates will alter the portfolio's sensitivity to interest rate changes. As duration is a measure of interest rate sensitivity, this involves increasing a portfolio's duration if interest rates are expected to fall and reducing duration if interest rates are expected to rise. For those managers whose benchmark is a bond index, this means increasing the portfolio duration relative to the index if interest rates are expected to fall and reducing it if interest rates are expected to rise. The degree to which the duration of the managed portfolio is permitted to diverge from that of the bond index may be limited by the client.

A portfolio's duration may be altered by swapping (or exchanging) bonds in the portfolio for new bonds that will achieve the target portfolio duration. Such swaps are commonly referred to as rate anticipation swaps. Alternatively, a more efficient means for altering the duration of a bond portfolio is to use interest rate futures contracts. As we explain in Chapter 9, buying futures increases a portfolio's duration, while selling futures decreases it.

Yield spread strategies involve positioning a portfolio to capitalize on expected changes in yield spreads between sectors of the bond market. Swapping (or exchanging) one bond for another when the manager believes that the prevailing yield spread between two bonds in the market is out of line with their historical yield spread, and that the yield spread will realign by the end of the investment horizon, are called *intermarket spread swaps*.

Individual security selection strategies involve identifying mispriced securities and taking a position in those securities so as to benefit when the market realigns. The most common strategy identifies an issue as undervalued because either (1) its yield is higher than that of comparably rated issues, or (2) its yield is expected to decline (and price therefore rise) because credit analysis indicates that its rating will improve.

A swap in which a manager exchanges one bond for another bond that is similar in terms of coupon, maturity, and credit quality, but offers a higher yield, is called a *substitution swap*. This swap depends on a capital market imperfection. Such situations sometimes exist in the bond market owing to temporary market imbalances and the fragmented nature of the non-Treasury bond market. The risk the manager faces in undertaking a substitution swap is that the bond purchased may not be truly identical to the bond for which it is exchanged.

What is critical in assessing yield spread and individual security selection strategies when an intermarket swap or substitution swap are being contemplated is to compare positions that have the same dollar duration. To understand why, consider two bonds, X and Y. Suppose that the price of bond X is 80 and has a modified duration of 5 while bond Y has a price of 90 and has a modified duration of 4. Since modified duration is the approximate percentage change per 100-basis-point change in yield, a 100-basis-point change in yield for bond X would change its price by about 5%. Based on a price of 80, its price will change by about \$4 per \$80 of market value. Thus, its dollar duration for a 100-basis-point change in yield is \$4 per \$80 of market value. Similarly, for bond Y, its dollar duration for a 100-basis-point change in yield per \$90 of market value can be determined. In this case it is \$3.6. So, if bonds X and Y are being considered as alternative investments in some strategy other than one based on anticipating interest rate movements, the amount of each bond in the strategy should be such that they will both have the same dollar duration.

To illustrate this, suppose that a manager owns \$10 million of par value of bond X, which has a market value of \$8 million. The dollar duration of bond X per 100-basis-point change in yield for the \$8 million market value is \$400,000. Suppose further that this manager is considering exchanging bond X that she owns in her portfolio for bond Y. If the manager wants to have the same interest rate exposure (i.e., dollar duration) for bond Y that she currently has for bond X, she will buy a market value amount of bond Y with the same dollar duration. If the manager purchased \$10 million of *par value* of bond Y and therefore \$9 million of *market value* of bond Y, the dollar value change per 100-basis-point change in yield would be only \$360,000. If, instead, the manager purchased \$10 million of *market value* of bond Y, the dollar duration per 100-basis-point change in yield would be \$400,000. Since bond Y is trading at 90, \$11.11 million of par value of bond Y must be purchased to keep the dollar duration of the position from bond Y the same as for bond X.

Mathematically, this problem can be expressed as follows:

$\$D_X$ = dollar duration per 100-basis-point change in yield for bond X for the market value of bond X held

MD_Y = modified duration for bond Y

MV_Y = market value of bond Y needed to obtain the same dollar duration as bond X

Then, the following equation sets the dollar duration for bond X equal to the dollar duration for bond Y:

$$\$D_X = (MD_Y/100)\ MV_Y$$

Solving for MV_Y,

$$MV_Y = \$D_X/(MD_Y/100)$$

Dividing by the price per \$1 of par value of bond Y gives the par value of Y that has an approximately equivalent dollar duration as bond X.

In our illustration, $\$D_X$ is \$400,000 and MD_Y is 4, then

$$MV_Y = \$400{,}000/(4/100) = \$10{,}000{,}000$$

Since the market value of bond Y is 90 per \$100 of par value, the price per \$1 of par value is 0.9. Dividing \$10 million by 0.9 indicates that the par value of bond Y that should be purchased is \$11.11 million.

Failure to adjust a portfolio repositioning based on some expected change in yield spread so as to hold the dollar duration the same means that the outcome of the portfolio will be affected by not only the expected change in the yield spread but also a change in the yield level. Thus, a manager would be making a conscious yield spread bet and possibly an undesired bet on the level of interest rates.

KEY POINTS

1. The price/yield relationship for an option-free bond is convex.

2. A property of an option-free bond is that for a small change in yield, the percentage price change is roughly the same whether the yield increases or decreases.

3. A property of an option-free bond is that for a large change in yield, the percentage price change is not the same for an increase in yield as it is for a decrease in yield.

4. A property of an option-free bond is that for a given change in basis points, the percentage price increase is greater than the percentage price decrease.

5. The coupon and maturity of an option-free bond affect its price volatility.

6. For a given term to maturity and initial yield, the lower the coupon rate the greater the price volatility of a bond.

7. For a given coupon rate and initial yield, the longer the term to maturity, the greater the price volatility.

8. For a given change in yield, price volatility is lower when yield levels in the market are high than when yield levels are low.

9. The percentage price change of a bond can be estimated by changing the yield by a small number of basis points and observing how the price changes.

10. Modified duration is the approximate percentage change in a bond's price for a 100-basis-point parallel shift in the yield curve assuming that the bond's cash flow does not change when the yield curve shifts.

11. Modified duration is the slope of a tangent line to the price/yield relationship.

12. The size of the interest rate shock is unimportant for approximating the duration of option-free bonds.

13. The dollar duration of a bond measures the dollar price change when the required yield changes.

14. Modified duration is not a useful measure of the price sensitivity for bonds with embedded options.

15. Effective duration is the approximate percentage price change of a bond for a 100-basis-point parallel shift in the yield curve allowing the cash flow to change in response to the change in yield.

16. The difference between modified duration and effective duration for bonds with embedded options can be significant.

17. The duration measure is only as good as the valuation model from which it is derived.

18. A portfolio's duration is obtained by calculating the weighted average of the durations of the bonds in the portfolio.

19. Empirical duration uses historical price series for MBS and data on Treasury yields to statistically estimate duration.

20. The estimate of a bond's price sensitivity based on duration can be improved by using a bond's convexity measure.

21. The convexity measure means nothing in isolation; it is the convexity adjustment that is important.

22. The price value of a basis point is the absolute value of the change in the price of a bond for a 1-basis-point change in yield.

23. The yield value of a price change is determined by calculating the difference between the yield to maturity if the bond's price was increased/decreased by one tick.

24. The greater the expected yield volatility, the greater the interest rate risk for a given duration and current value of the position.

25. A rate expectations strategy involves positioning the duration of a portfolio based on whether rates are expected to increase or decrease.

26. For a manager pursuing a rate expectations strategy, the portfolio duration relative to the bond index will be increased if interest rates are expected to fall and the duration will be reduced relative to the bond index if interest rates are expected to rise.

27. When contemplating an intermarket spread swap or a substitution swap, it is critical to keep the dollar duration of the portfolio constant.

CHAPTER 4

Measuring Yield Curve Risk

Duration is a useful metric for assessing a bond portfolio's sensitivity to a parallel shift in the reference yield curve (e.g., the Treasury yield curve). When the yield curve shift is not parallel, however, two bond portfolios with the same duration will not generally experience the same return performance. To evaluate differences in expected performance across portfolios, it is therefore necessary to quantify the price impact due to changes in the shape, as opposed to a parallel shift, of the yield curve. The purpose of this chapter is to discuss the various types of yield curve shifts, how different portfolios perform when the yield curve shifts, and how to measure a portfolio's exposure to yield curve risk.

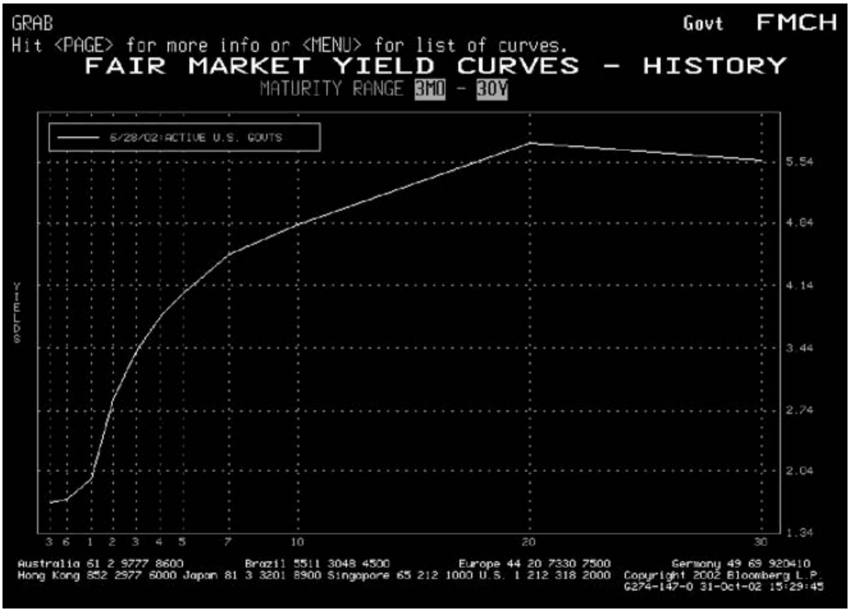
The objectives of this chapter are to:

1. Describe the types of shifts that have been observed for the yield curve.
2. Demonstrate why duration and convexity do not provide information about the interest rate risk of a portfolio if the yield curve does not shift in a parallel fashion.
3. Explain what tracking error is and how yield curve risk can be measured in terms of tracking error.
4. Describe the cash flow distribution analysis approach for measuring yield curve risk.
5. Explain key rate duration as a measure of yield curve risk.
6. Explain the slope elasticity measure of yield curve risk.
7. Explain the likely yield curve shift approach to managing yield curve risk.

TYPES OF YIELD CURVE SHIFTS

The yield curve has taken on multiple shapes over time. An upward sloping yield curve (also called a *positive* or *normal yield curve*) is one

EXHIBIT 4.1 Bloomberg Screen of Upward-Sloping Yield Curve



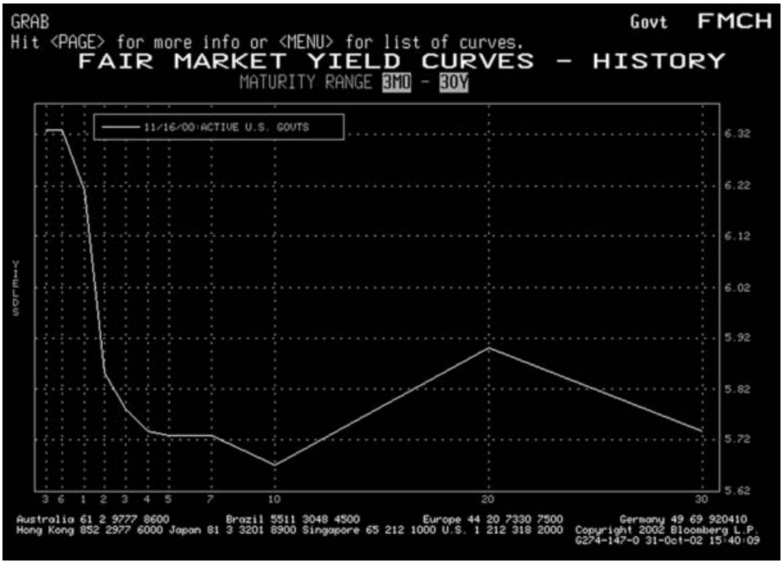
Source: Bloomberg Financial Markets

in which yield increases with term to maturity. Exhibit 4.1 shows a Bloomberg Fair Market Yield Curve (FMCH) screen for an upward sloping U.S. Treasury par curve on June 28, 2002. The yield curve exhibits this basic shape most of the time. Specifically, an upward sloping yield curve usually exhibits three common traits. First, the yield curve is very steeply sloped on the short end of the curve (maturities less than 5 years). Second, somewhere between the maturities of 5 and 10 years, a positively sloped yield curve starts bending over—this is called the "shoulder" of the yield curve. Third, the long end of the yield curve is usually relatively flat (maturities greater than 10 years).

While an upward sloping yield curve is by far the most common, three other shapes occasionally appear. A *downward sloping yield curve* (also called an *inverted yield curve*) is one in which the yield declines with maturity. Exhibit 4.2 shows a Bloomberg FMCH screen for a downward sloping U.S. Treasury par curve on November 16, 2000. For a *humped yield curve*, the yield increases with maturity initially and then subsequently declines with maturity.

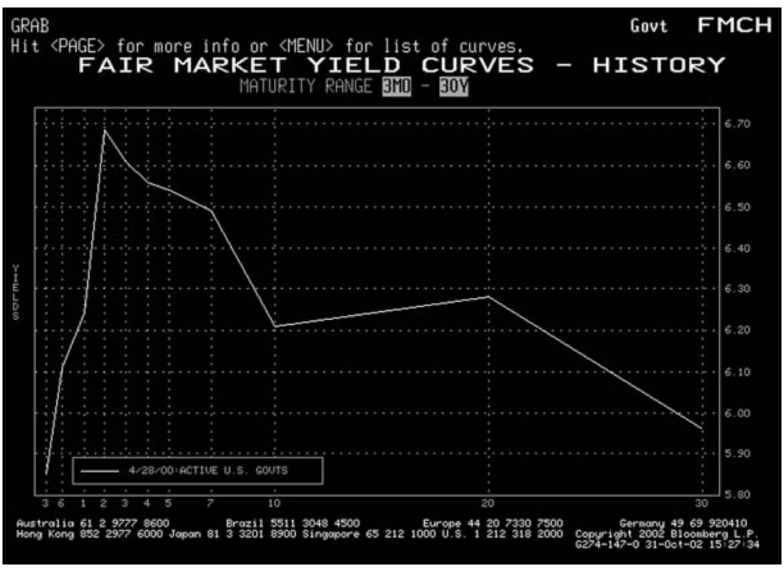
Exhibit 4.3 shows a Bloomberg FMCH screen for a humped U.S. Treasury par curve on April 26, 2000. There are two interesting points

EXHIBIT 4.2 Bloomberg Screen of Downward-Sloping Yield Curve



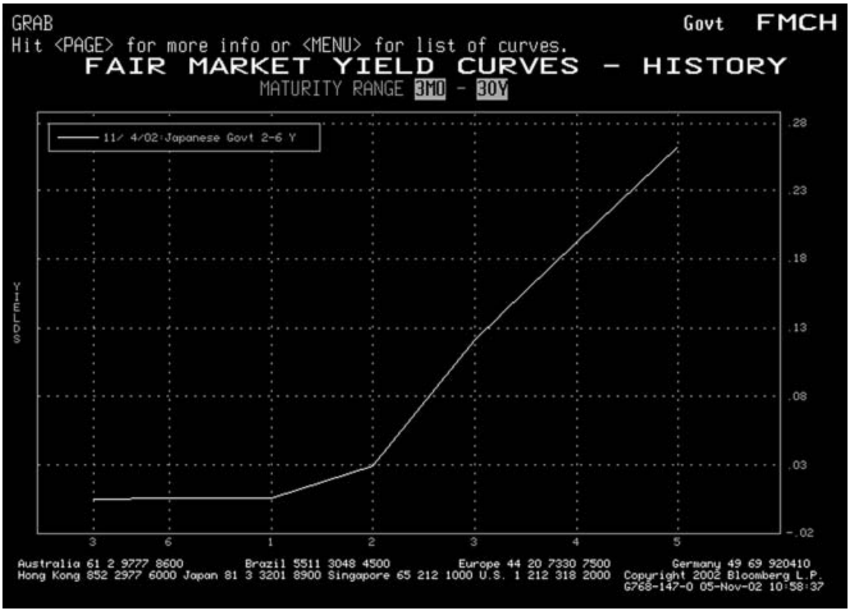
Source: Bloomberg Financial Markets

EXHIBIT 4.3 Bloomberg Screen of Humped Yield Curve



Source: Bloomberg Financial Markets

EXHIBIT 4.4 Bloomberg Screen of Flat Yield Curve



Source: Bloomberg Financial Markets

to note on humped yield curves. First, before a yield curve inverts, we normally observe a humped yield curve. Second, the short end of the yield curve is the last section of the yield curve to invert.

The last yield curve shape is a flat yield curve. As the name implies, the yield is approximately the same for each maturity. A flat yield curve is an elusive beast. Exhibit 4.4 shows a Bloomberg FMCH screen for a section of the Japanese governments yield curve (maturities 2 years through 6 years) on November 4, 2002. At first glance, this yield curve does not appear to be very flat. Look at the vertical axis on the left-hand side of the screen—it ranges between 0 and 28 basis points. This is about the flattest yield curve that one will ever observe.

The yield curve can change its shape on three different dimensions. The first dimension is a change in level (i.e., a parallel shift) such that the yield for all maturities changes by approximately the same number of basis points. A change in the slope of the yield curve (i.e., a flattening or steepening of the yield curve) is the second dimension and is usually measured as the spread between the yield on a longer maturity bond and the yield on a shorter maturity bond. The final dimension is a change in curvature which is also referred to a change in *humpedness* or a *butter-*

fly shift. Empirical evidence suggests that these changes in the yield curve's shape are not independent of each other. For example, a downward shift in level is typically accompanied with a steepening of the yield curve and an increase in curvature. Conversely, an upward shift in level tends to be associated with a flattening of the yield curve and a decrease in curvature.[1]

DURATION, CONVEXITY AND CHANGES IN THE YIELD CURVE'S SHAPE

As noted, a portfolio's duration indicates the approximate percentage price change for a parallel shift in the yield curve. Of course, this measure of interest rate risk assumes that all interest rates change by the same number of basis points. Two portfolios with similar durations may perform quite differently if the yield curve shifts in a nonparallel fashion. To illustrate this point, consider a butterfly trade.

A butterfly trade usually consists of three securities with different durations. In this illustration, we will employ three U.S. Treasury securities:

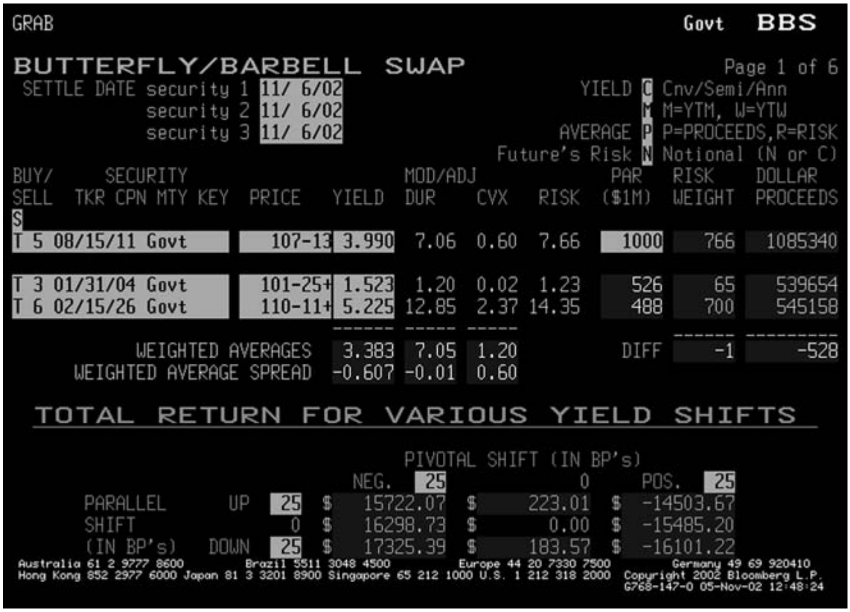
- The shortest duration instrument carries a coupon of 3% and matures on January 31, 2004.
- The intermediate duration instrument has a coupon of 5% and matures on August 15, 2011.
- The longest duration Treasury matures on February 15, 2026 and carries 6% coupon.

In a butterfly trade, the intermediate duration instrument is the butterfly's "body" (also called a "bullet portfolio") and the shorter and longer duration instruments are "wings" (also called a "barbell portfolio"). Accordingly, a portfolio manager engaging in a butterfly trade can buy the wings (barbell) and sell the body (bullet) or the reverse.

As an illustration, look at Exhibit 4.5 which shows the Bloomberg's Butterfly/Barbell Swap (BBS) screen. In this example, we will be using the three Treasuries described above to construct a butterfly trade where we will sell the bullet (i.e., the body) and simultaneously purchasing the

[1] For evidence on how changes in the yield curve's shape affect returns on U.S. Treasuries, see Frank J. Jones, "Yield Curve Strategies," *Journal of Fixed Income*, September 1991, pp. 43–61; Robert Litterman and José Scheinkman, "Common Factors Affecting Bond Returns," *Journal of Fixed Income*, June 1991, pp. 54–61; and Steven V. Mann and Pradipkumar Ramanlal, "The Relative Performance of Yield Curve Strategies," *Journal of Portfolio Management*, Summer 1997, pp. 64–70.

EXHIBIT 4.5 Bloomberg's Butterfly/Bullet Swap Screen



Source: Bloomberg Financial Markets

barbell (i.e., the wings). The bullet portfolio is a $1 million par value position in the 5% coupon Treasury that has a duration/convexity of 7.06/0.60. Next, the barbell portfolio is constructed by choosing the portfolio weights in the 3% coupon Treasury and the 6% coupon Treasury such that its duration is equal to the bullet's duration. As can be seen in the center of the screen, the weighted average duration of the barbell portfolio is 7.05. Note also that risk weights of the bullet and barbell also match. Recall from Chapter 3, Bloomberg's Risk is equal to the dollar value of a 1-basis-point change in yield multiplied by 100. Accordingly, "Risk" is tantamount to dollar duration.

While their durations are equal, there are two crucial differences between these two portfolios. First, the convexity measures of the two portfolios are not equal. Indeed, the weighted average convexity of the barbell is 1.20 compared to the bullet's convexity of 0.60. Second, the "yield" for the two portfolios is not the same. The yield for the bullet portfolio is simply the yield to maturity of the 5% coupon Treasury, 3.99%. The traditional (and incorrect) yield calculation for the barbell portfolio is found by taking a market-value weighted average of the yield to maturity of the two bonds included in the barbell portfolio.

EXHIBIT 4.6 Yield Curve Shifts for Bloomberg's Butterfly/Barbell Swap Screen

	Pivotal Shift in Basis Points								
	Negative Pivot			No Pivot			Positive Pivot		
	SD	ID	LD	SD	ID	LD	SD	ID	LD
Upward Parallel Shift in Basis Points	+50	+25	0	+25	+25	+25	0	+25	+50
No Shift	+25	0	–25	0	0	0	–25	0	+25
Downward Parallel Shift in Basis Points	0	–25	–50	–25	–25	–25	–50	–25	0

Using this method, the barbell's "yield" is 3.383%.[2] Thus, in order to obtain the benefits of the barbell's higher positive convexity, one must give up almost 61 basis points in yield over the duration-matched bullet portfolio. This yield difference is often called the "cost of convexity."

Now let's examine how our butterfly trade of buying the barbell and selling the bullet performs as the yield curve changes shape. This analysis is located at the bottom one-third of the screen under the heading "Total Return for Various Yield Shifts." The matrix tells the dollar gain or loss of our butterfly trade for nine different yield curve scenarios for an instantaneous shift in the yield curve. Exhibit 4.6 shows how the yields change for the shortest duration, intermediate duration, and long duration instruments. SD, ID, and LD are labels for the yield change on the shortest duration bond, the intermediate duration bond, and the longest duration bond. The default setting is parallel and pivotal shifts of positive and negative 25 basis points.[3]

Let us examine how our butterfly trade (buy barbell, sell bullet) performs when the yield curve shifts and changes slope. The middle column of the total return matrix in Exhibit 4.5 tells us the dollar change in the portfolio's value for an upward parallel shift of 25 basis points, no change in the yield curve, and a downward parallel shift of 25 basis

[2] To see why this approach is incorrect, consider a portfolio of ten Treasury coupon strips and the appropriate principal strip that exactly matches the cash flows of a 5-year Treasury note. Suppose further that the market value of the portfolio of strips is equal to the Treasury note's full price. The market value weighted average of the strip yields will almost never equal the yield to maturity of the 5-year Treasury note. The proper way to determine a portfolio yield is weight the yield to maturity of individual bonds using duration dollars. However, even properly calculated, yield is not a total return measure.

[3] For further details, see Kenwei Chong, "Bite the Bullet or Press the Barbell?" *Bloomberg Markets*, October 1998, pp. 79–84.

points. For an upward parallel shift of 25 basis points of the three bond yields, the barbell outperforms the bullet by a small amount, $223.01. Specifically, a long position in the shortest duration and longest duration bonds outperforms by $223.01 relative to a position in a same duration intermediate bond. For a downward parallel shift in yields, the barbell also outperforms the bullet by a small amount, $183.57. Both of these results are not surprising. Since these two portfolios have the same duration, they will experience about the same dollar price change for a parallel shift in yields with a slight advantage for the barbell owing to its higher positive convexity. Indeed, for a larger parallel shift in yields, the barbell should outperform the bullet by larger amounts. In other words, the barbell's higher positive convexity will have a higher payoff the larger the change in rates, all else equal.

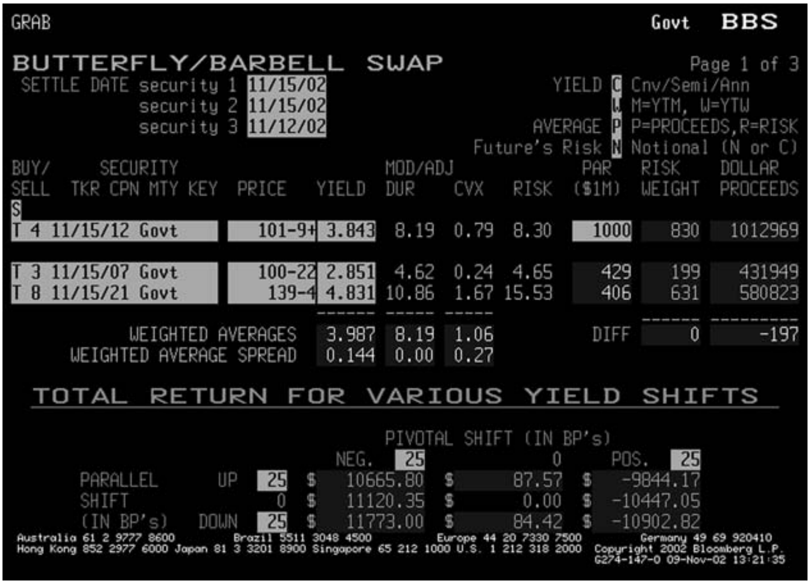
Now we turn our attention to a nonparallel shift in the yield curve. Let's combine a 25 basis point shift in the three yields with a "negative pivotal" shift. A negative pivotal shift adds an additional 25 basis points (in this example) to the yield of the shorter duration bond and subtracts an incremental 25-basis-points from the yield on the longer duration bond. Thus, an upward parallel shift of 25 basis points combined with a negative pivotal shift of 25 basis points results in net changes in the three bond yields of +50, +25, and 0 basis points, respectively. Conversely, a downward parallel shift of 25 basis points combined with a negative pivotal shift of 25 basis points results in changes in the three bond yields of 0, –25, and –50 basis points, respectively. Using the same logic, a negative pivotal shift without a parallel shift leads to a 25-basis-point increase in the yield of the shortest duration bond's yield and a 25-basis-point decrease in the yield of the longest duration bond's yield leaving the intermediate duration bond's yield unchanged.

All three of these nonparallel yield shifts flatten the yield curve. When this occurs, the barbell outperforms the bullet by a substantial amount. This result can be seen in the first column of the total return matrix of Exhibit 4.5. A flattening yield curve tends to favor barbells because the value of the longest duration bond is increasing and it contributes relatively more weight to the barbell's performance. Robin Grieves, in his empirical study of butterfly trades, finds that over short investment horizons that barbells outperform bullets when the yield curve flattens.[4]

Now let's steepen the yield curve using a positive pivotal shift—a shift that subtracts an incremental 25 basis points (in this example) from the yield on the shorter duration bond and adds an incremental 25

[4] See Robin Grieves, "Butterfly Trades," *Journal of Portfolio Management*, Fall 1999, pp. 87–96.

EXHIBIT 4.7 Bloomberg's Butterfly/Bullet Swap Screen



Source: Bloomberg Financial Markets

basis points to the yield on longer duration bond. The yield changes for the three bonds for a positive pivotal upward parallel shift, no shift, and a downward parallel shift are presented in the rightmost column in Exhibit 4.6. All three of these nonparallel yield shifts steepen the yield curve. When this occurs, the bullet outperforms the barbell by a substantial amount. This result appears in the third column of the total return matrix of Exhibit 4.5.

One additional statement about barbells and bullets is worthy of note. Namely, the statement made earlier that there is a tradeoff between convexity and yield does not necessarily arise in the market. This should not be surprising since the yield measure is not a good indicator of a portfolio's potential return. To illustrate, consider the bullet and barbell portfolios in Exhibit 4.7 which shows a Bloomberg BBS screen. As before, the durations of the two portfolios are equal but the barbell's convexity is higher the bullet's convexity (1.06 versus 0.79). Notice the weighted average yield for the barbell is also higher (3.987%) than the bullet portfolio's yield (3.843%).

In summary, looking at measures such as yield (either yield to maturity or some type of portfolio yield measure), duration or convexity tell us relatively little about the performance of bond portfolios over some investment horizon because performance depends on the magnitude of the change in yields and how the yield curve shifts. Accordingly, when one is constructing a portfolio to match the performance of a benchmark (e.g., a bond market index), care must be given to match the portfolio's risk profile to that of the benchmark. Matching by duration alone will not give any assurance about how either the portfolio or the benchmark index will respond to changes in the shape of the yield curve.

TRACKING ERROR AND YIELD CURVE RISK

When a portfolio manager's benchmark is a bond market index, risk is measured by the standard deviation of the return of the portfolio relative to the return of the benchmark. This risk measure is called *tracking error* and is computed as follows:

Step 1: Compute the total return for a portfolio for each period.

Step 2: Obtain the total return for the benchmark for each period.

Step 3: Obtain the difference between the values found in Step 1 and Step 2. The difference is referred to as the *active return*.

Step 4: Compute the standard deviation of the active returns. The resulting value is the tracking error.

The first panel of Exhibit 4.8 shows the calculation of the tracking error for a hypothetical portfolio assuming that the benchmark is the Lehman Brothers Aggregate Bond Index. The observations are monthly for 2001. The portfolio's monthly tracking error is 27.73 basis points.

The tracking error is unique to the benchmark used. For example, suppose the benchmark is the Salomon Smith Barney BIG Index. The monthly tracking error for the same portfolio is 31.15 basis points (see the second panel in Exhibit 4.8).

The tracking error measurement is in terms of the observation period. So, if monthly returns are used, the tracking error is a monthly tracking error. If weekly returns are used, the tracking error is a weekly tracking error. Tracking error is annualized as follows:

When observations are monthly:	Annual tracking error $= \text{Monthly tracking error} \times \sqrt{12}$
When observations are weekly:	Annual tracking error $= \text{Monthly tracking error} \times \sqrt{52}$

EXHIBIT 4.8 **Calculation of Tracking Error for a Hypothetical Portfolio: Benchmark is the Lehman Aggregate Bond Index and Benchmark Index Observation period = January 2001–December 2001**

Panel a: Benchmark index = Lehman Aggregate Bond Index

Month in 2001	Portfolio B's Return (%)	Benchmark Index Return (%)	Active Return (%)
Jan	1.59	1.64	–0.05
Feb	1.00	0.87	0.13
March	0.22	0.50	–0.28
April	–0.30	–0.42	0.12
May	0.95	0.60	0.35
June	0.15	0.38	–0.23
July	2.10	2.24	–0.14
Aug	1.10	1.15	–0.05
Sept	1.05	1.17	–0.12
Oct	1.95	2.09	–0.14
Nov	–0.98	–1.38	0.40
Dec	–0.42	–0.64	0.22
Sum			0.21
Mean			0.0175
Variance			0.0769
Standard Deviation = Tracking error			0.2773
Tracking error (in basis points) =			27.73

For example, when the Lehman index is used, the annual tracking error for the portfolio is

$$\text{Annual tracking error} = 27.73 \text{ basis points} \times \sqrt{12} = 96.06 \text{ basis points}$$

Backward-Looking versus Forward-Looking Tracking Error

We have just described how to calculate tracking error based on the actual active returns for a portfolio. Calculations computed for a portfolio based on a portfolio's actual active returns reflect the portfolio manager's decisions during the observation period with respect to the factors that affect tracking error. We call tracking error calculated from observed active returns for a portfolio *backward-looking tracking error*, *ex post tracking error,* or *actual tracking error.*

EXHIBIT 4.8 (Continued)
Panel b: Salomon Smith Barney BIG Index

Month in 2001	Portfolio B's Return (%)	Benchmark Index Return (%)	Active Return (%)
Jan	1.59	1.65	–0.06
Feb	1.00	0.89	0.11
March	0.22	0.52	–0.30
April	–0.30	–0.47	0.17
May	0.95	0.65	0.30
June	0.15	0.33	–0.18
July	2.10	2.31	–0.21
Aug	1.10	1.10	0.00
Sept	1.05	1.23	–0.18
Oct	1.95	2.02	–0.07
Nov	–0.98	–1.38	0.40
Dec	–0.42	–0.59	0.17
Sum			0.15
Mean			0.0125
Variance			0.0970
Standard Deviation = Tracking error			0.3115
Tracking error (in basis points) =			31.15

Notes:
Active return = Portfolio return – Benchmark Index return
Variance = Sum of the squares of the deviations from the mean/11
(Division by 11, which is number of observations minus 1)
Standard deviation = Tracking error = Square root of variance

A problem with using backward-looking tracking error in portfolio management is that it does not reflect the effect of current decisions by the portfolio manager on the future active returns and, for that reason, the future tracking error that may be realized. If, for example, the manager significantly changes the portfolio's duration or sector allocation, then the backward-looking tracking error, which is calculated using data from prior periods would not accurately reflect the current portfolio risks going forward. That is, the backward-looking tracking error will have little predictive value and can be misleading regarding portfolio risks going forward.

The portfolio manager needs a forward-looking estimate of tracking error to reflect the portfolio risk going forward. The way this is done in practice is by using the services of a commercial vendor or dealer firm that has modeled the factors that affect the tracking error associated with the bond market index that is the portfolio manager's benchmark. These models are called *multifactor risk models*. Given a manager's current portfolio holdings, the portfolio's current exposure to the various risk factors can be calculated and compared to the benchmark's exposures to the risk factors. Using the differential factor exposures and the risks of the factors, a *forward-looking tracking error* for the portfolio can be computed. This tracking error is also referred to as *predicted tracking error* and *ex ante tracking error*.

Given a forward-looking tracking error, a range for the future possible portfolio active return can be calculated assuming that the active returns are normally distributed. For example, assume the following:

Benchmark = Lehman Aggregate Bond Index
Expected return for Lehman Aggregate Bond Index = 10%
Forward-looking tracking error relative to Lehman Aggregate Bond Index = 100 basis points

Then from the properties of a normal distribution (see Chapter 5), we know that:

Number of standard deviations	Range for portfolio active return	Corresponding range for portfolio return	Probability
1	±1%	9%–11%	67%
2	±2%	8%–12%	95%
3	±3%	7%–13%	99%

It should be noted that there is no guarantee that the forward-looking tracking error at the start of, say, a year would exactly match the backward-looking tracking error calculated at the end of the year. There are two reasons for this. The first is that as the year progresses and changes are made to the composition of the portfolio, the forward-looking tracking error estimate would change to reflect the new exposure to risk factors. The second is that the accuracy of the forward-looking tracking error at the beginning of the year depends on the extent of the stability in the variances and correlations that commercial vendors use in their statistical models to estimate forward-looking tracking error. These problems notwithstanding, the average of forward looking tracking error estimates obtained at different times during the year will be reasonably close to the backward-looking tracking error estimate obtained at the end of the year.

Forward-Looking Tracking Error and Term Structure Risk

The forward-looking tracking error is useful in risk control and portfolio construction. The manager can immediately see the likely effect on tracking error of any intended change in the portfolio. Thus, scenario analysis can be performed by a portfolio manager to assess proposed portfolio strategies and eliminate those that would result in tracking error beyond a specified tolerance for risk.

The risk factors affecting the Lehman Brothers Aggregate Bond Index have been investigated by Lev Dynkin, Jay Hyman, and Wei Wu.[5] The risk factors are divided into two types: systematic risk factors and non-systematic risk factors. *Systematic risk factors* are forces that affect all securities in a certain category in the benchmark index. *Nonsystematic factor risk* is the risk that is not attributable to the systematic risk factors.

Systematic risk factors, in turn, are divided into two categories: term structure risk factors and nonterm structure risk factors. *Term structure risk factors* are risks associated with changes in the shape of the yield curve (level and shape changes). *Nonterm structure risk factors* include sector risk, quality risk, optionality risk, coupon risk, MBS sector risk, MBS volatility risk, and MBS prepayment risk. Our focus here is on term structure risk factors.

Give the risk factors associated with a benchmark, forward-looking tracking error can be estimated for a portfolio. The tracking error occurs because the portfolio constructed deviates from the exposures for the benchmark. For example, suppose that the duration for the Lehman Brothers Aggregate Bond Index is 4.3 and a portfolio manager constructs a portfolio with a duration of 4.9. Then there is different exposure to changes in the level of interest rates. This is one element of systematic term structure factor risk. What can be determined from the difference in durations is what the (forward-looking) tracking error due to the term structure risk factor will be.

The tracking error for a portfolio relative to a benchmark can be decomposed as follows:

- Tracking error due to systematic risk factors:
  - Tracking error due to term structure risk factor
  - Tracking error due to nonterm structure risk factors
- Tracking error due to nonsystematic risk factors

[5] Lev Dynkin, Jay Hyman, and Wei Wu, "Multi-Factor Risk Factors and Their Applications," in Frank J. Fabozzi (ed.), *Professional Perspectives on Fixed Income Portfolio Management*, Volume 2 (New Hope, PA: Frank J. Fabozzi Associates, 2001).

A portfolio manager provided with information about forwarding-looking tracking error for the current portfolio can quickly assess if (1) the risk exposure for the portfolio is one that is acceptable and (2) if the particular risk exposures are the ones that the manager wants. A client can in fact use forward-looking tracking error to communicate the degree of active portfolio management that it wants the portfolio manager it has retained to pursue.

Forward-looking tracking errors is obtained from vendors of multifactor risk models who, in turn, have constructed databases for the factors using statistical techniques. It is from this information that a portfolio manager can obtain the tracking error due to term structure risk factors.[6]

For example, Exhibit 4.9 shows a portfolio consisting of 57 bonds on September 30, 1998. Suppose that the benchmark for this portfolio is the Lehman Brothers Aggregate Bond Index. It can be shown that the tracking error for the portfolio is 52 basis points. Tracking error due to the term structure risk factor is the dominant source of total tracking error, 36.3 basis points.

EXHIBIT 4.9 **Portfolio Holdings (September 30, 1998)**

Issuer Name	Coup	Maturity	Moody	S&P	Par Val	%
BAKER HUGHES	8.000	05/15/04	A2	A	5,000	0.87
BOEING CO	6.350	06/15/03	Aa3	AA	10,000	1.58
COCA-COLA ENTERPRISES I	6.950	11/15/26	A3	A+	50,000	8.06
ELI LILLY CO	6.770	01/01/36	Aa3	AA	5,000	0.83
ENRON CORP	6.625	11/15/05	Baa2	BBB+	5,000	0.80
FEDERAL NATL MTG ASSN	5.625	03/15/01	Aaa+	AAA+	10,000	1.53
FEDERAL NATL MTG ASSN-G	7.400	07/01/04	Aaa+	AAA+	8,000	1.37
FHLM Gold 7-Years Balloon	6.000	04/01/26	Aaa+	AAA+	20,000	3.03
FHLM Gold Guar Single F.	6.500	08/01/08	Aaa+	AAA+	23,000	3.52
FHLM Gold Guar Single F.	7.000	01/01/28	Aaa+	AAA+	32,000	4.93
FHLM Gold Guar Single F.	6.500	02/01/28	Aaa+	AAA+	19,000	2.90
FIRST BANK SYSTEM	6.875	09/15/07	A2	A–	4,000	0.65
FLEET MORTGAGE GROUP	6.500	09/15/99	A2	A+	4,000	0.60
FNMA Conventional Long T.	8.000	05/01/21	Aaa+	AAA+	33,000	5.14
FNMA MTN	6.420	02/12/08	Aaa+	AAA+	8,000	1.23
FORD MOTOR CREDIT	7.500	01/15/03	A1	A	4,000	0.65
FORT JAMES CORP	6.875	09/15/07	Baa2	BBB–	4,000	0.63
GNMA I Single Family	9.500	10/01/19	Aaa+	AAA+	13,000	2.11
GNMA I Single Family	7.500	07/01/22	Aaa+	AAA+	30,000	4.66
GNMA I Single Family	6.500	02/01/28	Aaa+	AAA+	5,000	0.76
GTE CORP	9.375	12/01/00	Baa1	A	50,000	8.32

[6] In Chapter 15 we will see how tracking error due to quality risk is used in the analysis of credit risk.

EXHIBIT 4.9 (Continued)

Issuer Name	Coup	Maturity	Moody	S&P	Par Val	%
INT-AMERICAN DEV BANK-G	6.375	10/22/07	Aaa	AAA	6,000	1.00
INTL BUSINESS MACHINES	6.375	06/15/00	A1	A+	10,000	1.55
LEHMAN BROTHERS INC	7.125	07/15/02	Baa1	A	4,000	0.59
LOCKHEED MARTIN	6.550	05/15/99	A3	BBB+	10,000	1.53
MANITOBA PROV CANADA	8.875	09/15/21	A1	AA–	4,000	0.79
MCDONALDS CORP	5.950	01/15/08	Aa2	AA	4,000	0.63
MERRILL LYNCH & CO.-GLO	6.000	02/12/03	Aa3	AA–	5,000	0.76
NATIONSBANK CORP	5.750	03/15/01	Aa2	A+	3,000	0.45
NEW YORK TELEPHONE	9.375	07/15/31	A2	A+	5,000	0.86
NIKE INC	6.375	12/01/03	A1	A+	3,000	0.48
NORFOLK SOUTHERN CORP	7.800	05/15/27	Baa1	BBB+	4,000	0.71
NORWEST FINANCIAL INC.	6.125	08/01/03	Aa3	AA–	4,000	0.62
ONT PROV CANADA-GLOBA	7.375	01/27/03	Aa3	AA–	4,000	0.65
PUB SVC ELECTRIC + GAS	6.125	08/01/02	A3	A–	3,000	0.47
RAYTHEON CO	7.200	08/15/27	Baa1	BBB	8,000	1.31
RESOLUTION FUNDING CORP	8.125	10/15/19	Aaa+	AAA+	17,000	3.51
TIME WARNER ENT	8.375	03/15/23	Baa2	BBB–	5,000	0.90
ULTRAMAR DIAMOND SHAM	7.200	10/15/17	Baa2	BBB	4,000	0.63
US TREASURY BONDS	10.375	11/15/12	Aaa+	AAA+	10,000	2.17
US TREASURY BONDS	10.625	08/15/15	Aaa+	AAA+	14,000	3.43
US TREASURY BONDS	6.250	08/15/23	Aaa+	AAA+	30,000	5.14
US TREASURY NOTES	8.875	02/15/99	Aaa+	AAA+	9,000	1.38
US TREASURY NOTES	6.375	07/15/99	Aaa+	AAA+	4,000	0.61
US TREASURY NOTES	7.125	09/30/99	Aaa+	AAA+	17,000	2.59
US TREASURY NOTES	5.875	11/15/99	Aaa+	AAA+	17,000	2.62
US TREASURY NOTES	6.875	03/31/00	Aaa+	AAA+	8,000	1.23
US TREASURY NOTES	6.000	08/15/00	Aaa+	AAA+	11,000	1.70
US TREASURY NOTES	8.000	05/15/01	Aaa+	AAA+	9,000	1.50
US TREASURY NOTES	7.500	11/15/01	Aaa+	AAA+	10,000	1.67
US TREASURY NOTES	6.625	03/31/02	Aaa+	AAA+	6,000	0.96
US TREASURY NOTES	6.250	08/31/02	Aaa+	AAA+	10,000	1.60
US TREASURY NOTES	5.750	08/15/03	Aaa+	AAA+	1,000	0.16
US TREASURY NOTES	6.500	05/15/05	Aaa+	AAA+	1,000	0.17
US TREASURY NOTES	6.125	08/15/07	Aaa+	AAA+	1,000	0.17
WELLS FARGO + CO	6.875	04/01/06	A2	A–	5,000	0.80
WESTPAC BANKING CORP	7.875	10/15/02	A1	A+	3,000	0.49

Source: Exhibit 9 in Lev Dynkin, Jay Hyman, and Wei Wu, "Multi-Factor Risk Models and Their Applications," *Professional Perspectives on Fixed Income Portfolio Management*, Volume 3 (2001), pp. 101–145.

YIELD CURVE RISK MEASURES

The weakness of duration and convexity measures, as we have illustrated earlier in this chapter, is that it assumes that when yields along the yield curve change, each yield changes by the same number of basis points. Simply put, all bond yields are the same regardless of when the cash flows are delivered across time and changes in yields are perfectly correlated. In this section, we describe several approaches that have been used to measure the exposure of a portfolio or a position to changes in the yield curve. They include:

1. Cash flow distribution analysis versus a benchmark.
2. Key rate duration.
3. Slope elasticity measure.
4. Analysis of likely shifts in the yield curve.

Cash Flow Distribution Analysis versus a Benchmark

The most straightforward approach to assessing a portfolio's risk exposure to yield curve shifts is by looking at the distribution of the present value of the cash flows for the portfolio being managed versus its benchmark. The benchmark will be either a bond index or a liability structure. The steps are as follows:

Step 1: Determine the discrete time periods for the analysis. The shortest and longest time is determined by the shortest and longest cash flows for the portfolio and the benchmark. Each time period is referred to as a *cash flow vertex*.

Step 2: Compute the cash flows for the portfolio and the benchmark for each cash flow vertex.

Step 3: Compute the present value of the cash flows for the portfolio and the benchmark for each cash flow vertex. The spot rate used to compute the present value is the spot rate for the cash flow vertex. For example, if the cash flow vertex is year 5, the 5-year spot rate is used.

Step 4: Compute the duration contribution at each cash flow vertex for the portfolio and the benchmark.

Step 5: Compute the duration contribution as a percentage of duration for both the portfolio and the benchmark for each cash flow vertex.

Step 6: Compute the difference in the portfolio percentage and benchmark percentage computed in Step 5 for each cash flow vertex.

EXHIBIT 4.10 **Cash Flow Distribution Analysis Lehman Aggregate Index (as of 6/30/99)**

Time Period	Percent of PV	Duration Contribution	Percent of Duration
0	2.5	0.00	0.0
0.5	5.8	0.03	0.6
1	5.9	0.06	1.2
1.5	10.6	0.16	3.2
2	12.1	0.24	4.9
3	12.7	0.38	7.8
4	10.7	0.43	8.7
5	7.6	0.38	7.8
6	5.7	0.34	7.0
7	4.7	0.33	6.8
8	3.9	0.32	6.5
9	3.8	0.35	7.1
10	4.6	0.46	9.5
15	3.8	0.57	11.6
20	2.4	0.47	9.7
25	1.0	0.26	5.3
30	0.3	0.10	2.1
40	0.0	0.02	0.3
Total	[a]	4.88	100.0

Source: Exhibit 5 in Kenneth E. Volpert, "Managing Indexed and Enhanced Indexed Bond Portfolios," Chapter 4 in Frank J. Fabozzi (ed.), *Fixed Income Readings for the Chartered Financial Analyst Program* (New Hope, PA: Frank J. Fabozzi Associates, 2000).

[a] There was considerable rounding in reporting the values in this column. For this reason, the total does not add to 100%.

Steps 1–5 are shown in Exhibit 4.10 for the Lehman Brothers Aggregate Index as of June 30, 1999. It is the last column in the exhibit that a portfolio is compared against to identify exposure to yield curve risk.

The application is not straightforward because of the inclusion of bonds with embedded options and mortgage-backed and asset-backed securities. Suppose a bond is a 7-year bond that is callable in three years. The cash flows for this bond depends on the portfolio manager's assessment of the probability that it will be called in three years. For mortgage-backed and asset-backed securities, the cash flows depend on the prepayment assumption.

Another mechanical aspect of the process is the allocation of cash flows to the cash flow vertices when a cash flow is not exactly on a cash flow vertex date. For example, consider a bond whose coupon payment of $1 million is to be received 4.75 years from now and that there is a 4-year and 5-year cash flow vertex such as in Exhibit 4.10. How should the $1 million coupon payment be allocated? The procedure would be to allocate 25% to the 4-year cash flow vertex and 75% to the 5-year cash flow vertex.

Despite its simplicity, the cash flow distribution analysis is commonly used as a measure of yield curve risk for index fund managers according to Kenneth Volpert, Principal and Senior Portfolio Manager of The Vanguard Group.[7]

Key Rate Duration

One approach to measuring the sensitivity of a bond to changes in the shape of the yield curve is to change the yield for a particular maturity of the yield curve and determine the sensitivity of a security or portfolio to this change holding all other yields constant. The sensitivity of the bond's value to a particular change in yield is called *rate duration*. There is a rate duration for every point on the yield curve. Consequently, there is not one rate duration but a vector of rate durations representing each maturity on the yield curve. The total change in value if all rates move by the same number of basis points simultaneously is simply the duration of a security or portfolio to a parallel shift in rates.

The most popular version of this approach was developed by Thomas Ho in 1992.[8] This approach examines how changes in Treasury yields at different points on the spot curve affect the value of a bond portfolio. Ho's methodology has three basic steps. The first step is to select several key maturities or "key rates" of the spot rate curve. Ho's approach focuses on 11 key maturities on the spot rate curve. These rate durations are called *key rate durations*. The specific maturities on the spot rate curve for which a key rate duration is measured are 3 months, 1 year, 2 years, 3 years, 5 years, 7 years, 10 years, 15 years, 20 years, 25 years, and 30 years. However, in order to illustrate Ho's methodology, we will select only three key rates: 1 year, 10 years, and 30 years.

[7] Kenneth E. Volpert, "Managing Indexed and Enhanced Indexed Bond Portfolios," Chapter 4 in Frank J. Fabozzi (ed.), *Fixed Income Readings for the Chartered Financial Analyst Program* (New Hope, PA: Frank J. Fabozzi Associates, 2000), p. 91.

[8] Thomas S. Y. Ho, "Key Rate Durations: Measures of Interest Rate Risk," *The Journal of Fixed Income,* September 1992, pp. 29–44.

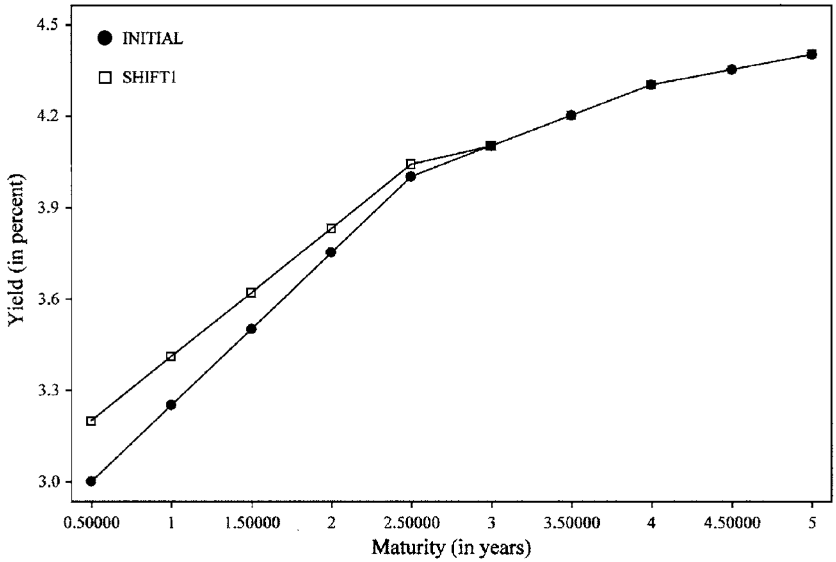
EXHIBIT 4.11 Valuation of 5-Year 6% Coupon Bond Using Spot Rates

Years	Period	Spot Rate (in percent)	Cash Flow (in dollars)	Present Value (in dollars)
0.5	1	3.00	3	2.96
1.0	2	3.25	3	2.90
1.5	3	3.50	3	2.85
2.0	4	3.75	3	2.79
2.5	5	4.00	3	2.72
3.0	6	4.10	3	2.66
3.5	7	4.20	3	2.59
4.0	8	4.30	3	2.53
4.5	9	4.35	3	2.47
5.0	10	4.40	103	82.86
			Total	107.32

The next step is to specify how other rates on the spot curve change in response to key rate changes. Ho's rule is that a key rate's effect on neighboring rates declines linearly and reaches zero at the adjacent key rates. For example, suppose the 10-year key rate increases by 40 basis points. All spot rates between 10 years and 30 years will increase but the amount each changes will be different and the magnitude of the change diminishes linearly. Specifically, there are 40 semiannual periods between 10 and 30 years. Each spot rate starting with 10.5 years increases by 1 basis point less than the spot rate to its immediate left (i.e., 39 basis points) and so forth. The 30-year rate which is the adjacent key rate is assumed to be unchanged. Thus, only one key rate changes at a time. Spot rates between 1 year and 10 years change in an analogous manner such that all rates change but by differing amounts. Changes in the 1-year key rate affect spot rates between 1 and 10 years, while spot rates 10 years and beyond are assumed to be unaffected by changes in the 1-year spot rate. In a similar vein, changes in the 30-year key rate affect all spot rates between 30 years and 10 years while spot rates shorter than 10 years are assumed to be unaffected by changes in the 30-year rate. This process is illustrated in Exhibit 4.11. Note that if we add the three rate changes together we obtain a parallel yield curve shift of 40 basis points.

The third and final step is to calculate the percentage change in the bond's portfolio value when each key rate and neighboring spot rates are changed. There will be as many key rate durations as there are pre-

EXHIBIT 4.12 Graph of the Initial Spot Curve and the Spot Curve After the 0.5-Year Key Rate Shift



selected key rates. Let's illustrate this process by calculating the key rate duration for a coupon bond. Our hypothetical 6% coupon bond has a maturity value of $100 and matures in five years. The bond delivers coupon payments semiannually. Valuation is accomplished by discounting each cash flow using the appropriate spot rate. The bond's current value is $107.32 and the process is illustrated in Exhibit 4.12. The initial hypothetical (and short) spot curve is contained in column (3).[9] The present values of each of the bond's cash flows is presented in the last column.

To compute the key rate duration of the 5-year bond, we must select some key rates. We assume the key rates are 0.5, 3, and 5 years. To compute the 0.5-year key rate duration, we shift the 0.5-year rate upwards by 20 basis points and adjust the neighboring spot rates between 0.5 and 3 years as described earlier. (The choice of 20 basis points is arbitrary.) Exhibit 4.13 is a graph of the initial spot curve and the spot curve after the 0.5-year key rate and neighboring rates are shifted. The next step is to com-

[9] The spot rates are annual rates and are reported as bond-equivalent yields. When present values are computed, we use the appropriate semiannual rates that are taken to be one half the annual rate.

EXHIBIT 4.13 Valuation of the 5-Year 6% Coupon Bond After 0.5-Year Key Rate and Neighboring Spot Rates Change

Years	Period	Spot Rate (in percent)	Cash Flow (in dollars)	Present Value (in dollars)
0.5	1	3.20	3	2.95
1.0	2	3.41	3	2.90
1.5	3	3.62	3	2.84
2.0	4	3.83	3	2.78
2.5	5	4.04	3	2.71
3.0	6	4.10	3	2.66
3.5	7	4.20	3	2.59
4.0	8	4.30	3	2.53
4.5	9	4.35	3	2.47
5.0	10	4.40	103	82.86
			Total	107.30

pute the bond's new value as a result of the shift. This calculation is shown in Exhibit 4.14. The bond's value subsequent to the shift is \$107.30. To estimate the 0.5-year key rate duration, we divide the percentage change in the bond's price as a result of the shift in the spot curve by the change in the 0.5-year key rate. Accordingly, we employ the following formula:

$$\text{Key rate duration} = \frac{P_0 - P_1}{P_0 \Delta y}$$

where

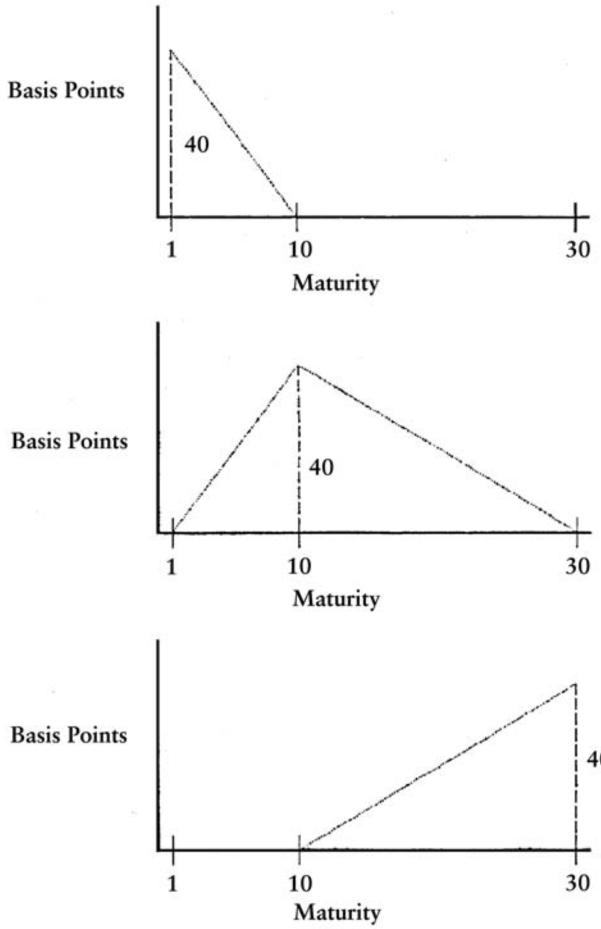
P_1 = the bond's value after the shift in the spot curve
P_0 = the bond's value using the initial spot curve
Δy = shift in the key rate (in decimal)

Substituting in numbers from our illustration presented above, we can compute the 0.5-year key rate duration as follows:

P_1 = 107.30
P_0 = 107.32
Δy = 0.002

$$\begin{aligned}\text{0.5-year key rate duration} &= \frac{107.32 - 107.30}{107.32(0.002)} \\ &= 0.0932\end{aligned}$$

EXHIBIT 4.14 **Graph of How Spot Rates Change when Key Rates Change**



To compute the 3-year key rate duration, we repeat this process. We shift the 3-year rate by 20 basis points and adjust the neighboring spot rates as described earlier. Exhibit 4.15 shows a graph of the initial spot curve and the spot curve after the 3-year key rate and neighboring rates are shifted. Note that in this case the only two spot rates that do not change are the 0.5-year and the 5-year key rates. Then, we compute the bond's new value as a result of the shift. The bond's post-shift value is \$107.25 and the calculation appears in Exhibit 4.16. Accordingly, the 3-year key rate duration is computed as follows:

EXHIBIT 4.15 Graph of the Initial Spot Curve and the Spot Curve After the 3-Year Key Rate Shift

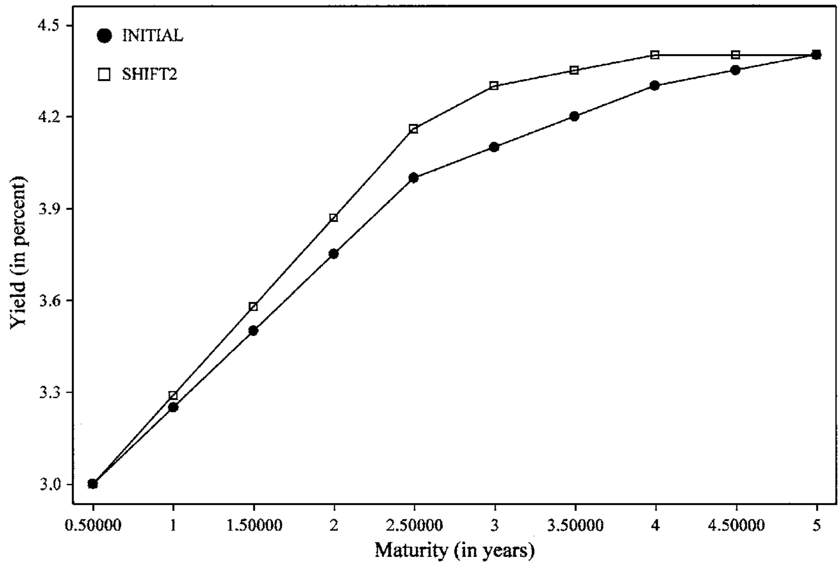
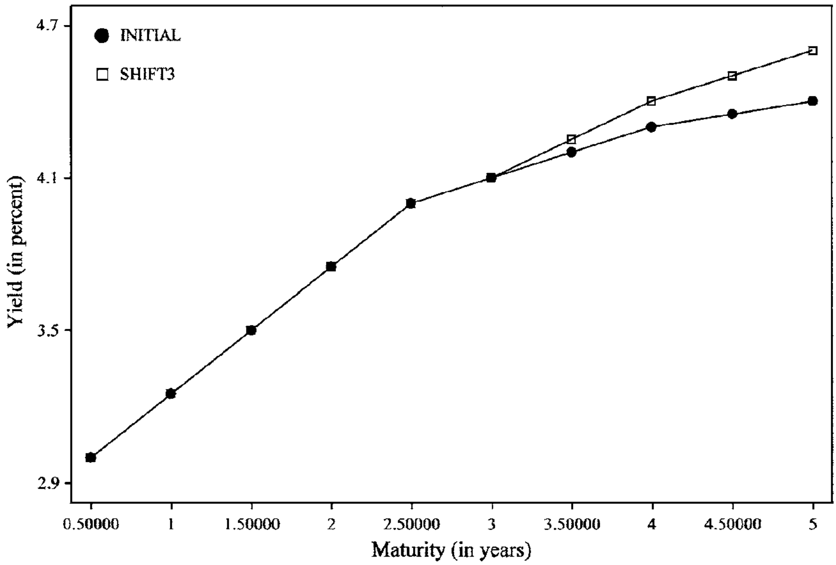


EXHIBIT 4.16 Valuation of the 5-Year 6% Coupon Bond After 3-Year Key Rate and Neighboring Spot Rates Change

Years	Period	Spot Rate (in Percent)	Cash Flow (in Dollars)	Present Value (in Dollars)
0.5	1	3.00	3	2.96
1.0	2	3.29	3	2.90
1.5	3	3.58	3	2.84
2.0	4	3.87	3	2.78
2.5	5	4.16	3	2.71
3.0	6	4.30	3	2.64
3.5	7	4.35	3	2.58
4.0	8	4.40	3	2.52
4.5	9	4.40	3	2.47
5.0	10	4.40	103	82.86
			Total	107.25

EXHIBIT 4.17 **Graph of the Initial Spot Curve and the Spot Curve After the 5-Year Key Rate Shift**



$$\text{3-year key rate duration} = \frac{107.32 - 107.25}{107.32(0.002)}$$
$$= 0.3261$$

The final step is to compute the 5-year key duration. We shift the 5 year rate by 20 basis points and adjust the neighboring spot rates. Exhibit 4.17 presents a graph of the initial spot curve and the spot curve after the 5-year key rate and neighboring rates are shifted. The bond's post-shift value is $106.48 and the calculation appears in Exhibit 4.18. Accordingly, the 5-year key rate duration is computed as follows:

$$\text{5-year key rate duration} = \frac{107.32 - 106.48}{107.32(0.002)}$$
$$= 3.9135$$

What information can be gleaned from these key rate durations? Each key rate duration by itself means relatively little. However, the dis-

EXHIBIT 4.18 Valuation of the 5-Year 6% Coupon Bond After 5-Year Key Rate and Neighboring Spot Rates Change

Years	Period	Spot Rate (in percent)	Cash Flow (in dollars)	Present Value (in dollars)
0.5	1	3.00	3	2.96
1.0	2	3.25	3	2.90
1.5	3	3.50	3	2.85
2.0	4	3.75	3	2.79
2.5	5	4.00	3	2.72
3.0	6	4.10	3	2.66
3.5	7	4.25	3	2.59
4.0	8	4.40	3	2.52
4.5	9	4.50	3	2.46
5.0	10	4.60	103	82.05
			Total	106.48

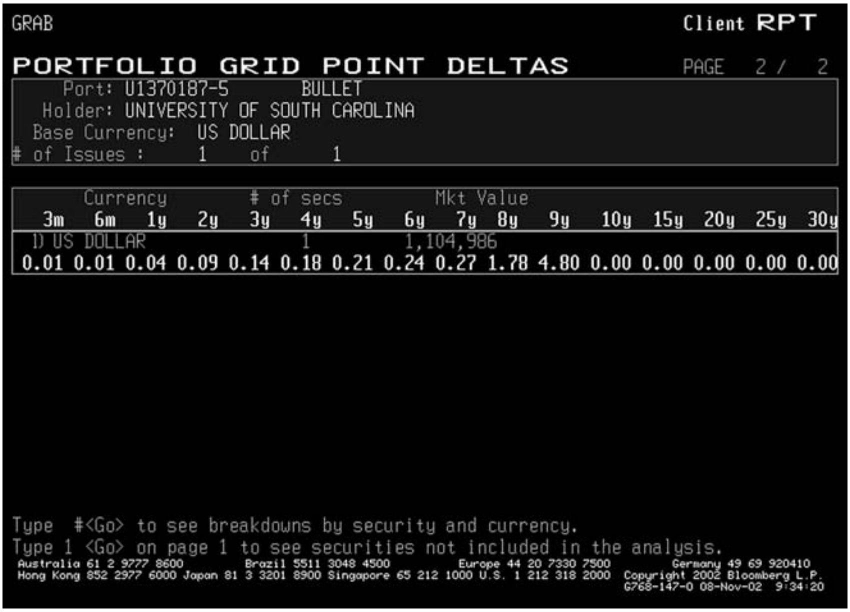
tribution of the bond's key rate durations helps us assess its exposure to yield curve risk. Intuitively, the sum of the key rate durations is approximately equal to a bond's duration.[10] As a result, it is useful to think of a set of key rate durations as a decomposition of duration into sensitivities to various portions of the yield curve. In our illustration, it is not surprising that the lion's share of the yield curve risk exposure of the coupon bond in our illustration is due to the bond's terminal cash flow, so the 5-year key rate duration is the largest of the three. Simply put, the 5-year bond's value is more sensitive to movements in longer spot rates and less sensitive to movements in shorter spot rates.

Key rate durations are most useful when comparing two (or more) bond portfolios that have approximately the same duration. If the spot curve is flat and experiences a parallel shift, these two bond portfolios can be expected to experience approximately the same percentage change in value. However, the performance of the two portfolios will generally not be the same for a nonparallel shift in the spot curve. The key rate duration profile of each portfolio will give the portfolio manager some clues about the relative performance of the two portfolios when the yield curve changes shape and slope.

As an illustration of the information that can be gleaned from key rate durations, let's employ the bullet and barbell portfolios constructed

[10] The reason it is only approximate is because modified duration assumes a flat yield curve whereas key rate duration takes the spot curve as given.

EXHIBIT 4.19 Bloomberg's Screen of Key Rate Durations for the Bullet Portfolio

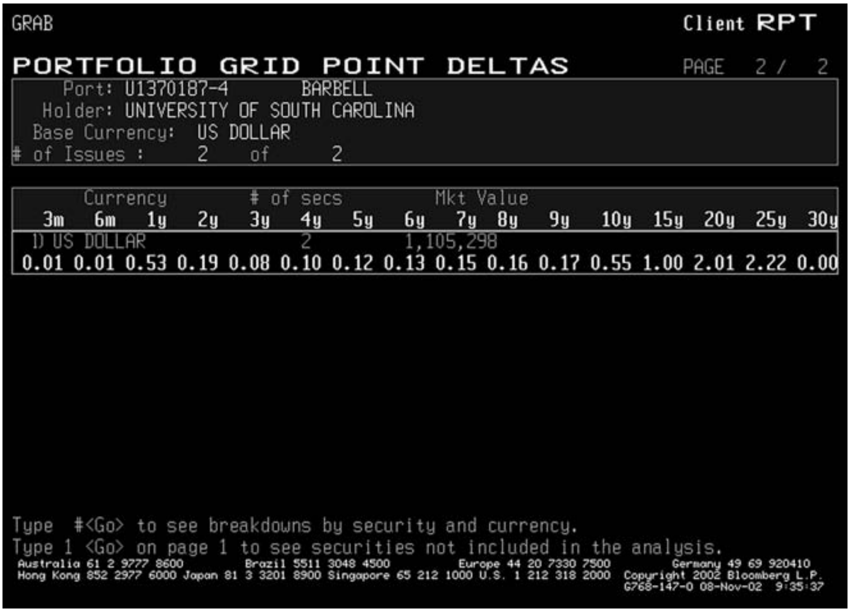


Source: Bloomberg Financial Markets

earlier. Recall, the bullet portfolio is a \$1 million par value position in the 5% coupon Treasury maturing in August 2011 that has a duration of 7.06. Conversely, the barbell portfolio is constructed by choosing the portfolio weights in the 3% coupon Treasury maturing in January 2004 and the 6% coupon Treasury maturing February 2026 such that its duration is equal to the bullet's duration.

Exhibit 4.19 shows a Bloomberg screen of key rate durations for the bullet portfolio. The key rate durations are called "Portfolio Grid Point Deltas." Bloomberg allows the user to chose up to 15 key rates. Accordingly, there are 15 key rate durations in this illustration. To calculate the key rate duration, each key rate is changed by 10 basis points (the default) and neighboring rates on the curve are adjusted as described earlier. The set of key rate durations indicates how interest rate risk for a particular portfolio is allocated across the yield curve. For the bullet portfolio, interest rate risk is concentrated in 8- and 9-year maturities. Thus, the bullet portfolio is most sensitive to movements in the middle of the curve and less sensitive to movements in the short and long end of the curve.

EXHIBIT 4.20 Bloomberg's Screen of Key Rate Durations for the Barbell Portfolio



Source: Bloomberg Financial Markets

Exhibit 4.20 shows a Bloomberg screen of key rate durations for the barbell portfolio. For this portfolio, interest rate risk is concentrated in the short and long end of the yield curve. Furthermore, it is relatively insensitive to the movement of yields for the intermediate maturities. This finding is consistent with the results of our barbell and bullet simulation. Namely, the barbell is very sensitive to a flattening yield curve.

Slope Elasticity Measure

The *slope elasticity measure*, introduced by Schumacher, Dektar, and Fabozzi for managing the yield curve risk of portfolios of collateralized mortgage obligation bonds, also looks at the sensitivity of a position or portfolio to changes in the slope of the yield curve.[11] Schumacher, Dektar, and Fabozzi define the yield curve slope as the spread between the 30-year on-the-run Treasury yield and the 3-month Treasury bill yield

[11] Michael P. Schumacher, Daniel C. Dektar, and Frank J. Fabozzi, "Yield Curve Risk of CMO Bonds," in Frank J. Fabozzi (ed.), *CMO Portfolio Management* (New Hope, PA: Frank J. Fabozzi Associates, 1994). An adaptation of this work appears as Chapter 14 in this book.

(i.e., basically the longest and the shortest points on the Treasury yield curve). They find that while this is not a perfect definition, it captures most of the effect of changes in yield curve slope.

They then define changes in the yield curve as follows: Half of any basis point change in the yield curve slope results from a change in the 3-month yield and half from a change in the 30-year yield. For example, with a 200-basis-point steepening of the yield curve, the assumption is that 100 basis points of that steepening come from a rise in the 30-year yield, and another 100 basis points come from a fall in the 3-month yield.

The sensitivity of a bond's price to changes in the yield curve is simply its slope elasticity. They define slope elasticity as the approximate negative percentage change in a bond's price resulting from a 100 basis point change in the slope of the curve. Slope elasticity is calculated as follows: increase and decrease the yield curve slope, calculate the price change for these two scenarios after adjusting for the price effect of a change in the level of yields, and compare the prices to the initial price. More specifically, the slope elasticity for each scenario is calculated as follows:

$$\frac{\text{Price effect of a change in slope/Base price}}{\text{Change in yield curve slope}}$$

The slope elasticity is then the average of the slope elasticity for the two scenarios.

A bond or bond portfolio that benefits when the yield curve flattens is said to have *positive slope elasticity*; a bond or a bond portfolio that benefits when the yield curve steepens is said to have *negative slope elasticity*. The definition of yield curve risk follows from that of slope elasticity. It is defined as the exposure of the bond to changes in the slope of the yield curve.

In Chapter 14, we show to use the slope elasticity methodology for controlling the yield curve risk of a mortgage-backed securities portfolio.

Analysis of Likely Yield Curve Shifts

While key rate duration is a useful measure for identifying the exposure of a portfolio to different potential shifts in the yield curve, it is difficult to employ this approach to yield curve risk in hedging a portfolio. An alternative approach is to investigate how yield curves have changed historically and incorporate typical yield curve change scenarios into the hedging process. This approach of using likely yield curve changes

EXHIBIT 4.21 Typical Monthly Yield Curve Shifts (bps)

	Level		Change due to	
Years	Up	Down	Flattening	Steepening
2	23.0	–23.0	17.2	–17.2
5	25.8	–25.8	11.2	–11.2
10	24.3	–24.3	3.4	–3.4

Source: Kenneth Dunn, Roberto Sella, and Frank J. Fabozzi, "Hedging Mortgage Securities," forthcoming in Frank J. Fabozzi (ed.), *Fixed Income Readings for the Chartered Financial Analyst Program: Second Edition* (New Hope, PA: Frank J. Fabozzi Associates, 2004).

obtained from principal component analysis has been suggested by Richard and Gord,[12] Golub and Tilman,[13] and Axel and Vankudre.[14]

Empirically, studies have found that yield curve changes are not parallel. Rather, when the level of interest rates changes, studies have found that short-term rates move more than longer-term rates. Some firms develop their own proprietary models that decompose historical movements in the rate changes of Treasury strips with different maturities in order to analyze typical or likely rate movements. The statistical technique used to decompose rate movements is principal component analysis.

Most empirical studies, published and proprietary, find that more than 95% of historical movements in rate changes can be explained by changes in (1) the overall level of interest rates and (2) twists in the yield curve (i.e., steepening and flattening). For example, Miller Anderson & Sherrerd's proprietary model of the movement of monthly Treasury strip rates finds the "typical" monthly rate change in basis points for three maturities as of early 2003 based on historical returns shown in Exhibit 4.21.

"Typical" means one standard deviation in the change in the monthly rate. The last two columns in Exhibit 4.21 indicate the change in the monthly rate found by the principal component analysis that is due to a flattening or steepening of the yield curve. From Exhibit 4.21,

[12] Scott F. Richard and Benjamin J. Gord, "Measuring and Managing Interest-Rate Risk," Chapter 2 in Frank J. Fabozzi (ed.), *Managing Fixed Income Portfolios* (Hoboken, NJ: John Wiley & Sons, 1997).
[13] Bennett W. Golub and Leo M. Tilman, "Measuring Plausibility of Hypothetical Interest Rate Shocks," Chapter 6 in *Managing Fixed Income Portfolios*.
[14] Ralph Axel and Prashant Vankudre, "Managing the Yield Curve with Principal Component Analysis," *Professional Perspectives on Fixed Income Portfolio Management*, Volume 3 (2000), pp. 37–49.

the impact on the yield curve for a typical rise in the overall level of interest rates and a flattening of the yield curve are found as follows.

To find the typical change in the slope of the 10-year–2-year, the difference between the 17.2 basis points for the 2-year and the 3.4 basis points is computed. The difference of 13.8 basis points means that the typical monthly flattening is 13.8 basis points. The typical monthly steepening is 13.8 basis points.

KEY POINTS

1. Four shapes have been observed for the Treasury yield curve: upward sloping, inverted, flat, and humped.

2. Historically, the types of yield curve shifts that have been observed are a parallel shift, a change in slope of the yield curve, and a change in the curvature of the yield curve.

3. A parallel shift in the yield curve means that the yield for all maturities change by the same number of basis points.

4. When using a portfolio's duration and convexity to measure the exposure to interest rates, it is assumed that the yield curve shifts in a parallel fashion.

5. For a nonparallel shift in the yield curve, duration and convexity do not provide adequate information about the interest rate risk exposure.

6. Exposure of a portfolio or position to a shift in the yield curve is called *yield curve risk*.

7. Tracking error measures the standard deviation of the active returns of a portfolio relative to a benchmark.

8. Backward-looking tracking error measures the tracking error based on actual active returns; forward-looking tracking error measures the potential tracking error of a portfolio.

9. One of the systematic factors that affects forward-looking tracking error is term structure factor risk and it is this risk that measures a bond portfolio's exposure to yield curve risk.

10. A simple approach to measuring yield curve risk, an approach commonly used by index managers, is an analysis of the cash flow distribution of a portfolio relative to a benchmark.

11. Key rate duration measures how changes in Treasury yields at different points on the spot rate curve affect the value of a bond.

12. Slope elasticity looks at the sensitivity of a position or portfolio to changes in the slope of the yield curve and is defined as the approximate negative percentage change in a bond's price resulting from a 100-basis-point change in the slope of the curve.

13. With slope elasticity, changes in the yield curve are defined as follows: Half of any basis point change in the yield curve slope results from a change in the 3-month yield and half from a change in the 30-year yield.

14. Using principal component analysis, a portfolio manager can determine likely yield curve shifts and use those shifts to assess the exposure of a portfolio to yield curve risk.

CHAPTER 5

Probability Distributions and Their Properties

Several concepts from probability theory and statistics are essential for measuring a portfolio's or a position's exposure to interest rate and credit risk. In this chapter, we introduce the concept of a probability distribution and examine its two key parameters: expected value and standard deviation. We will also describe one particular probability distribution known as the normal distribution. The normal distribution plays a critical role in statistical inference and many phenomena (financial and otherwise) generate random variables with probability distributions that are well approximated by the normal probability distribution. For example, properties of the normal distribution will be indispensable when we introduce the value-at-risk framework later in the book.

The objectives of this chapter are to:

1. Explain what is meant by a random variable.
2. Describe what a probability distribution is.
3. Explain how to calculate the variance and standard deviation.
3. Describe the fundamental properties of the normal probability distribution.
4. Demonstrate several applications of the normal probability distribution.
5. Describe what a skewed distribution is.
6. Describe how a probability distribution can be obtained using Monte Carlo simulation.

RANDOM VARIABLE AND PROBABILITY DISTRIBUTION

A *random* or *stochastic* variable is a variable that can take on many possible values and is unknown prior to the time it is observed. Moreover, each possible value can be assigned a probability of observing that particular outcome. A *probability distribution* or *probability function* is a graphical description of all the values that the random variable can take on and the probability associated with each.[1]

Let us employ some notation to develop these concepts. If we let X denote a random variable, then we use a subscript to denote particular values of the random variable. For example, X_i refers to the ith value for the random variable X. The probability of a particular value for the random variable X is typically denoted by stating the specific value, $P(X =$ specific value$)$, or by using the more compact subscript notation, $P(X_i)$.

To illustrate these concepts, consider a long position in a Treasury strip that matures on August 15, 2022. Suppose that the par value of the position is \$50 million. On the settlement date of August 28, 2002, the strip is priced to yield 5.5960%. The price is \$33.225 per \$100 of par value, so the full price of \$50 million face value position is \$16,612,500. Suppose a portfolio manager is concerned with the potential loss that would be realized from this position two weeks hence. The loss will depend on the yield on this zero-coupon bond on September 11, 2002.

Exhibit 5.1 shows the nine possible yields that the manager believes can occur two weeks hence. The exhibit shows the probability of realizing each possible yield at that time. The random variable in this illustration is the yield two weeks hence and it can take on nine possible outcomes. This is the probability distribution for the yield. Notice the sum of the probabilities is one.

Rather than defining the random variable as the yield, the random variable could just as easily be the profit/loss of the position over the next two weeks. There is one-to-one correspondence between each yield and a profit/loss. This profit/loss is shown in the last column of Exhibit 5.1. The probability distribution for the profit/loss is the same as the probability distribution for the yield. For example, the probability that the loss will be \$284,594 is 15%. The probability of obtaining a particular outcome is called a *marginal probability*. If X denotes the profit/loss, then $P(X = \$284{,}594)$ or $P(X_7)$ is 15%. The probability that there will be loss on the position is the probability of a yield higher than 5.5960%. In our illustration it is 39% and this is called the *cumulative probability*.

[1] It is also called a *probability density function*.

EXHIBIT 5.1 **Probability for Yield Distribution and Profit/Loss Distribution in Two Weeks for a Position in a Treasury Strip that Matures on August 15, 2022**
Purchase price: $33.224628
Par position: $50,000,000
Dollar position: $16,612,314

		Two weeks from now on September 11, 2002			
	Yield (%)	Probability (%)	Bond Price ($)	Market Value ($)	Profit/Loss ($)
1	5.1960	1.5	35.98146	17,990,730	1,378,416
2	5.2960	10.0	35.28953	17,644,765	1,032,451
3	5.3960	12.5	34.61124	17,305,620	693,306
4	5.4960	15.0	33.94631	16,973,155	360,841
5	5.5960	22.0	33.29446	16,647,230	34,916
6	5.6960	15.0	32.65544	16,327,720	–284,594
7	5.7960	12.5	32.02899	16,014,495	–597,819
8	5.8960	10.0	31.41485	15,707,425	–904,889
9	5.9960	1.5	30.81278	15,405,390	–1,205,924
	Total	100.0			

STATISTICAL MEASURES OF A PROBABILITY DISTRIBUTION

Various measures are used to summarize the probability distribution of a random variable. The two most often used measures are the expected value and the variance (or standard deviation).

Expected Value

The *expected value* of a probability distribution is the weighted average of the distribution. The weights in this case are the probabilities associated with the random variable X. The expected value of a random variable is denoted by $E(X)$ and is computed as follows:

$$E(X) = P_1X_1 + P_2X_2 + ... + P_nX_n$$

where P_i is the probability associated with the outcome X_i.

Exhibit 5.2 shows how to calculate the expected value for the profit/loss of $50 million par value position in a U.S. Treasury strip that mature on August 15, 2022 whose probability distribution is shown in Exhibit 5.1. The expected value of the profit/loss at the end of the anticipated two-week holding period is $46,398.03.

EXHIBIT 5.2 Calculation of Expected Value

	Yield (%)	Probability (%)	Profit/Loss ($)	Probability × Profit/Loss ($)
1	5.1960	1.5	1,378,416	20,676.24
2	5.2960	10.0	1,032,451	103,245.10
3	5.3960	12.5	693,306	86,663.25
4	5.4960	15.0	360,841	54,126.15
5	5.5960	22.0	34,916	7,681.52
6	5.6960	15.0	–284,594	–42,689.10
7	5.7960	12.5	–597,819	–74,727.38
8	5.8960	10.0	–904,889	–90,488.90
9	5.9960	1.5	–1,205,924	–18,088.86
			Expected Value	46,398.03

Variance

A manager is interested not only in the expected value of a probability distribution but also in the dispersion of the random variable around the expected value. A measure of dispersion of the probability distribution is the *variance* of the distribution. The variance of a random variable X, denoted by var(X), is computed from the following formula:

$$\text{var}(X) = [X_1 - E(X)]^2 P_1 + [X_2 - E(X)]^2 P_2 + ... + [X_n - E(X)]^2 P_n$$

Notice that the variance is simply a weighted average of the deviations of each possible outcome from the expected value, where the weight is the probability of an outcome occurring. The greater the variance, the greater the distribution of the possible outcomes for the random variable. The reason that the deviations from the expected value are squared is to avoid outcomes above and below the expected value from cancelling each other out.

The problem with using the variance as a measure of dispersion is that it is in terms of squared units of the random variable (e.g., squared dollars, squared percent, etc.) Consequently, the square root of the variance, called the *standard deviation*, is often used instead because it is a more easily interpretable measure of dispersion since it is in the same units as the mean. Mathematically this can be expressed as follows:

$$std(X) = \sqrt{var(X)}$$

where std(X) denotes the standard deviation of the random variable X.

EXHIBIT 5.3 **Calculation of Variance and Standard Deviation**

	Yield (%)	Probability (%)	Profit/Loss ($)	Expected Value ($)	(Profit/Loss – EV)^2 × Probability
1	5.1960	1.5	1,378,416	46,398.03	26,614,078,286
2	5.2960	10.0	1,032,451	46,398.03	97,230,046,951
3	5.3960	12.5	693,306	46,398.03	52,311,241,015
4	5.4960	15.0	360,841	46,398.03	14,831,157,679
5	5.5960	22.0	34,916	46,398.03	29,004,118
6	5.6960	15.0	–284,594	46,398.03	16,433,358,092
7	5.7960	12.5	–597,819	46,398.03	51,876,946,912
8	5.8960	10.0	–904,889	46,398.03	90,494,700,393
9	5.9960	1.5	–1,205,924	46,398.03	23,524,656,815
				Variance	373,345,145,992

Note: Standard deviation = $[373{,}345{,}145{,}992]^{1/2}$ = 610,019.80

Exhibit 5.3 shows how to calculate the variance for the profit/loss of the U.S. Treasury strip position whose probability distribution is shown in Exhibit 5.1.

Discrete versus Continuous Probability Distributions

A probability distribution can be classified according to the values that a random variable can realize. When the value of the random variable can only take on specific values, then the probability distribution is referred to as a *discrete probability distribution*. For example, in our illustration, we assumed only nine specific values for the random variable. Hence, to this point we have been working with a discrete probability distribution. If, instead, the random variable can take on any possible value within the range of outcomes, then the probability distribution is said to a *continuous probability distribution*.

When a random variable is either the price, yield, or return on a financial asset, the distribution can be assumed to be a continuous probability distribution. This means that it is possible to obtain, for example, a price of 95.43231 or 109.34872 and any value in between. In practice, we know that financial assets are not quoted in such a way. Nevertheless, there is no loss in describing the distribution as continuous. However, what is important in using a continuous distribution is that in moving from one price to the next, there is no major jump. For example, if the price declines from 95.14 to 70.50, it is assumed that there are trades that are executed at prices at small increments below 95.14

before getting to 70.50. In contrast, if the price can just "jump" from 95.14 to 70.50, then the distribution is referred to as a *jump process*.

NORMAL PROBABILITY DISTRIBUTION

In many applications involving probability distributions, it is assumed that the underlying probability distribution is a *normal distribution*. An example of a normal distribution is shown in Exhibit 5.4. A normal distribution is an example of a continuous probability distribution.

The area under the normal distribution or normal curve between any two points on the horizontal axis is the probability of obtaining a value between those two values. For example, the probability of realizing a value for the random variable X that is between X_1 and X_2 in Exhibit 5.4 is shown by the shaded area. Mathematically, the probability of realizing a value for X between these two variables can be written as follows:

$$P(X_1 < X < X_2)$$

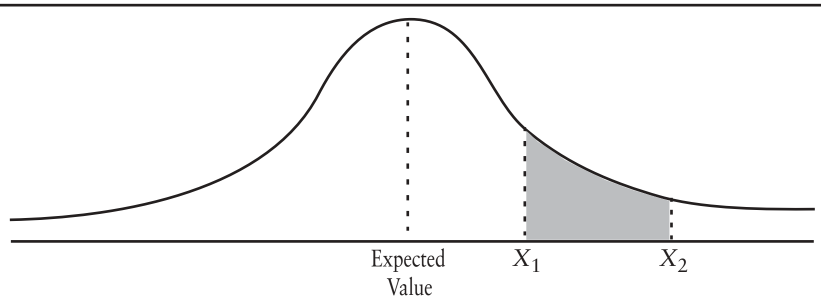
The entire area under the normal curve is equal to 1 which means the sum of the probabilities is 1.

Properties of the Normal Distribution

The normal distribution has the following properties:

1. The point in the middle of the normal curve is the expected value for the distribution.

EXHIBIT 5.4 **Normal Distribution**



Note: Probability of realizing a value between X_1 and X_2 is shaded area.

2. The distribution is symmetric around the expected value. That is, half of the distribution is to the left of the expected value and the other half is to the right. Thus, the probability of obtaining a value less than the expected value is 50%. The probability of obtaining a value greater than the expected value is also 50%.
3. The probability that the actual outcome will be within a range of one standard deviation above the expected value and one standard deviation below the expected value is 68.3%.
4. The probability that the actual outcome will be within a range of two standard deviations above the expected value and two standard deviations below the expected value is 95.5%.
5. The probability that the actual outcome will be within a range of three standard deviations above the expected value and three standard deviations below the expected value is 99.7%.

Exhibit 5.5 graphically presents these properties.

Suppose that a manager estimates a position one week from now has an expected profit of \$40,000 with a standard deviation of \$100,000, and that the probability distribution can be approximated well by a normal distribution. The probability is 68.3% that one week from now the profit will be between –\$60,000 (the expected value of \$40,000 minus one standard deviation of \$100,000) and \$140,000 (the expected value of \$40,000 plus one standard deviation of \$100,000). The probability is 95.5% that the profit will be between –\$160,000 (the expected value minus two standard deviations) and \$240,000 (the expected value plus two standard deviations).

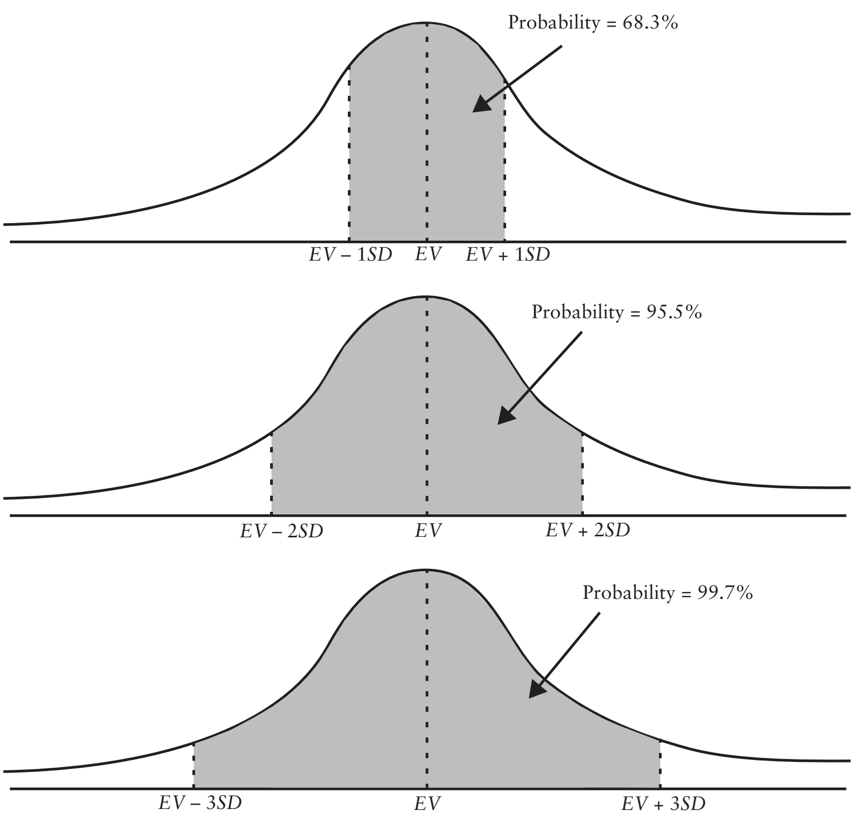
Suppose that the standard deviation is believed to be \$70,000 rather than \$100,000. Then the probability is 68.3% that the profit will be between –\$30,000 and \$110,000; the probability is 95.5% that the profit will be between –\$100,000 and \$180,000. *Notice that the smaller the standard deviation, the narrower the range for the possible outcome for a given probability.*

Using Normal Distribution Tables

Tables are available that give the probability of obtaining a value between any two values of a normal probability distribution. All that must be known in order to determine the probability is the expected value and the standard deviation.

The normal distribution table is constructed for a normal distribution that has an expected value of 0 and a standard deviation of 1. In order to use the table it is necessary to convert the normal distribution under consideration into a distribution that has an expected value of 0 and a standard deviation of 1. This is done by standardizing the values of the distribution under consideration.

EXHIBIT 5.5 Properties of a Normal Distribution



EV = Expected Value
SD = Standard Deviation

The procedure is as follows. Suppose that a normal distribution for some random variable X has an expected value $E(X)$ and a standard deviation denoted by std(X). To standardize any particular value, say X_1, the following is computed:

$$z_1 = \frac{X_1 - E(X)}{std(X)}$$

where z_1 is the *standardized value* for X_1. The standardized value is also called the *normal deviate*.

Exhibit 5.6 is an abridged table that shows the area under the normal curve, which, as stated before, represents probability. This particular table shows the probability of obtaining a value greater than some specified value in standardized form in the right-hand tail of the distribution. This is the shaded area shown in the normal curve at the top of Exhibit 5.6.

The illustrations to follow demonstrate how to use the table. We will use the same example as earlier: the expected value of the profit of the position is \$40,000 and the standard deviation is \$100,000.

Suppose that the manager wants to know the probability of realizing a value greater than \$90,000. The standardized value (z) corresponding to \$90,000 is 0.5, as shown below:

$$\frac{\$90{,}000 - \$40{,}000}{\$100{,}000} = 0.5$$

The probability of obtaining a value greater than \$90,000 is the same as a standardized value greater than 0.5. From Exhibit 5.6, the probability of obtaining a standardized value greater than 0.5 is 0.3085 or 30.85%.

Suppose the probability of obtaining a loss is sought by the manager. This is equivalent to realizing a value of X that is less than zero. The standardized value is –0.4, as shown below:

$$\frac{\$0 - \$40{,}000}{\$100{,}000} = -0.4$$

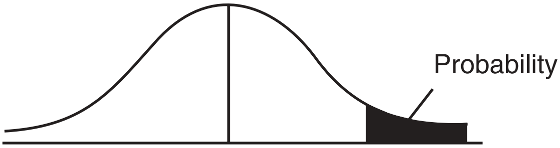
The negative value indicates that the manager is looking for values in the left-hand tail. From Exhibit 5.6, the probability of obtaining a value greater than 0.4 is 0.3446 or 34.46%. Since the normal distribution is symmetric, the probability of realizing a standardized value greater than 0.4 is the same as the probability of realizing a standardized value less –0.4. Thus, the probability of realizing a loss is 34.46%.

Suppose the manager wants to know the probability of realizing a loss greater than \$150,000. The value of X is then –\$150,000 and the corresponding standardized value is

$$\frac{-\$150{,}000 - \$40{,}000}{\$100{,}000} = -1.1$$

From Exhibit 5.6 it can be seen that the probability of getting a standardized value greater than 1.1 is 13.57%. Thus, the probability of realizing a standardized value of less than –1.1 or equivalently a loss of \$150,000 is 13.57%.

EXHIBIT 5.6 Normal Distribution Table



z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641
0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
0.8	0.2110	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1449	0.1423	0.1401	0.1379
1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
1.5	.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
1.6	.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
1.7	.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
1.8	.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
1.9	.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010

The standardized value is nothing more than the number of standard deviations above the expected value since the expected value of z is zero. From an examination of Exhibit 5.6, we can see the properties of a normal distribution that we discussed earlier. For example, look at the value in the table for a standardized value equal to 2. The probability is 2.28%. This is the probability of realizing a value in each of the tails of the normal distribution. Doubling this probability gives 4.56%, which is the probability of realizing a value in either of the two tails. This means that the probability of getting a value between the two tails is 95.44%. This agrees with the third property of the normal probability distribution that we stated above—there is a 95.5% probability of getting a value between two standard deviations below and above the expected value.

The Appropriateness of Using a Normal Distribution

In a normal distribution, the expected value (i.e., mean) and the standard deviation are the only parameters needed to make statements about the probabilities of possible outcomes. When using the normal distribution to make statements about probabilities, it is necessary to determine whether the historical or empirical distribution (i.e., a distribution created from observed data) is well approximated by the normal distribution.

For example, one property of the normal distribution is that it is perfectly symmetric around its expected value. Simply put, if one sawed the distribution apart at the expected value and compared the two parts, each part would be exactly the same. To see this, let us introduce another measure of central tendency—the median.[2] If the observations are arranged in ascending (or descending) order of magnitude, the median is the value chosen such that half the observations are above it and the remaining half of the observations are below it.

With this tool at our disposal, a good indicator of a distribution's symmetry involves comparing the expected value and the median. If the expected value and the median are equal, the distribution is symmetric. Since the normal distribution is symmetric, its expected value and mean are the same. When the expected value is not equal to the mean, the distribution is asymmetric and is referred to as a *skewed distribution*.[3] Specifically, if the median is less than the mean, the distribution is skewed to the right and has a long tail on the right-hand side of the distribution. Such a distribution is referred to as *positively skewed* and is shown Exhibit 5.7. Conversely, if the median is greater than the mean, the distribution is skewed to the left and has a long tail on the left-hand side of

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[2] The central tendency is a set of measurements that describe the data's tendency to cluster around particular values and include such measures as the mean, median, and mode.

[3] The normal distribution has a skewness of zero.

EXHIBIT 5.7 Distribution Skewed to the Right (Positively Skewed)

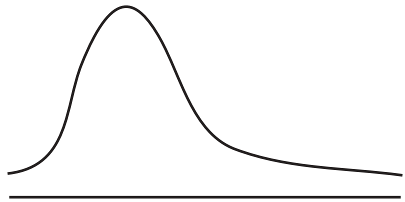
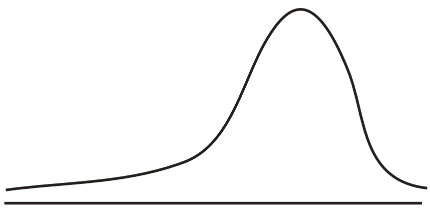


EXHIBIT 5.8 Distribution Skewed to the Left (Negatively Skewed)



the distribution. This type of distribution is referred to as *negatively skewed* and is shown in Exhibit 5.8.

In addition to skewness, a historical distribution may have more or fewer outliers than the normal distribution predicts. If the distribution has very thick tails (more observations in the tails), the distribution is said to be *leptokurtic*. Conversely, if the distribution has very thin tails (fewer observations in the tails), the distribution is said to be *platykurtic*. Kurtosis is a measure of the thickness of the tails of a distribution.

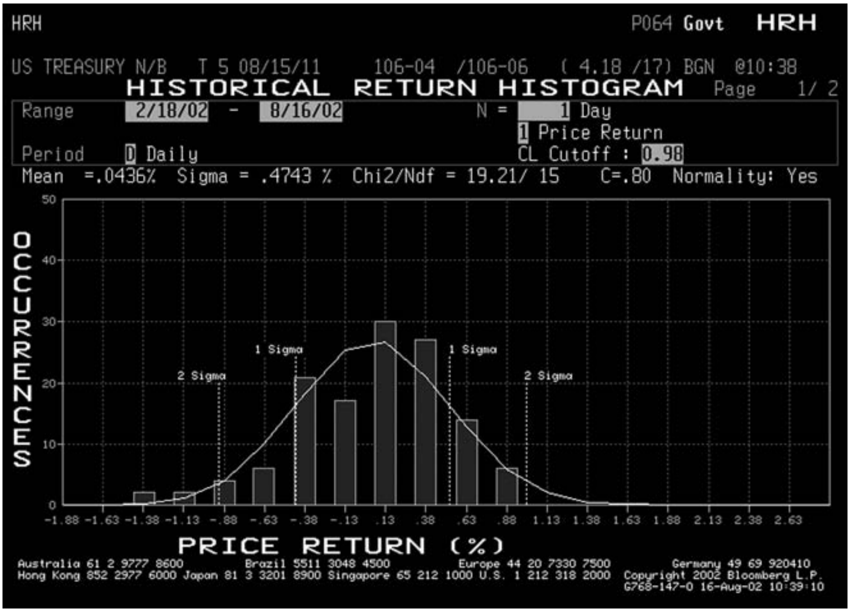
The following two questions must be answered to assess whether a historical distribution can be approximated by a normal distribution:

1. Does the data fit the values predicted by the normal distribution?
2. Are the returns today uncorrelated with the returns from prior periods?

Goodness of Fit

Most introductory statistics texts detail how to test if the historical data for some random variable (e.g., daily bond returns) can be approximated with a normal distribution. Basically the test involves breaking the historical observations into intervals (sometimes called "buckets"). For each interval, the number of predicted observations based on the normal probability distribution is determined. Then the number predicted for the interval and the number actually observed are compared. This operation is carried out for all intervals. Statistical tests can then be used to determine if the historically observed distribution differs significantly from a normal distribution.

EXHIBIT 5.9 Bloomberg's Historical Return Histogram



Source: Bloomberg Financial Markets

As an illustration, let us examine a time series of daily returns for a 10-year Treasury note with 5% coupon that matures on August 15, 2011. Exhibit 5.9 presents a histogram created using Bloomberg's Historical Return Histogram (HRH) function for our 10-year Treasury's daily returns for the period February 18, 2002 to August 16, 2002. A histogram is a graphical representation of the frequency distribution for a random variable. A normal distribution is superimposed over the histogram. Just above the graph is the mean and the standard deviation of the daily returns for the sample period. This screen also presents the results of a statistical test for whether or not the historical distribution of the daily returns over this sample period is well-approximated by the normal distribution. For this security, the data over this sample period are approximately normal and this is indicated over on the right-hand side of the screen by "Normality: Yes."

Exhibit 5.10 presents the Frequency Distribution Table used to create the histogram. The first column includes the return intervals in increments of 25 basis points. The actual number of daily returns that fall into each interval is shown in the second column. Finally, the predicted number of daily returns that should fall into each interval if the sample is well-approximated by the normal distribution is shown in the last column.

EXHIBIT 5.10 Bloomberg's Frequency Distribution Table

Page P064 Govt HRH

FREQUENCY DISTRIBUTION TABLE Page 2/ 2

RANGE OF RETURN (%)			ACTUAL OCCURRENCES	THEORETICAL OCCURRENCES
-2.00	-	-1.75	0	0.01
-1.75	-	-1.50	0	0.06
-1.50	-	-1.25	2	0.31
-1.25	-	-1.00	2	1.30
-1.00	-	-0.75	4	4.16
-0.75	-	-0.50	6	10.04
-0.50	-	-0.25	21	18.38
-0.25	-	0.00	17	25.47
0.00	-	0.25	30	26.73
0.25	-	0.50	27	21.25
0.50	-	0.75	14	12.80
0.75	-	1.00	6	5.84
1.00	-	1.25	0	2.02
1.25	-	1.50	0	0.53
1.50	-	1.75	0	0.10
1.75	-	2.00	0	0.02
2.00	-	2.25	0	0.00
2.25	-	2.50	0	0.00
2.50	-	2.75	0	0.00
2.75	-	3.00	0	0.00

Source: Bloomberg Financial Markets

In general, we expect that bond return distributions will be skewed. To see this, let's consider a long position in Treasury securities. There is a lower limit on the loss, and this limit depends on how high rates can rise. Since Treasury rates have never exceeded 15%, this places a *de facto* lower bound on a negative return from a long position in a Treasury security. Of course, just because rates have never exceeded 15% in the past does mean that rates higher than this threshold are impossible in the future. However, assuming that negative nominal interest rates are impossible, there is a maximum return.[4] The maximum price of a bond is the undiscounted value of the cash flows (i.e., the sum of the coupon payments and maturity value.) Thus, the maximum return is obtained when interest rates fall to zero. Empirical evidence suggests government bond return distributions are negatively skewed. JP Morgan reports that return distributions for both Treasury securities and swaps exhibit negative skewness.[5] Moreover, Treasury return distributions exhibit fatter tails than predicted by a normal distribution.

[4] Two additional points should be noted. First, nominal interest rates cannot be negative as long as investors can hold cash. Second, negative real rates are possible.
[5] *RiskMetrics* ™—Technical Document, JP Morgan, May 26, 1995, New York, p. 48.

One way to overcome the problem of negative skewness of bond returns is to convert the returns into the logarithm of returns. The transformation to the logarithm of returns tends to pull in the outlier negative returns such that the distribution after the transformation is approximately normal. The resulting probability distribution of the logarithm of returns is said to be *lognormally distributed*.

One would expect a skewed return distribution for options and derivatives with option like features such as caps and floors. For example, a long position in an option contract has a maximum loss equal to price paid for the option and a relatively large upside potential. Conversely, a short position in an option has a maximum profit equal to price received for the option and a relatively large downside potential. Option positions, as a result, guarantee a skewed return distribution. It is difficult to measure the riskiness of such positions using the standard deviation measure.

Independence of Returns

For any probability distribution, it is important to assess whether the value of a random variable in one period is affected by the value of the random variable observed in a prior period. Casting this in terms of returns, it is critical to know whether the return that can be realized today is influenced by the return in a prior period. The terms *serial correlation* and *autocorrelation* are used to describe the correlation between the return in different periods. JP Morgan's analysis suggests that there is only a small positive serial correlation for government bond returns.[6]

CONFIDENCE INTERVALS

When a range for the possible values of a random variable and a probability associated with that range are calculated, the range is referred is as a *confidence interval*. In general, for a normal distribution, the confidence interval is calculated as follows:

$$\text{(Expected value} - \text{Standardized value} \times \text{Standard deviation) to}$$

$$\text{(Expected value} + \text{Standardized value} \times \text{Standard deviation)}$$

The standardized value indicates the number of standard deviations way from the expected value and corresponds to a particular probability. For example, suppose a manager wants a confidence interval of

[6] *RiskMetrics™—Technical Document*, p. 48.

95%. This means that there will be 2.5% in each tail. From Exhibit 5.6, we see that a standardized value with a 2.5% probability is 1.96. Thus, a 95% confidence interval is

$$(\text{Expected value} - 1.96 \times \text{Standard deviation}) \text{ to}$$

$$(\text{Expected value} + 1.96 \times \text{Standard deviation})$$

For example, suppose that a manager wants to construct a confidence interval for the change in the value of a position over the next four days. Assuming that the change in value is normally distribution with an expected value of zero and a standard deviation of \$20,000, then a 95% confidence interval would be

$$(\$0 - 1.96 \times \$20{,}000) \text{ to } (\$0 + 1.96 \times \$20{,}000) \text{ or } -\$39{,}200 \text{ to } \$39{,}200$$

MONTE CARLO SIMULATION

The probability distribution for the change in the value of a bond or derivative instrument such as an option may depend on the outcome of a number of random variables. For example, the change in the value of a bond will depend on the sensitivity of the bond's value to rate changes, changes in the shape of the yield curve, and its yield volatility. In the case of mortgage-backed securities, it will also depend on the change in prepayment speeds. Each random variable will have its own probability distribution and there may be some random variables that may not be normally distributed. Moreover, each of the random variables that affects the change in value of a bond may not be independent. That is, there may be a significant correlation between the random variables. (We'll discuss correlation in Chapter 6.)

One way to evaluate the risk of a bond position is to evaluate all possible combinations of potential outcomes for the random variables and develop a probability distribution based on the change in the bond's value from all combinations. However, since each random variable may have a substantial number of possible outcomes, evaluation of all possible combinations of outcomes is usually impractical. Rather than evaluate all possible combinations of potential outcomes, a large number of combinations of outcomes can be evaluated. This approach is called scenario analysis. Scenario analysis, however, has a major drawback: The assessment of risk will depend on the scenarios analyzed.

An alternative to complete enumeration of the outcomes and scenario analysis for developing a probability distribution is *Monte Carlo* *simulation*. We described this methodology in Chapter 2 where we explained how it is used for valuing mortgage-backed securities.

There are ten steps in a Monte Carlo simulation. Each step is described below.

Step 1: The performance measure must be specified. For risk measurement, the appropriate performance measure is the change in the value of a bond.

Step 2: The problem under investigation must be expressed mathematically. The mathematical description of the problem must include all important variables and their interactions. The variables in the mathematical model will be either *deterministic* or *random*. A deterministic variable can take only one value; a random variable can take on more than one value.

Step 3: For those variables that are random variables, a probability distribution for each must be specified.

Step 4: For each random variable, representative numbers must be assigned to each possible outcome based on the probability distribution.

Step 5: A random number must be attained for each random variable.[7]

Step 6: For each random number, the corresponding value of the random variable must be determined.

Step 7: The corresponding value of each random variable found in the previous step must be used to determine the value of the performance measure.

Step 8: The value of the performance measure found in step 7 is recorded.

Step 9: Steps 5 through 8 must be repeated many times.[8] The repetition of steps 5 through 8 is known as a *trial*.

Step 10: On the basis of the value for the performance measure for each trial recorded in step 8, a probability distribution is constructed.

[7] Most computers have a built-in random number generator.
[8] The number of trials is determined by a technique called variance reduction.

In practice, some of the deterministic variables are actually unknown but are assigned some assumed value in the Monte Carlo simulation. The simulation is repeated with different assumed values for the deterministic variables that are unknown in order to assess the impact of these variables on the probability distribution.

When there is more than one asset in a position, Monte Carlo simulation can consider the interaction (or correlation) among the prices and rates for all assets. The correlations are estimated using historical data. The probability distribution generated for a position will then depend on the correlation between the price or rates of each asset. The sensitivity of the probability distribution can be examined by repeating the Monte Carlo simulation using a different set of correlations.

KEY POINTS

1. A random variable is a variable for which a probability can be assigned to each possible value that can be taken by the variable.

2. A probability distribution describes all the values that the random variable can take on and the probability associated with each.

3. The expected value of a probability distribution is the weighted average of the distribution.

4. Variance is a measure of the dispersion of the random variable around its expected value.

5. The standard deviation is the square root of the variance.

6. The greater the standard deviation, the greater the variability of the random variable around the expected value.

7. A discrete probability distribution is one in which the random variable can only take on specific values, while a random variable can take on any possible value within the range of outcomes for a continuous probability distribution.

8. In jump process, a random variable can realize large movements without taking on interim values.

9. A normal distribution is a symmetric probability distribution that is used in many business applications.

10. The area under the normal distribution or normal curve between any two points on the horizontal axis is the probability of obtaining a value between those two values.

11. If a random variable follows a normal distribution then the expected value and the standard deviation are the only two parameters that are needed to make statements about the probability of outcomes for that random variable.

12. In order to apply the normal distribution to make statements about probabilities, it is necessary to assess whether a historical distribution is properly characterized as normally distributed.

13. There are statistical tests that can be used to determine whether a historical distribution can be characterized as a normal distribution.

14. A skewed distribution is a probability distribution that is not symmetric around the expected value.

15. A positively skewed distribution is one in which there is a long tail to the right; a negatively skewed distribution is one in which there is a long tail to the left.

16. Serial correlation or autocorrelation is the correlation between returns over time.

17. There is only a small positive serial correlation for government bond returns.

18. A confidence interval gives a range for possible values of a random variable and a probability associated with that range.

19. For complex bonds and bond positions, Monte Carlo simulation can be used to obtain a probability distribution.

CHAPTER 6

Correlation Analysis and Regression Analysis

In the previous two chapters we have dealt with a single random variable. In this chapter, we will look at the relationship between random variables. The two statistical analyses that we shall describe are correlation analysis and regression analysis.

The objectives of this chapter are to:
1. Describe what is meant by the correlation coefficient between two random variables and how it is calculated.
2. Describe what the covariance is and its relationship to the correlation coefficient.
3. Describe how the variance of the return of a portfolio of assets is calculated and the important role that the correlation plays.
4. Explain the role of correlation in selecting hedging instruments.
5. Explain what regression analysis is and how to estimate a regression.
6. Explain what the coefficient of determination of a regression measures.

CORRELATION ANALYSIS

The *correlation coefficient* measures the association between two random variables. No cause and effect are assumed when a correlation coefficient is computed. After we describe how the correlation between two random variables is calculated from historical data, we will look at the role played by this measure in risk management.

The formula for calculating the correlation coefficient, or simply correlation, between two random variables X and Y is

$$\text{Correlation} = \frac{T\sum_{t=1}^{T} X_t Y_t - \left(\sum_{t=1}^{T} X_t\right)\left(\sum_{t=1}^{T} Y_t\right)}{\sqrt{\left(T\sum_{t=1}^{T} X_t^2 - \left(\sum_{t=1}^{T} X_t\right)^2\right)\left(T\sum_{t=1}^{T} Y_t^2 - \left(\sum_{t=1}^{T} Y_t\right)^2\right)}}$$

where the subscript t denotes the t-th observation and T is the total number of observations.

The correlation can have a value between –1 and 1. A positive value means that the two random variables tend to move together. In this case, the two random variables are said to be *positively correlated*. A negative value means that the two random variables tend to move in the opposite direction. Two random variables that exhibit this characteristic are said to be *negatively correlated*. A correlation close to zero means that the two random variables tend not to track each other.

To illustrate how to use the above formula, we will calculate the correlation between the rate of return on two hypothetical assets: asset 1 and asset 2. Let

X = rate of return on asset 1
Y = rate of return on asset 2

Sixty pairs of monthly returns for the two assets are provided in Exhibit 6.1. The last row of the exhibit indicates that

$$\sum_{t=1}^{60} X_t = -36.516 \qquad \sum_{t=1}^{60} Y_t = 123.288 \qquad \sum_{t=1}^{60} X_t Y_t = 627.3633$$

$$\sum_{t=1}^{60} X_t^2 = 3{,}479.3256 \qquad \sum_{t=1}^{60} Y_t^2 = 3{,}402.0807$$

Substituting these values into the formula:

$$\begin{aligned}&\text{Correlation}\\ &= \frac{60(627.3633) - (-36.516)(123.288)}{\sqrt{[60(3{,}479.3256) - (-36.516)^2][60(3{,}402.0807) - (123.288)^2]}}\\ &= 0.21\end{aligned}$$

EXHIBIT 6.1 **Calculation of the Correlation Between the Monthly Rate of Return Between Asset 1 and Asset 2**

X_t = monthly return on asset 1 (%)
Y_t = monthly return on asset 2 (%)

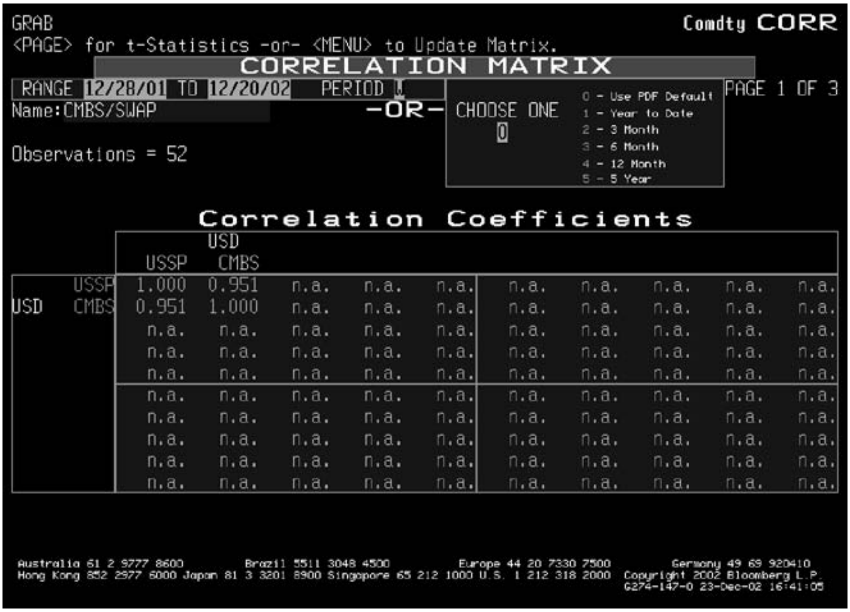
t	X_t	Y_t	X_t^2	Y_t^2	X_tY_t
1	7.1790	27.2730	51.5380	743.8165	195.7929
2	–6.1440	–8.6490	37.7487	74.8052	53.1395
3	–10.1850	1.4290	103.7342	2.0420	–14.5544
4	4.4670	8.4510	19.9541	71.4194	37.7506
5	–2.7760	8.5560	7.7062	73.2051	–23.7515
6	2.0520	1.8020	4.2107	3.2472	3.6977
7	2.7930	16.8170	7.8008	282.8115	46.9699
8	2.9000	–4.9620	8.4100	24.6214	–14.3898
9	–6.7240	–0.5330	45.2122	0.2841	3.5839
10	–8.2380	–7.2370	67.8646	52.3742	59.6184
11	–1.4110	3.3530	1.9909	11.2426	–4.7311
12	–3.5850	5.0560	12.8522	25.5631	–18.1258
13	4.7810	–11.4990	22.8580	132.2270	–54.9767
14	6.5500	–1.3290	42.9025	1.7662	–8.7050
15	2.1660	4.6180	4.6916	21.3259	10.0026
16	2.7090	–1.1760	7.3387	1.3830	–3.1858
17	11.2020	11.7860	125.4848	138.9098	132.0268
18	–2.0830	6.1480	4.3389	37.7979	–12.8063
19	–5.1060	0.7580	26.0712	0.5746	–3.8703
20	–7.5470	–8.3520	56.9572	69.7559	63.0325
21	4.4170	–1.3680	19.5099	1.8714	–6.0425
22	–0.9400	5.2760	0.8836	27.8362	–4.9594
23	8.9770	6.8180	80.5865	46.4851	61.2052
24	–0.5500	1.9850	0.3025	3.9402	–1.0918
25	12.1680	9.7330	148.0602	94.7313	118.4311
26	2.5330	11.2750	6.4161	127.1256	28.5596
27	–11.5530	7.6000	133.4718	57.7600	–87.8028
28	–9.5500	–2.6020	91.2025	6.7704	24.8491
29	4.2090	–0.4120	17.7157	0.1697	–1.7341
30	–8.4810	2.3080	71.9274	5.3269	–19.5741
31	4.2470	2.6320	18.0370	6.9274	11.1781
32	–3.1260	1.0700	9.7719	1.1449	–3.3448
33	6.9680	–6.9090	48.5530	47.7343	–48.1419

EXHIBIT 6.1 (Continued)

t	X_t	Y_t	X_t^2	Y_t^2	X_tY_t
34	–5.1870	4.2970	26.9050	18.4642	–22.2885
35	–4.6210	–2.6070	21.3536	6.7964	12.0469
36	–3.7840	17.3750	14.3187	301.8906	–65.7470
37	1.1240	–0.6580	1.2634	0.4330	–0.7396
38	–2.1280	–3.6290	4.5284	13.1696	7.7225
39	–3.8850	–1.0340	15.0932	1.0692	4.0171
40	8.6830	–6.6200	75.3945	43.8244	–57.4815
41	1.3330	–0.3580	1.7769	0.1282	–0.4772
42	7.8510	1.8800	61.6382	3.5344	14.7599
43	–3.1930	5.9040	10.1952	34.8572	–18.8515
44	–7.2980	7.3310	53.2608	53.7436	–53.5016
45	–6.7820	–0.3260	45.9955	0.1063	2.2109
46	–17.1830	5.2290	295.2555	27.3424	–89.8499
47	3.8650	10.0000	14.9382	100.0000	38.6500
48	–26.1900	–1.1330	685.9161	1.2837	29.6733
49	2.2330	–8.5960	4.9863	73.8912	–19.1949
50	6.6310	–1.8180	43.9702	3.3051	–12.0552
51	–6.4370	3.5260	41.4350	12.4327	–22.6969
52	–4.4230	–5.8820	19.5629	34.5979	26.0161
53	9.5940	12.8950	92.0448	166.2810	123.7146
54	–6.3980	–5.5560	40.9344	30.8691	35.5473
55	–9.8730	–6.8110	97.4761	46.3897	67.2450
56	3.3710	3.7210	11.3636	13.8458	12.5435
57	–8.1970	–3.8590	67.1908	14.8919	31.6322
58	9.5240	13.7120	90.7066	188.0189	130.5931
59	17.6630	–3.7180	311.9816	13.8235	–65.6710
60	4.8720	0.3070	23.7364	0.0942	1.4957
Total	–36.5160	123.2880	3,479.3256	3,402.0807	627.3633

As an illustration, let us employ Bloomberg's correlation screen, which is presented in Exhibit 6.2. This screen allows the user to calculate a correlation matrix for numerous random variables (e.g., interest rates, returns, etc.) over a specified time period. In this illustration, the two variables selected are the 10-year swap spreads (USSP) and the spreads (over Treasuries) for AAA commercial mortgage-backed securi-

EXHIBIT 6.2 **Bloomberg Screen of a Correlation Matrix for 10-Year Swap Spreads and 10-Year CMBS Spreads**



Source: Bloomberg Financial Markets

ties with an average life of ten years (CMBS).[1] We examine 52 weekly observations of these two variables over the sample period of December 28, 2001 to December 20, 2002. Over this sample period, the correlation coefficient is 0.951. Since the correlation coefficient is reasonably close to 1, this result suggests that the movements in 10-year swap spreads and 10-year CMBS spreads are very highly correlated.

Covariance

The covariance also measures how two random variables vary together. The covariance is related to the correlation coefficient as follows:

$$\text{Covariance} = \text{std}(X)\ \text{std}(Y)\ (\text{correlation})$$

Since the standard deviations are positive, the covariance will have the same sign as the correlation. Thus, if two random variables are positively

[1] The CMBS spreads are derived from a Morgan Stanley index which is updated every Friday.

correlated they will have a positive covariance. Similarly, the covariance will be negative if the two random variables are negatively correlated.

The covariance between the rates of return for asset 1 and asset 2 for the 60-month period reported in Exhibit 6.1 is found as follows. The standard deviation for the rate of return of asset 1 is 7.765. For asset 2 it is 7.305. The correlation is 0.21. Therefore, the covariance is

$$\text{Covariance} = 7.675\ (7.305)\ (0.21) = 11.77$$

Measuring the Variance of a Two-Asset Portfolio

As explained in previous chapters, the variance or standard deviation can be viewed as a measure of risk for an individual security. The risk of a portfolio or position in several assets is not simply the weighted average of the variance of the component assets. The basic principle of modern portfolio theory is that the variance of a portfolio of assets depends not only on the variance of the assets, but also their covariances.[2] Specifically, the variance of a two-asset portfolio is equal to

$$\text{var}(P) = W_X^2\text{var}(X) + W_Y^2\text{var}(Y) + 2W_X W_Y\text{cov}(X, Y)$$

where

$\text{var}(P)$	=	variance of the rate of return of a portfolio comprised of asset 1 and asset 2
$\text{var}(X)$	=	variance of the rate of return of asset 1
$\text{var}(Y)$	=	variance of the rate of return of asset 2
$\text{cov}(X,Y)$	=	covariance between the rate of return on asset 1 and asset 2
W_X	=	market value of asset 1/market value of portfolio
W_Y	=	market value of asset 2/market value of portfolio

In words, the formula says that the variance of the portfolio return is the sum of the weighted variances of the two assets plus the weighted covariance between the two assets.

For our two hypothetical assets, suppose that 60% is invested in asset 1 and 40% in asset 2. Then the inputs for calculating the variance of a portfolio consisting of these two assets are

$$\begin{aligned}
\text{var}(X) &= (7.675)^2 = 58.9056\\
\text{var}(Y) &= (7.305)^2 = 53.3630\\
\text{cov}(X,Y) &= 11.77
\end{aligned}$$

[2] Harry M. Markowitz, "Portfolio Selection," *Journal of Finance*, March 1952, pp. 71–91.

$$W_X = 0.6$$
$$W_Y = 0.4$$

Then

$$\begin{aligned}\mathrm{var}(P) &= (0.6)^2(58.9056) + (0.4)^2(53.3630) + 2(0.6)(0.4)(11.77)\\ &= 21.2060 + 8.5381 + 5.6496 = 35.3937\end{aligned}$$

The portfolio's standard deviation is then 5.9494 (the square root of 35.3937). Notice that the portfolio's standard deviation is less than that of the standard deviation of either asset.

The key in the risk of a portfolio or position as measured by the standard deviation or variance is the correlation (or covariance) between the two assets. Exhibit 6.3 shows the portfolio standard deviation for several assumed correlations and different weights for asset 1 and asset 2 in the portfolio. For a given allocation of the two assets in the portfolio, the more negatively correlated, the lower the portfolio standard deviation. The minimum variance (for a given allocation) occurs when the correlation is –1.

Consequently, for a manager seeking to measure and then control the risk of a portfolio or position in two assets, the correlation and the relative amounts of the two assets determines the standard deviation of the portfolio or position. It is critical to have a good estimate of the correlation to measure risk.

There is another point to note from the results reported in Exhibit 6.3. Suppose that a manager wants to hedge a position in asset 1. By hedging it is meant that the manager seeks to employ some hedging instrument such that the combined position of asset 1 and the hedging instrument will produce a portfolio standard deviation of zero. Look at the last column of Exhibit 6.3. If a hedging instrument, say asset 2, can be identified that has a –1 correlation with asset 1 and the manager takes a position in asset 2 such that the portfolio has 48.77% of asset 1 and 51.23% of asset 2, then the standard deviation of the portfolio will be approximately zero.

Consequently, hedging involves identifying one or more instruments that have a correlation of close to –1 with the position that the manager seeks to protect and selecting the appropriate amount of the hedging instrument. If the position in asset 1 is a long position, then this typically involves shorting a position in the hedging instrument, asset 2.

Measuring the Variance of a Portfolio with More than Two Assets

Thus far we have given the portfolio variance and standard deviation for a portfolio consisting of two assets. The extension to three assets—asset 1, asset 2, and asset 3—is as follows:

EXHIBIT 6.3 Portfolio Standard Deviation for Different Correlations and Weights for Asset 1 and Asset 2

Assumptions:

X = rate of return of asset 1 (%)
Y = rate of return of asset 2 (%)
W_X = weight of asset 1
W_Y = weight of asset 2

Standard deviation for asset 1 = 7.675
Standard deviation for asset 2 = 7.305

Correlation	Covariance	W_X: 0.6 W_Y: 0.4	0.5 0.5	0.4 0.6	0.4877 0.5123
1.0	56.07	7.5270	7.4900	7.4530	7.4854
0.8	44.85	7.1605	7.1059	7.0827	7.1013
0.6	33.64	6.7743	6.6998	6.6920	6.6952
0.4	22.43	6.3646	6.2674	6.2770	6.2628
0.2	11.21	5.9268	5.8029	5.8325	5.7982
0.0	0.00	5.4538	5.2978	5.3512	5.2930
–0.2	–11.21	4.9358	4.7393	4.8222	4.7342
–0.4	–22.43	4.3565	4.1054	4.2274	4.0999
–0.6	–33.64	3.6874	3.3537	3.5339	3.3476
–0.8	–44.85	2.8661	2.3750	2.6658	2.3671
–1.0	–56.07	1.6830	0.1850	1.3130	0.0007

$$\begin{aligned}\text{var}(P) = {} & W_X^2\text{var}(X) + W_Y^2\text{var}(Y) + W_Z^2\text{var}(Z) + 2W_XW_Y\text{cov}(X, Y) \\ & + 2W_XW_Z\text{cov}(X, Z) + 2W_YW_Z\text{cov}(Y, Z)\end{aligned}$$

where

var(P) = variance of the rate of return of a portfolio comprised of assets 1, 2 and 3
var(X) = variance of the rate of return of asset 1
var(Y) = variance of the rate of return of asset 2
var(Z) = variance of the rate of return of asset 3
cov(X,Y) = covariance between the rate of return on asset 1 and asset 2
cov(X,Z) = covariance between the rate of return on asset 1 and asset 3
cov(Y,Z) = covariance between the rate of return on asset 2 and asset 3
W_X = market value of asset 1/market value of portfolio
W_Y = market value of asset 2/market value of portfolio
W_Z = market value of asset 3/market value of portfolio

In words, the portfolio's variance is the sum of the weighted variances of the individual assets plus the sum of the weighted covariances of the assets.

In general, for a portfolio with J assets, the portfolio variance is:

$$\text{var}(P) = \sum_{j=1}^{J} W_j^2 \text{var}(j) + \sum_{\substack{j=1 \\ \text{for } j \neq k}}^{J} \sum_{k=1}^{J} W_j W_k \text{cov}(j, k)$$

REGRESSION ANALYSIS

In correlation analysis, neither random variable is assumed to effect the other random variable. In some situations in managing risk it is necessary to estimate the relationship between two random variables in which it is assumed that one random variable affects the other random variable. *Regression analysis* is a statistical technique that can be used to estimate relationships between variables. Regression analysis will be explained with an illustration.

The Simple Linear Regression Model

Suppose that a manager believes that the return on asset 3 affects the return on asset 2 and wants to estimate the relationship. Assume that the manager believes that the relationship can be expressed as follows:

$$\text{Return on asset 2} = \alpha + \beta \text{ (return on asset 3)}$$

The values α and β are called the *parameters* of the model. The objective of regression analysis is to estimate the parameters.

There are several points to note about this relationship. First there are only two variables in the relationship—the return on asset 3 and the return on asset 2. Because there are only two variables and the relationship is linear, this regression model is called a *simple linear regression model*. Since the return on asset 2 is assumed to depend on the return on asset 3, the return on asset 2 is referred to as the *dependent variable*. The return on asset 3 is referred to as the *explanatory* or *independent variable* because it is used to explain the return on asset 2. Second, it is highly unlikely that the estimated relationship will describe the true relationship between the two returns exactly because other factors may influence the return on asset 2. Consequently, the relationship may be more accurately described by adding a random error term to the relationship. That is, the relationship can be expressed as follows:

$$\text{Return on asset 2} = \alpha + \beta(\text{Return on asset 3}) + \text{Random error term}$$

The expression can be simplified as follows:

$$Y = \alpha + \beta X + e$$

where

Y = rate of return on asset 2
X = rate of return on asset 3
e = random error term

Estimating the Parameters of the Simple Linear Regression Model

In order to estimate the parameters of the simple linear regression model, historical information on the returns of asset 3 and asset 2 are needed. We will use the 60 monthly returns in Exhibit 6.4.

One possible way of estimating the relationship between the two returns is simply to plot the observations on a graph and then draw a line through the observations which it is believed best represent the relationship. Selecting two points on this line will determine the estimated relationship. The obvious pitfall is that there is no specified criterion for drawing the line, and hence different individuals would obtain different estimates of the relationship based on the same observations.

The regression method specifies a logical criterion for estimating the relationship. To understand this criterion, first rewrite the simple linear regression so that it shows the estimated relationship for each observation. This is done as follows:

$$Y_t = \alpha + \beta X_t + e_t$$

where the subscript t denotes the observation for the t-th month. For example, for the fourth observation ($t = 4$), the above expression is

$$4.467 = \alpha + \beta(5.13) + e_4$$

For observation 18 ($t = 18$), the expression is

$$-2.083 = \alpha + \beta(-0.60) + e_{18}$$

The values for e_4 and e_{18} are referred to as the *observed error term for the observation*. Note that the value of the observed error term for both observations will depend on the values selected for α and β. This suggests a criterion for selecting the two parameters. The parameters

EXHIBIT 6.4 **Worksheet for the Estimation of the Parameters of the Simple Linear Regression: Relationship Between Monthly Return on Asset 3 and Asset 2**
X_t = monthly return on asset 3 (%)
Y_t = monthly return on asset 2 (%)

t	X_t	Y_t	X_tY_t	X_t^2	Y_t^2
1	7.2100	7.1790	51.7606	51.9841	51.5380
2	–2.5000	–6.1440	15.3600	6.2500	37.7487
3	2.3600	–10.1850	–24.0366	5.5696	103.7342
4	5.1300	4.4670	22.9157	26.3169	19.9541
5	4.0400	–2.7760	–11.2150	16.3216	7.7062
6	–0.5500	2.0520	–1.1286	0.3025	4.2107
7	8.9800	2.7930	25.0811	80.6404	7.8008
8	1.9300	2.9000	5.5970	3.7249	8.4100
9	–0.3900	–6.7240	2.6224	0.1521	45.2122
10	–2.3600	–8.2380	19.4417	5.5696	67.8646
11	2.0700	–1.4110	–2.9208	4.2849	1.9909
12	2.3900	–3.5850	–8.5682	5.7121	12.8522
13	–6.7200	4.7810	–32.1283	45.1584	22.8580
14	1.2900	6.5500	8.4495	1.6641	42.9025
15	2.6200	2.1660	5.6749	6.8644	4.6916
16	–2.4800	2.7090	–6.7183	6.1504	7.3387
17	9.7500	11.2020	109.2195	95.0625	125.4848
18	–0.6900	–2.0830	1.4373	0.4761	4.3389
19	–0.3200	–5.1060	1.6339	0.1024	26.0712
20	–9.0400	–7.5470	68.2249	81.7216	56.9572
21	–4.9200	4.4170	–21.7316	24.2064	19.5099
22	–0.3700	–0.9400	0.3478	0.1369	0.8836
23	6.4300	8.9770	57.7221	41.3449	80.5865
24	2.7500	–0.5500	–1.5125	7.5625	0.3025
25	4.3600	12.1680	53.0525	19.0096	148.0602
26	7.1500	2.5330	18.1110	51.1225	6.4161
27	2.4200	–11.5530	–27.9583	5.8564	133.4718
28	0.2400	–9.5500	–2.2920	0.0576	91.2025
29	4.3200	4.2090	18.1829	18.6624	17.7157
30	–4.5800	–8.4810	38.8430	20.9764	71.9274
31	4.6600	4.2470	19.7910	21.7156	18.0370
32	2.3700	–3.1260	–7.4086	5.6169	9.7719
33	–1.6700	6.9680	–11.6366	2.7889	48.5530

EXHIBIT 6.4 (Continued)

t	X_t	Y_t	X_tY_t	X_t^2	Y_t^2
34	1.3400	–5.1870	–6.9506	1.7956	26.9050
35	–4.0300	–4.6210	18.6226	16.2409	21.3536
36	11.4400	–3.7840	–43.2890	130.8736	14.3187
37	–1.8600	1.1240	–2.0906	3.4596	1.2634
38	1.3000	–2.1280	–2.7664	1.6900	4.5284
39	–1.9500	–3.8850	7.5758	3.8025	15.0932
40	2.9400	8.6830	25.5280	8.6436	75.3945
41	0.4900	1.3330	0.6532	0.2401	1.7769
42	–1.4900	7.8510	–11.6980	2.2201	61.6382
43	4.0900	–3.1930	–13.0594	16.7281	10.1952
44	–2.0500	–7.2980	14.9609	4.2025	53.2608
45	1.1800	–6.7820	–8.0028	1.3924	45.9955
46	0.3500	–17.1830	–6.0141	0.1225	295.2555
47	3.4100	3.8650	13.1797	11.6281	14.9382
48	1.2300	–26.1900	–32.2137	1.5129	685.9161
49	0.7300	2.2330	1.6301	0.5329	4.9863
50	1.3600	6.6310	9.0182	1.8496	43.9702
51	2.1500	–6.4370	–13.8396	4.6225	41.4350
52	–2.4200	–4.4230	10.7037	5.8564	19.5629
53	2.6800	9.5940	25.7119	7.1824	92.0448
54	0.2900	–6.3980	–1.8554	0.0841	40.9344
55	–0.4000	–9.8730	3.9492	0.1600	97.4761
56	3.7900	3.3710	12.7761	14.3641	11.3636
57	–0.7700	–8.1970	6.3117	0.5929	67.1908
58	2.0700	9.5240	19.7147	4.2849	90.7066
59	–0.9500	17.6630	–16.7799	0.9025	311.9816
60	1.2100	4.8720	5.8951	1.4641	23.7364
Total	72.0100	–36.5160	401.8848	909.5345	3,479.3256

should be estimated in such a way that the sum of the observed error terms for all observations is as small as possible.

Although this is a good standard, it presents one problem. Some observed error terms will be positive, and others will be negative. Consequently, positive and negative observed error terms will offset each other. To overcome this problem, each error term could be squared. On the basis of that criterion, the objective would then be to select parameters so as to minimize the sum of the square of the observed error terms. This is precisely the criterion used to estimate the parameters in regression analysis. Because of this property, regression analysis is sometimes referred to as the *method of least squares*.

The formulas that can be used to estimate the parameters on the basis of this criterion are derived using differential calculus. Their use will be illustrated. If a hat (ˆ) over the parameter denotes the estimated value and T denotes the total number of observations, then the estimated parameters for α and β are computed from the observations using the following formulas:

$$\hat{\beta} = \frac{\sum_{t=1}^{T} X_t Y_t - \frac{1}{T}\sum_{t=1}^{T} X_t \sum_{t=1}^{T} Y_t}{\sum_{t=1}^{T} X_t^2 - \frac{1}{T}\left(\sum_{t=1}^{T} X_t\right)^2} \quad \text{and} \quad \hat{\alpha} = \frac{1}{T}\sum_{t=1}^{T} Y_t - \frac{1}{T}(\hat{\beta})\sum_{t=1}^{T} X_t$$

Although the formulas look complicated, they are easy to apply. In actual problems with a large number of observations, there are regression analysis programs that will compute the value of the parameters using the above formulas. Most electronic spread sheets are preprogrammed to perform simple linear regression analysis.

The above formulas may be used to compute the estimated parameters on the basis of the 60 observations given in Exhibit 6.4. The worksheet for the sums needed to apply the formula is shown as Exhibit 6.4 and summarized below:

$$\sum_{t=1}^{60} X_t = 72.03 \qquad \sum_{t=1}^{60} Y_t = -36.516$$

$$\sum_{t=1}^{60} X_t Y_t = 401.8848 \qquad \sum_{t=1}^{60} X_t^2 = 909.5345$$

We then have

$$\hat{\beta} = \frac{401.8848 - \frac{1}{60}(72.01)(-36.516)}{909.5345 - \frac{1}{60}(72.01)^2}$$

and

$$\hat{\alpha} = \frac{1}{60}(-36.516) - \frac{1}{60}(0.5415)(72.01) = -1.2585$$

The estimated relationship between the monthly return on asset 3 and asset 2 is then

$$Y = -1.2585 + 0.5415X$$

Goodness of Fit

The manager will be interested in knowing how "good" the estimated relationship is. Statistical tests determine in some sense how good the relationship is between the dependent variable and the explanatory variable. A measure of the "goodness of fit" of the relationship is the *coefficient of determination*.

The explanatory or independent variable X is being used to try to explain movements in the dependent variable Y. But what movements is it trying to explain? The variable X is trying to explain why the variable Y would deviate from its mean. It can be shown that if no explanatory variable is used to try to explain movements in Y, the method of least squares would give the mean of Y as the value estimate of Y. Thus the ability of X to explain deviations of Y from its mean is of interest. In regression analysis, when we refer to the variation in a variable we mean its deviation from its mean.

The coefficient of determination indicates the percentage of the variation of the dependent variable that is explained by the explanatory variable (i.e., explained by the regression).[3] That is,

$$\text{Coefficient of determination} = \frac{\text{Variation of } Y \text{ explained by } X}{\text{Variation of } Y}$$

The coefficient of determination is commonly referred to as "R-squared" and denoted by R^2.

The coefficient of determination can take on a value between 0 and 1. If all the variation of Y is explained by X, then the coefficient of determination is 1. When none is explained by X, the coefficient of determination is 0. Hence, the closer the coefficient of determination is to 1, the stronger the relationship between the variables.

[3] In statistics textbooks, the terms "total sum of squares" and "explained sum of squares" are used instead of variation in Y and variation in Y explained by X.

Another interpretation of the coefficient of determination is that it measures how close the observed points are to the regression line. The closer the observed points are to the regression line, the closer the coefficient of determination will be to 1. On the other hand, the greater the scatter of the observed points from the regression line, the closer the coefficient of determination will be to 0.

Computation of the coefficient of determination is as follows. To compute variation of *Y*, the following formula is used:

$$\text{Variation of } Y = \sum_{t=1}^{T} Y_t^2 - \frac{1}{T}\left(\sum_{t=1}^{T} Y_t\right)^2$$

The variation of *Y* explained by *X* is computed using the following formula:

$$\text{Variation of } Y \text{ explained by } X = \hat{\beta}\left(\sum_{t=1}^{T} X_t Y_t - \frac{1}{T}\sum_{t=1}^{T} X_t \sum_{t=1}^{T} Y_t\right)$$

The coefficient of determination is then found by dividing the variation of *Y* explained by *X* by the variation of *Y*.

From the worksheet shown as Exhibit 6.4,

$$\sum_{t=1}^{60} X_t = 72.03 \qquad \sum_{t=1}^{60} Y_t = -36.516$$

$$\sum_{t=1}^{60} X_t Y_t = 401.8848 \qquad \sum_{t=1}^{60} Y_t^2 = 3{,}479.326$$

Then,

$$\text{Variation of } Y = 3{,}479.326 - \frac{1}{60}(-36.516)^2 = 3{,}457.142362$$

and

$$\begin{aligned}&\text{Variation of } Y \text{ explained by } X\\ &= (0.5415)\left[401.8848 - \frac{1}{60}(72.01)(-36.516)\right]\\ &= 241.3500215\end{aligned}$$

The coefficient of determination is therefore

$$\text{Coefficient of determination} = \frac{241.3500215}{3{,}457.142362} = 0.07$$

A coefficient of determination of 0.07 means that approximately 7% of the variation in the monthly return of asset 2 is explained by the monthly return of asset 3.

There are tests that can be performed to determine whether the coefficient of determination is statistically significant. Alternatively, the statistical significance of the estimated β parameter can be tested. The test involves determining whether the estimated β is statistically different from zero. If there is no relationship between the two random variables, the estimated β would not be statistically different from zero. A discussion of these tests is provided in statistics textbooks.

Extension of the Simple Linear Regression Model

In many applications, a dependent variable may be best explained by more than one explanatory variable. When such a relationship is estimated, it is referred to as a *multiple regression*. The computations for obtaining the parameters of a multiple regression are difficult to perform by hand. Fortunately, there are numerous multiple regression analysis programs for computing the parameters of a multiple regression.

The interpretation of the coefficient of determination is the same in a multiple regression as it is in a simple linear regression. In the latter case, it is the total sum of squares explained by the explanatory variable X. In a multiple regression, the coefficient of determination is the variation in Y explained by all the explanatory variables. By adding an explanatory variable to a regression model, the belief is that the new explanatory variable will significantly increase the variation in Y explained by the regression. For example, suppose that a simple linear regression is estimated and that the variation in Y is 1,000 and the variation explained by the single explanatory variable X is 600. Suppose that another explanatory variable is added to the regression model and that the inclusion of this explanatory variable increased the variation in Y explained from 600 to 750. Thus, it would increase the coefficient of determination from 60% to 75% (750/1,000). This new explanatory variable would appear to have contributed substantially to explaining the variation in Y. On the other hand, had the variation in Y explained by the regression increased from 600 to 610, the coefficient of determination would have increased from 60% to only 61%. Thus it appears that the new explanatory variable did not do much to help explain the dependent variable.

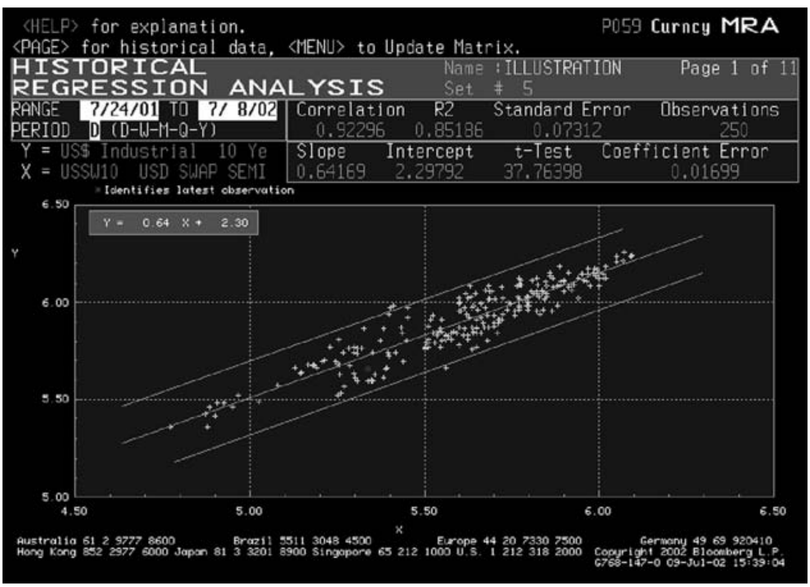
Relationship Between Correlation Coefficient and Coefficient of Determination

The coefficient of determination turns out to be equal to the square of the correlation coefficient. Thus, the square root of the coefficient determination is the correlation coefficient. Since the correlation coefficient can be between –1 and 1, the coefficient of determination will be between 0 and 1. The sign of the correlation coefficient will be the same as the sign of the slope of the regression, β. For example, the coefficient of determination between the monthly return of asset 3 and asset 2 is 0.07. The correlation coefficient is therefore 0.26.

Illustration of the Simple Linear Regression Method

Now that we have the major elements of the process in place, let us examine another illustration using Bloomberg's MRA (multiple regression analysis) screen. The dependent variable is the 10-year Aa industrial yields while the independent variable is the 10-year swap rate (discussed in Chapter 10). We examine 250 daily observations of these two variables over the sample period July 24, 2001 to July 8, 2002. The regression results, scatter plot and fitted regression line are presented in Exhibit 6.5. The estimated parameters for the intercept and the slope

EXHIBIT 6.5 Bloomberg Multiple Regression Analysis Screen



Source: Bloomberg Fianancial Markets

EXHIBIT 6.6 **Bloomberg Time Series Plots of the Correlation Coefficient, Coefficient of Determination, and Residuals**

Source: Bloomberg Fianancial Markets

are 2.29792 and 0.64169, respectively. Over this sample period, the correlation between 10-year Aa industrial bond yields and 10-year swap rates is 0.92296. Correspondingly, the coefficient of determination for our linear regression is 0.85186 which means that approximately 85% of the variation in the level of the daily 10-year Aa industrial bond yields is explained by the level of the daily 10-year swap rates. The coefficient of determination is reported at the top of the screen and is labeled "R2."

Exhibit 6.6 presents a time series plot with two panels. The top panel presents a plot of the levels of the correlation coefficient and the coefficient of determination. As noted, the coefficient of determination is equal to the square of the correlation coefficient so the coefficient of determination will always lie below the correlation coefficient. The bottom panel is a plot of the residuals for this regression.

KEY POINTS

1. The correlation coefficient measures the association between two random variables with no cause and effect assumed.
2. The correlation coefficient can have a value between –1 and 1.
3. A positive value for the correlation coefficient means that the two random variables tend to move together and are said to be positively correlated.
4. A negative value for the correlation coefficient means that the two random variables tend to move in the opposite direction and are said to be negatively correlated.
5. The covariance is related to the correlation, being the product of the standard deviation of the random variables and their correlation.
6. The variance of a portfolio's return is not simply the weighted average of the variance of the return of the component assets.
7. The variance of a portfolio's return depends not only on the variance of the assets, but also upon the correlation between the assets.
8. The variance of a portfolio's return is reduced the lower the correlation, with the maximum reduction when the correlation is –1.
9. For a manager to measure the risk of a portfolio, it is critical to have a good estimate of the correlation of returns between each pair of assets in the portfolio.
10. The correlation is important in selecting hedging instruments.
11. Hedging involves identifying one or more instruments that have a correlation close to –1 with the position that the manager seeks to protect, and selecting the appropriate amount of the hedging instrument.
12. Regression analysis is a statistical technique that can be used to estimate relationships between variables.

13. In regression analysis, one random variable is assumed to be affected by one or more other random variables.

14. In a simple linear regression, there is one dependent variable and one explanatory variable.

15. In a multiple linear regression, there is more than one explanatory variable.

16. The procedure for estimating the parameters of a regression is the method of least squares.

17. The coefficient of determination, or R-squared, is a measure of how good the relationship is between the dependent variable and the explanatory variables.

18. The coefficient of determination can take on a value between 0 and 1.

19. The coefficient of determination indicates the percentage of the variation in the dependent variable explained by the explanatory variable or variables.

20. The coefficient of determination between two random variables is equal to the square of the correlation coefficient.

CHAPTER 7

Measuring and Forecasting Yield Volatility

The standard deviation is a measure of dispersion of a random variable around its mean or expected value. Consequently, the standard deviation is commonly used as a measure of the volatility of prices or yields. Because volatility plays such a key role in interest rate risk control, we shall discuss how the standard deviation is calculated from historical data and derived from the market prices of derivative products. Moreover, we will examine approaches to forecasting the standard deviation.

The objectives of this chapter are to:
1. Explain how the standard deviation is estimated from historical yield data.
2. Demonstrate how the daily standard deviation is affected by the number of observations used in the calculation and the time period selected.
3. Explain the different ways the daily standard deviation can be annualized.
4. Explain what implied volatility is.
5. Describe the different approaches for forecasting volatility.

HISTORICAL VOLATILITY

The variance of a random variable using historical data is calculated using the following formula:

$$\text{Variance} = \sum_{t=1}^{T} \frac{(X_t - \overline{X})^2}{T-1} \tag{7.1}$$

and then

$$\text{Standard deviation} = \sqrt{\text{Variance}}$$

where

X_t = observation t on variable X
$\overline{X}$ = the sample mean for variable X
T = the number of observations in the sample

Our focus is on yield volatility. More specifically, we are interested in the percentage change in daily yields. So, X_t will denote the percentage change in yield from day t and the prior day, t–1. If we let y_t denote the yield on day t and y_{t-1} denote the yield on day t–1, then X_t which is the natural logarithm of percentage change in yield between two days, can be expressed as:

$$X_t = 100[\text{Ln}(y_t/y_{t-1})]$$

For example, on 6/25/02 the 2-year Constant Maturity Treasury (CMT) yield was 2.92% and on 6/26/02 it was 2.77%.[1] Therefore, the natural logarithm of X on 6/26/02 was:

$$X = 100[\text{Ln}(2.77/2.92)] = -5.27363$$

To illustrate how to calculate a daily standard deviation from historical yield data, consider the data in Exhibit 7.1 that shows the yield on a 2-year CMT from 6/20/02 to 7/26/02 in the third column. From the 26 observations, 25 days of percentage yield changes are calculated in the fourth column. The fifth column shows the square of the deviations of the observations from the mean. The bottom of Exhibit 7.1 shows the calculation of the daily mean for the 25 observations, the variance, and the standard deviation. The computed daily standard deviation is 2.70296%.

The daily standard deviation will vary depending on the 25 days selected. For example, the daily yields from 7/3/01 to 8/7/01 were used to generate 25 daily percentage yield changes. The computed daily standard deviation was 1.1389%.

[1] At the end of each trading day, primary U.S. Treasury securities dealers report closing prices of the most actively traded bills, notes, and bonds to the Federal Reserve Bank of New York. CMT indexes are computed from yields on these securities. For example, the 1-year CMT yield is the average yield of the actively traded securities with a constant maturity of one year. The Federal Reserve publishes this index in its weekly statistical release H.15.

EXHIBIT 7.1 **Calculation of the Daily Standard Deviation Based on 25 Daily Observations for the 2-year Constant Maturity Treasury (June 21, 2002 to July 26, 2002)**

t	Date	y_t	$X_t = 100[\text{Ln}(y_t/y_{t-1})]$	$(X_t - \overline{X})^2$
0	20-Jun-02	2.930		
1	21-Jun-02	2.880	–1.72121	0.33586
2	24-Jun-02	2.930	1.72121	8.19616
3	25-Jun-02	2.920	–0.34188	0.63968
4	26-Jun-02	2.770	–5.27363	17.07301
5	27-Jun-02	2.850	2.84717	15.91090
6	28-Jun-02	2.900	1.73917	8.29932
7	01-Jul-02	2.880	–0.69204	0.20217
8	02-Jul-02	2.780	–3.53394	5.72289
9	03-Ju1-02	2.780	0.00000	1.30343
10	05-Jul-02	2.900	4.22598	28.81178
11	08-Jul-02	2.840	–2.09067	0.90058
12	09-Jul-02	2.740	–3.58461	5.96792
13	10-Jul-02	2.610	–4.86077	13.83163
14	11-Jul-02	2.610	0.00000	1.30343
15	12-Jul-02	2.560	–1.93430	0.62824
16	15-Jul-02	2.550	–0.39139	0.56294
17	16-Jul-02	2.660	4.22328	28.78276
18	17-Jul-02	2.630	–1.13423	0.00006
19	18-Jul-02	2.550	–3.08905	3.79225
20	19-Jul-02	2.480	–2.78348	2.69551
21	22-Jul-02	2.400	–3.27898	4.56806
22	23-Jul-02	2.340	–2.53178	1.93238
23	24-Jul-02	2.380	1.69496	8.04650
24	25-Jul-02	2.280	–4.29250	9.92770
25	26-Jul-02	2.200	–3.57181	5.90552
		Total	–28.655	175.3407

Note:

$$\text{Sample mean} = \overline{X} = \frac{-28.655}{25} = -1.14618$$

$$\text{Variance} = \frac{175.3407}{25-1} = 7.305863\%$$

$$\text{Std} = \sqrt{7.305863} = 2.70296\%$$

Source for daily yields: Federal Reserve Statistical Release H.15.

EXHIBIT 7.2 Comparison of Daily and Annual Volatility for a Different Number of Observations (Ending Date July 26, 2002) for Various Constant Maturity Treasury Rates and 6-month Libor

Number of Observations	Daily Standard Deviations (%)	Annualized Standard Deviation (%)		
		250 Days	260 Days	365 Days
10-Year Constant Maturity Treasury				
265	1.3562	21.4434	21.8681	25.9102
60	1.2678	20.0457	20.4427	24.2213
25	1.3799	21.8181	22.2502	26.3630
10	1.1906	18.8250	19.1979	22.7464
5-Year Constant Maturity Treasury				
265	1.7636	27.8850	28.4372	33.6935
60	1.5966	25.2445	25.7444	30.5030
25	1.7789	28.1269	28.6839	33.9858
10	1.5636	24.7227	25.2123	29.8725
2-Year Constant Maturity Treasury				
265	2.6305	41.5919	42.4156	50.2556
60	2.4791	39.1980	39.9743	47.3631
25	2.7029	42.7366	43.5830	51.6388
10	2.6836	42.4315	43.2718	51.2701
6-month Libor				
265	1.6775	26.5236	27.0489	32.0486
60	1.1643	18.4092	18.7738	22.2439
25	1.1758	18.5910	18.9592	22.4636
10	1.3991	22.1217	22.5598	26.7298

The selection of the number of observations can have a significant effect on the calculated daily standard deviation. This can be seen in Exhibit 7.2, which presents the daily standard deviation for a 10-year CMT yield, 5-year CMT yield, 2-year CMT yield, and 6-month Libor for 265 days, 60 days, 25 days, and 10 days ending 7/26/02.

Annualizing the Standard Deviation

The daily standard deviation can be annualized by multiplying it by the square root of the number of days in a year.[2] That is,

$$\text{Daily standard deviation} \times \sqrt{\text{Number of days in a year}}$$

Market practice varies with respect to the number of days in the year that should be used in the annualizing formula above. Typically, either 250 days, 260 days, or 365 days are used.

Thus, in calculating an annual standard deviation, the investor must decide on:

1. The number of daily observations to use.
2. The number of days in the year to use to annualize the daily standard deviation.

Exhibit 7.2 shows the difference in the annual standard deviation for the daily standard deviation based on the different number of observations and using 250 days, 260 days, and 365 days to annualize. Exhibit 7.3 compares the 25-day annual standard deviation for two different time periods for a 10-year CMT yield, 5-year CMT yield, 2-year CMT yield, and 6-month Libor.

Volatility Changes Over Time

As can be seen from the data in Exhibits 7.2 and 7.3, the volatility estimates differ considerably depending on the number of observations employed in the calculation and the sample period. To illustrate how volatility changes over time, we will employ Bloomberg's HVG (historical volatility graph) function which is presented in Exhibit 7.4. This graph displays the historical volatility of the 2-year CMT rate for the period February 1, 2002 to July 31, 2002. The time series of four annualized volatility measures are plotted and differ by the number of daily observations used in the calculation (e.g., 10, 30, 50, and 100 days). Daily volatility estimates are annualized assuming 250 days in the year and this appears at the bottom of the screen. The summer months of 2002 were particularly turbulent in the U.S. financial markets. During June and July 2002, the stock market expe-

[2] For any probability distribution, it is important to assess whether the value of a random variable in one period is affected by the value that the random variable took on in a prior period. Casting this in terms of yield changes, it is important to know whether the yield today is affected by the yield in a prior period. The term *serial correlation* is used to describe the correlation between the yield in different periods. Annualizing the daily yield by multiplying the daily standard deviation by the square root of the number of days in a year assumes that serial correlation is not significant.

rienced several days of violent intraday price swings. As investors rebalanced their portfolios away from common stocks and into safer securities, this higher volatility spilled over to the U.S. Treasury market.

Interpreting the Standard Deviation

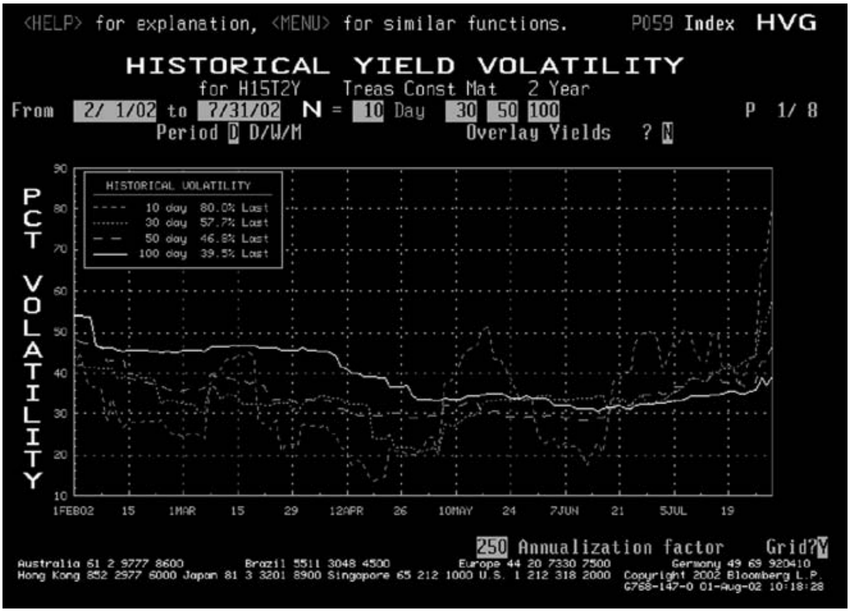
What does it mean if the annual standard deviation for the 2-year CMT yield is 30%. It means that if the prevailing yield is 2.5%, then the annual standard deviation is 75 basis points (2.5% times 30%).

Assuming that the yield volatility is approximately normally distributed, we can use this probability distribution to construct an interval or range for what the future yield will be. For example, we know that for a normal distribution there is a 68.3% probability that the yield will be between one standard deviation below and above the expected value (i.e., the mean). The expected value is the prevailing yield. If the annual standard deviation is 75 basis points and the prevailing yield is 2.5%, then there is a 68.3% probability that the yield next year will be between 1.75% (2.5% minus 75 basis points) and 3.25% (2.5% plus 75 basis points). For three standard deviations below and above the prevailing yield, there is a 99.7% that that yield next year will be in this interval. Using the numbers above, three standard deviations is 225 basis points (3 times 75 basis points). The interval is then 0.25% (2.5% minus 225 basis points) and 4.75% (2.5% plus 225 basis points).

EXHIBIT 7.3 Comparison of Daily Standard Deviations Calculated for Two 25-Day Periods

Dates		Daily standard deviation (%)	Annualized standard deviation (%)		
From	To		250 days	260 days	365 days
10-Year Constant Maturity Treasury					
7/3/01	8/7/01	0.7799	12.3313	12.5755	14.9000
6/20/02	7/26/02	1.3799	21.8181	22.2502	26.3630
5-Year Constant Maturity Treasury					
7/3/01	8/7/01	0.9794	15.4857	15.7924	18.7114
6/20/02	7/26/02	1.7789	28.1269	28.6839	33.9858
2-Year Constant Maturity Treasury					
7/3/01	8/7/01	1.1389	18.0076	18.3642	21.7587
6/20/02	7/26/02	2.7029	42.7366	43.5830	51.6388
6-month Libor					
7/3/01	8/7/01	0.5181	8.1919	8.3541	9.8983
6/20/02	7/26/02	1.1758	18.5910	18.9592	22.4636

EXHIBIT 7.4 Bloomberg Screen of Historical Yield Volatility for the 2-Year CMT



Source: Bloomberg Financial Markets

The interval or range constructed is called a *confidence interval*. Our first interval of 1.75%–3.25% is 68.3% confidence interval. Our second interval of 0.25%–4.75% is a 99.7% confidence interval. A confidence interval with any probability can be constructed using a normal probability distribution table.

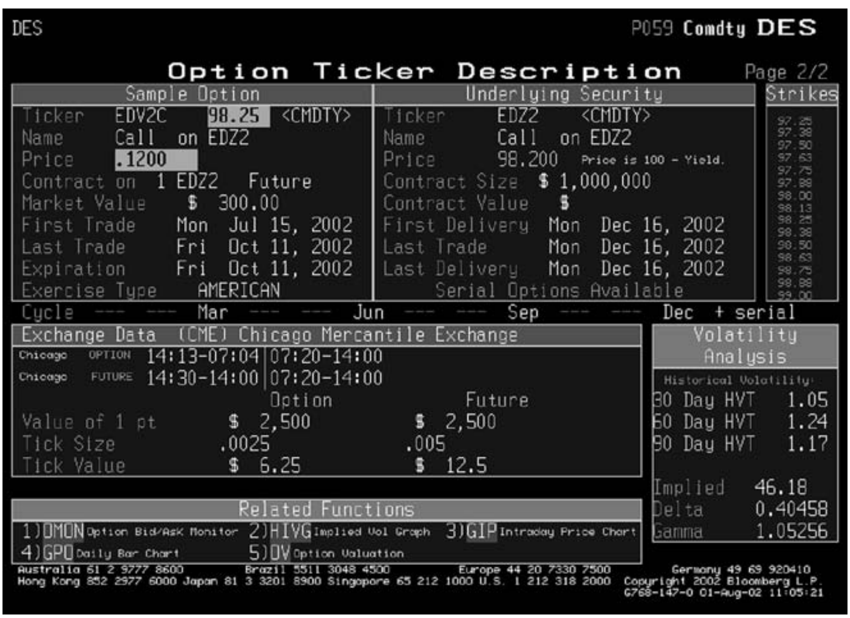
HISTORICAL VERSUS IMPLIED VOLATILITY

Market participants estimate yield volatility in one of two ways. The first way is by estimating historical yield volatility. This is the method that we have thus far described in this chapter. The resulting volatility is called *historical volatility*. The second way is to estimate yield volatility based on the observed prices of interest rate options or caps. Yield volatility calculated using this approach is called *implied volatility*.

The implied volatility is based on some option pricing model. One of the inputs to any option pricing model in which the underlying is a Treasury security or Treasury futures contract is expected yield volatility. If the observed price of an option is assumed to be the fair price and the option pricing model is assumed to be the model that would generate that fair price, then the implied yield volatility is the yield volatility that when used as an input into the option pricing model would produce the observed option price.

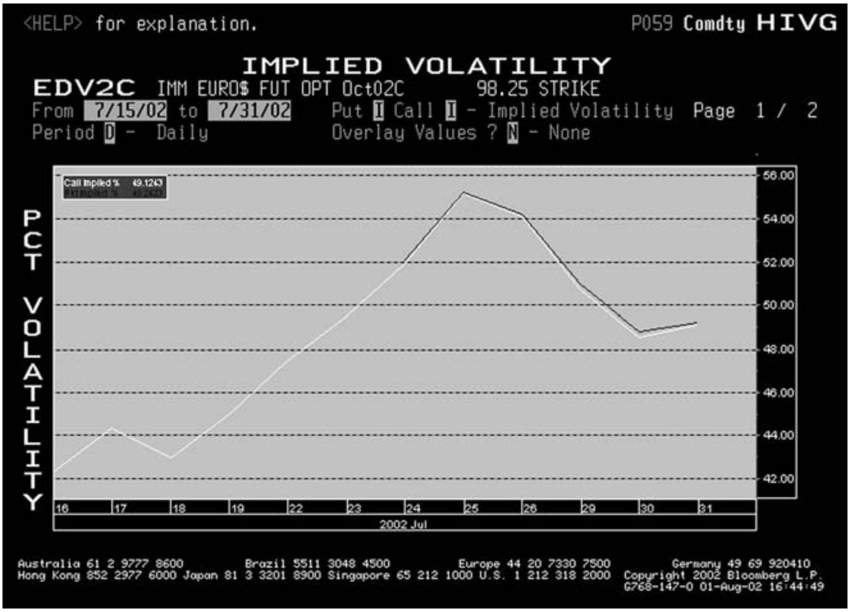
To illustrate the implied volatility, let us consider an option on a Eurodollar CD futures contract which is one of the most actively traded interest rate futures contracts in the world. As explained in Chapter 11, an option on a futures contract (or simply futures option) gives its owner the right to buy from or sell to the writer a designated futures contract at the strike price at any time during the option's life. A call (put) option on a futures contract gives the buyer the right to establish a long (short) futures position in the underlying contract. Exhibit 7.5 presents the Bloomberg Description screen for a call option on a 3-month Eurodollar CD futures contract. The exercise price is 98.25 and this option expires on October 11, 2002. A box in the lower left-hand corner of the screen contains the volatility analysis which includes the historical volatilities for the last 30, 60, and 90 days as well as the implied volatility of 46.18%. The option pricing model used in this calculation is the trinomial model.

EXHIBIT 7.5 **Bloomberg Description Screen of a Call Option on a 3-Month Eurodollar CD Futures Contract**



Source: Bloomberg Financial Markets

EXHIBIT 7.6 Bloomberg Screen of the Implied Volatility of Options on a 3-Month Eurodollar CD Futures Contract



Source: Bloomberg Financial Markets

Exhibit 7.6 presents a Bloomberg time series plot (function HIVG) of the implied volatilities for an October call and put option on a Eurodollar CD futures contract with an exercise price of 98.25. The sample period is July 15, 2002 to July 31, 2002. These daily implied volatilities are calculated using the prices of the three options whose exercise price is closest to the price of the underlying futures contract. In other words, the three option contracts that are closest to being at-the-money. The implied volatility displayed in the graph is a weighted average of the implied volatilities of these three options. The implied volatility of the options that are nearest to being at-the-money are weighted more heavily.

There are several problems with using implied volatility. First, it is assumed the option pricing model is correct. Second, option pricing models typically assume that volatility is constant over the life of the option. Therefore, interpreting an implied volatility becomes difficult. Third and perhaps most importantly, implied volatilities of options on the same underlying instrument should be the same regardless of the type of option (i.e., call or put), the time to expiration, and exercise price. In practice, implied volatilities do differ by the type of option, time to expiration, and exercise price.[3] This begs the question of which of the many implied volatilities should be used. To help them answer this question, many practitioners construct a three-dimensional plot of implied volatility against time to expiration and exercise price, which is called the *implied volatility surface*. The implied volatility surface represents the constant value of volatility that equates each traded option's model price to its market price.[4]

FORECASTING YIELD VOLATILITY[5]

As can be seen, the yield volatility as measured by the standard deviation can vary based on the time period selected and the number of observations. Now we turn to the issue of forecasting yield volatility. There are several methods. Before describing these methods, let's address the question of what mean should be used in the calculation of the forecasted standard deviation.

Suppose at the end of 7/11/02 an investor is interested in a forecast for volatility using the ten most recent days of trading and updating that forecast at the end of each trading day. What mean value should be used?

The investor can calculate a 10-day moving average of the daily percentage yield change. Exhibit 7.1 shows the percentage change in yield for a 2-year CMT from 6/20/02 to 7/26/2002. To calculate a moving average of the daily percentage change on 7/11/02, the trader would use the ten trading days from 6/26/02 to 7/11/02. At the end of 7/12/02, the trader will calculate the 10-day average by using the percentage yield change on 6/27/02 and would exclude the percentage yield change on 6/26/02. In other words, the trader will use the ten trading days from 6/27/02 to 7/12/02. Exhibit 7.7 shows the 10-day moving average calculated from 7/11/02 to 7/26/02. The 10-day moving average ranges from –0.5547% to –1.5781%.

Rather than using a moving average, it is more appropriate to use an expectation of the average. It has been argued that it would be more appropriate to use a mean value of zero.[6] In that case, the variance as given by equation (7.1) simplifies to

[3] The relationship between the implied volatility and the exercise price is called the *smile*.

[4] For more details, see Paul Wilmott, *Derivatives* (West Sussex, England: John Wiley & Sons Ltd., 1998).

[5] For a more extensive and rigorous discussion of forecasting yield volatility, see Frank J. Fabozzi and Wai Lee, "Forecasting Yield Volatility," in Frank J. Fabozzi (ed.), *Perspectives on Interest Rate Risk Management for Money Managers and Traders* (New Hope, PA: Frank J. Fabozzi Associates, 1997).

[6] Jacques Longerstacey and Peter Zangari, *Five Questions about RiskMetrics™,* JP Morgan Research Publication, 1995.

EXHIBIT 7.7 10-Day Moving Daily Average for a 2-Year Constant Maturity Treasury

10-Trading Days Ending	Daily Average (%)
11-Jul-02	–0.5950
12-Jul-02	–1.0731
15-Jul-02	–1.2862
16-Jul-02	–0.7946
17-Jul-02	–0.5547
18-Jul-02	–0.8636
19-Jul-02	–1.5645
22-Jul-02	–1.6834
23-Jul-02	–1.5781
24-Jul-02	–0.9225
25-Jul-02	–1.3518
26-Jul-02	–1.5155

$$\text{Variance} = \sum_{t=1}^{T} \frac{X_t^2}{T-1} \tag{7.2}$$

Now let's look at the various methods for forecasting daily volatility.

Equally-Weighted Average Method

The daily standard deviation given by equation (7.2) assigns an equal weight to all observations. So, if an investor is calculating volatility based on the most recent ten days of trading, each day is given a weight of 10%. For example, suppose that an investor is interested in the daily volatility of a 2-year CMT yield and decides to use the ten most recent trading days. Exhibit 7.8 reports the 10-day volatility for various days using the data in Exhibit 7.1 and the formula for variance given by equation (7.2) and then taking the square root. For the period 7/11/02 to 7/26/02, the 10-day volatility ranged from 2.5893% to 3.1844%. These high volatilities reflect the great turbulence in global financial markets during the summer of 2002.

Weighted Average Method

To give greater importance to more recent information, observations further in the past should be given less weight. This can be done by revising the variance as given by equation (7.2) as follows:

EXHIBIT 7.8 Moving Daily Standard Deviation Based on 10-Days of Observations Assuming a Mean of Zero and Equal Weighting

10-Trading Days Ending	Daily Standard Deviation (%)
11-Jul-02	3.0332
12-Jul-02	2.9522
15-Jul-02	2.8977
16-Jul-02	3.2133
17-Jul-02	3.0134
18-Jul-02	3.1844
19-Jul-02	3.0029
22-Jul-02	3.1187
23-Jul-02	3.0018
24-Jul-02	2.5893
25-Jul-02	2.9584
26-Jul-02	3.1231

$$\text{Variance} = \sum_{t=1}^{T} \frac{W_t X_t^2}{T-1} \tag{7.3}$$

where W_t is the weight assigned to observation t such that the sum of the weights is equal to 1 (i.e., $\Sigma\, W_t = 1$) and the further the observation from today, the lower the weight.

The weights should be assigned so that the forecasted volatility reacts faster to a recent major market movement and declines gradually as we move away from any major market movement. One approach is to use an *exponential moving average*.[7] The formula for the weight W_t in an exponential moving average is

$$W_t = (1 - \beta)\beta^t$$

where β is a value between 0 and 1. The observations are arrayed so that the closest observation is $t = 1$, the second closest is $t = 2$, and so on.

For example, if β is 0.90, then the weight for the closest observation ($t = 1$) is

$$W_1 = (1 - 0.90)\,(0.90)^1 = 0.09$$

[7] This approach is suggested by JP Morgan *RiskMetrics*™.

For $t = 5$ and β equal to 0.90, the weight is:

$$W_5 = (1 - 0.90)\,(0.90)^5 = 0.05905$$

The parameter β is measuring how quickly the information contained in past observations is "decaying" and hence is referred to as the "decay factor." The smaller the β, the faster the decay. What decay factor to use depends on how fast the mean value for the random variable X changes over time. A random variable, whose mean value changes slowly over time, will have a decay factor close to 1. A discussion of how the decay factor should be selected is beyond the scope of this book.[8]

ARCH Method and Variants

A times series characteristic of financial assets suggests that a period of high volatility is followed by a period of high volatility. Furthermore, a period of relative stability in returns appears to be followed by a period that can be characterized in the same way. This suggests that volatility today may depend upon recent prior volatility. This can be modeled and used to forecast volatility.

The statistical model used to estimate this time series property of volatility is called an *a*uto*r*egressive *c*onditional *h*eteroscedasticity or ARCH model.[9] The term "conditional" means that the value of the variance depends on or is conditional on the value of the random variable. The term heteroscedasticity means that the variance is not equal for all values of the random variable.

The simplest ARCH model is

$$\sigma_t^2 = a + b(X_{t-1} - \bar{X})^2 \qquad (7.4)$$

where

σ_t^2 = variance on day t
$X_{t-1} - \bar{X}$ = deviation from the mean on day t–1

and a and b are parameters.

The parameters a and b must be estimated statistically. The statistical technique of regression analysis is used to estimate the parameters.

Equation (7.4) states that the estimate of the variance on day t depends on how much the observation on day t–1 deviates from the

[8] A technical description is provided in *RiskMetrics™—Technical Document,* pp. 77–79.

[9] See Robert F. Engle, "Autoregressive Conditional Heteroskedasticity with Estimates of Variance of U.K. Inflation," *Econometrica* 50 (1982), pp. 987–1008.

mean. Thus, the variance on day t is "conditional" on the deviation from day t–1. The reason for squaring the deviation is that it is the magnitude, not the direction of the deviation, that is important for forecasting volatility.[10] By using the deviation on day $t - 1$, recent information (as measured by the deviation) is being considered when forecasting volatility.

The ARCH model can be generalized in two ways. First, information for days prior to t–1 can be included into the model by using the squared deviations for several prior days. For example, suppose that four prior days are used. Then equation (7.4) can be generalized to:

$$\sigma_t^2 = a + b_1(X_{t-1} - \overline{X})^2 + b_2(X_{t-2} - \overline{X})^2 + b_3(X_{t-3} - \overline{X})^2 + b_4(X_{t-4} - \overline{X})^2 \tag{7.5}$$

where a, b_1, b_2, b_3, and b_4 are parameters to be estimated statistically.

A second way to generalize the ARCH model is to include not only squared deviations from prior days as a random variable that the variance is conditional on but also the estimated variance for prior days. For example, the following equation generalizes equation (7.4) for the case where the variance at time t is conditional on the deviation squared at time t–1 and the variance at time t–1:

$$\sigma_t^2 = a + b(X_{t-1} - \overline{X})^2 + c\sigma_{t-1}^2 \tag{7.6}$$

where a, b, and c are parameters to be estimated statistically.

Suppose that the variance at time t is assumed to be conditional on four prior periods of squared deviations and three prior variances, then equation (7.4) can be generalized as follows:

$$\sigma_t^2 = a + b_1(X_{t-1} - \overline{X})^2 + b_2(X_{t-2} - \overline{X})^2 + b_3(X_{t-3} - \overline{X})^2 + b_4(X_{t-4} - \overline{X})^2 + c_1\sigma_{t-1}^2 + c_2\sigma_{t-2}^2 + c_3\sigma_{t-3}^2 \tag{7.7}$$

where the parameters to be estimated are a, the b_i's (i = 1, 2, 3, 4), and c_j's (j = 1, 2, 3).

Equations (7.5), (7.6), and (7.7) are referred to as *generalized* ARCH or GARCH models. GARCH models are conventionally denoted as follows: GARCH(i,j) where i indicates the number of prior squared deviations included in the model and j the number of prior variances in

[10] The variance for the unconditional variance (i.e., a variance that does not depend on the prior day's deviation) is

$$\sigma_t^2 = a/(1 - b)$$

the model. Equations (7.5), (7.6), and (7.7) would be denoted GARCH(4,0), GARCH(l,l), and GARCH(4,3), respectively.

There have been further extensions of ARCH models but these extensions are beyond the scope of this chapter.[11]

KEY POINTS

1. The standard deviation is commonly used as a measure of volatility.
2. Yield volatility can be estimated from daily yield observations.
3. The observation used in the calculation of the standard deviation is the natural logarithm of the percentage change in yield between two dates.
4. The selection of the number of observations and the time period can have a significant effect on the calculated daily standard deviation.
5. A daily standard deviation is annualized by multiplying it by the square root of the number of days in a year.
6. Typically, 250 days, 260 days, or 365 days are used to annualize the daily standard deviation.
7. Yield volatility varies considerably over time.
8. Assuming that the yield volatility is approximately normally distributed, the annual standard deviation can be used to construct a confidence interval for the yield one year from now.
9. Implied volatility can also be used to estimate yield volatility.
10. Implied volatility depends on the option pricing model employed as well as features of the option itself.

[11] For an overview of these extensions as well as the GARCH models, see Robert F. Engle, "Statistical Models for Financial Volatility," *Financial Analysts Journal*, January–February 1993, pp. 72–78.

11. In forecasting volatility, it is more appropriate to use an expectation of zero for the mean value.

12. The simplest method for forecasting volatility is weighting all observations equally.

13. A forecasted volatility can be obtained by assigning greater weight to more recent observations such that the forecasted volatility reacts faster to a recent major market movement and declines gradually as we move away from any major market movement.

14. Generalized autoregressive conditional heteroscedasticity (GARCH) models can be used to capture the time series characteristic of yield volatility in which a period of high volatility is followed by a period of high volatility and a period of relative stability appears to be followed by a period that can be characterized in the same way.

CHAPTER 8

Measuring Interest Rate Risk with Value-at-Risk

The purpose of this chapter is twofold. First, we introduce the risk measurement tool known as *Value-at-Risk* (VaR). Specifically, we discuss the general principles of the three main methodologies used to calculate VaR, as well as some of the key assumptions used in the calculations—probability distribution of returns, volatility, and correlations. Second, we illustrate how the VaR methodology can be used to measure the interest rate risk exposure for a bond portfolio. In Chapter 15, we will illustrate how VaR can be used to measure credit risk.

The objectives of this chapter are to:

1. Introduce the concept of value-at-risk as a measurement tool for risk.
2. Define value-at-risk (VaR).
3. Discuss the three different approaches to measuring VaR: variance-covariance; historical simulation; Monte Carlo simulation.
4. Calculate the VaR of a single fixed-income security using the variance-covariance method.
5. Illustrate the use of VaR to measure the interest rate risk of a porfolio of fixed-income securities using the variance-covariance method.
6. Discuss the issues in implementing VaR.

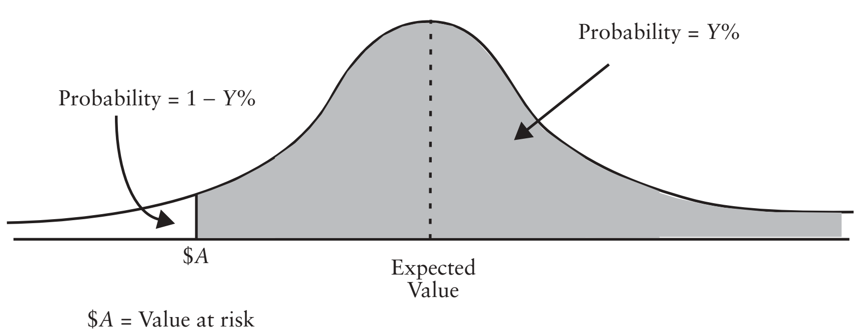
INTRODUCING VALUE-AT-RISK

Value-at-Risk (VaR) is a widely used methodology for quantifying risk (e.g., interest rate and credit risk) and its adoption by bank regulators is an indicator of its importance as a risk management tool. The application of VaR has been extended from its initial use in investment banks to commercial banks and corporations, following its introduction in October 1994 when JP Morgan launched RiskMetrics™. The basic idea of VaR is a simple one. VaR is a measure of the worst expected loss that a portfolio may suffer over a period of time that has been specified by the user, under normal market conditions and a specified level of confidence. Specifically, VaR is the expected loss of a portfolio over a specified time period for a set level of probability. For example, suppose a daily VaR is stated as $100,000 for a 95% level of confidence. This means there is a only a 5% chance that the loss the next day will be greater than $100,000. Stated another way, we expect this portfolio to lose more than $100,000 in one out of twenty days. This language emphasizes that VaR is not the maximum loss that will occur. Rather, we expect the actual loss to be greater than the VaR a certain percentage of the time—in this case, 5%.

VaR can be exhibited graphically assuming a normal probability distribution along with an expected value and standard deviation. Exhibit 8.1 shows a normal probability distribution for the change in the value of a portfolio over the next T days. The VaR is the loss of $\$A$ where the probability to the right of that value is $Y\%$. Correspondingly, the VaR is where the probability to the left of that value (i.e., the probability in the tail) is equal to $1 - Y\%$.

To implement VaR, all of a portfolio's positions data must be gathered into one centralized database. The overall risk calculated by aggregating the risks from individual instruments across the entire portfolio. The potential move in each instrument (that is, each risk factor) is inferred from past daily price movements over a given observation period (e.g., one year).

EXHIBIT 8.1 **Graphical Depiction of VaR**



There is no one VaR number for a single portfolio, because different methodologies used for calculating VaR produce different results. Moreover, the VaR number captures only those risks that can be measured in quantitative terms; it does not capture risk exposures such as operational risk, liquidity risk, regulatory risk or country risk. It is important to recognize precisely what VaR attempts to capture and what it does not. Moreover, using such a tool in no way compensates for inadequate procedures and rules in the management of a portfolio.

CALCULATION METHODS

There are three different methods for calculating VaR. They are

- Variance-covariance
- Historical simulation
- Monte Carlo simulation

Variance-Covariance Method

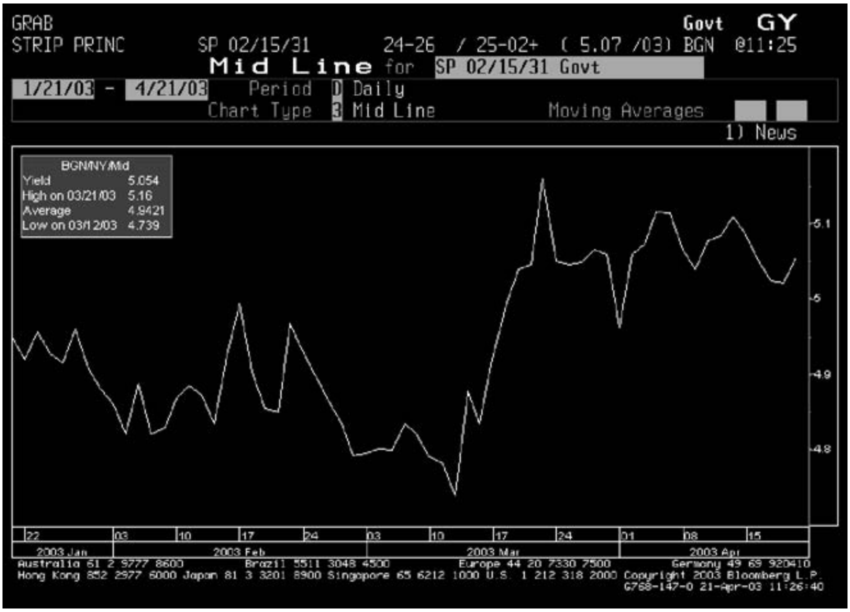
The variance-covariance method assumes the returns on risk factors are normally distributed, the correlations between risk factors are constant, and the delta (or price sensitivity to changes in a risk factor) of each portfolio constituent is constant. Since our focus in this chapter is using VaR to measure interest rate risk, delta would be an interest rate sensitivity measure (e.g., duration). The volatility of each risk factor is extracted from the historical observation period. Historical data on security returns is therefore required. The potential effect of each component of the portfolio on the overall portfolio value is then worked out from the component's delta (with respect to a particular risk factor) and that risk factor's volatility.

To calculate the VaR for a single asset with the variance-covariance method, the first step is to calculate the standard deviation of its returns using its historical volatility. If a 95% confidence level is required, meaning we wish to have 5% of the observations in the left-hand tail of the normal distribution, this means that the observations in that area are 1.645 standard deviations away from the mean (see Chapter 6).

Calculating the VaR of Single Bond

Suppose on April 21, 2003, we want to calculate the daily VaR for $1,000,000 face value position in U.S. Treasury principal strip that matures on February 15, 2031. The market value of the position is $250,550.62. Exhibit 8.2 presents a Bloomberg screen of the time series

EXHIBIT 8.2 Time Series of the U.S. Treasury Strips' Yields



Source: Bloomberg Financial Markets

of the Strip's yields each day from the period January 21, 2003 to April 21, 2003. The daily standard deviation over this period is 1.0718% using these daily yield observations computed using Bloomberg's historical volatility function. Suppose a 95% confidence level is required. Accordingly, the extreme loss tail is 1.645 standard deviations below the mean as noted. When computing VaR using daily returns, the mean is commonly assumed to be zero. In this example, the extreme loss tail is 1.645 standard deviations below the mean, which is calculated as follows: $1.645 \times 0.010718 = 0.0176$. This number tells us that there is only a 5% chance of getting a daily return of –1.76% or lower over the next day. The last step is to convert the extreme loss tail into a dollar value by multiplying by the market value of the position as follows: $0.0176 \times \$250{,}550.62 = \$4{,}417.49$. This number tells over the next day there is a only a 5% chance that the loss on the long position in the principal strip will be greater than $4,417.49.

While this method for computing historical volatility is the most straightforward approach, the effects of a large one-time market move can significantly distort estimated volatilities over the required forecasting period. For example if using 30-day historic volatility, a market shock will stay in the volatility figure for 30 days until it drops out of the sample range and correspondingly causes a sharp drop in (historic) volatility 30 days after the event. This is because each past observation is equally weighted in the volatility calculation.

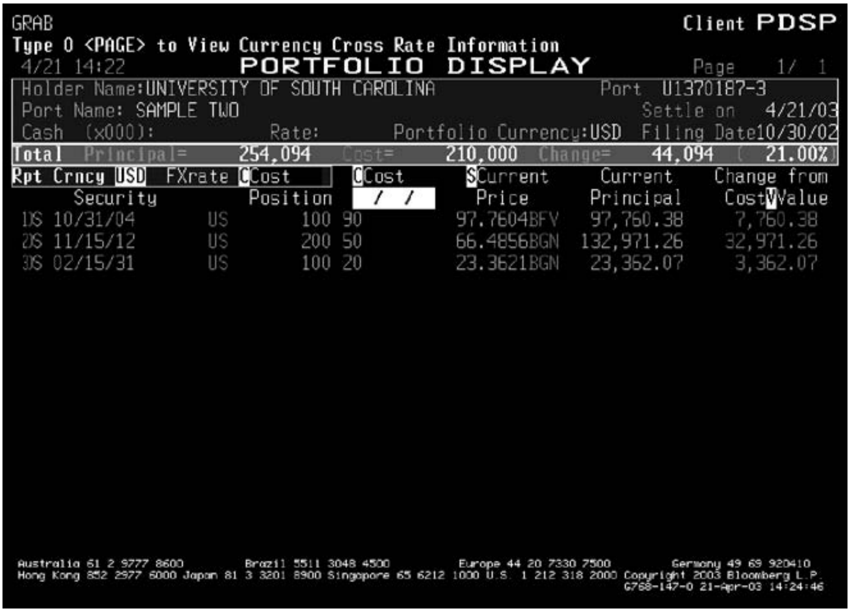
An alternative method is to weight past observations unequally. The logic of this approach is to give more weight to recent observations so that large jumps in volatility are not caused by events that occurred some time ago. One method is to use exponentially weighted moving averages as discussed in Chapter 7.

Calculating the VaR of a Bond Portfolio

As we increase the number of assets in the portfolio, the volatility used in the VaR calculation requires the variances of the individual asset returns as while as the correlation between asset returns. In a two-asset portfolio, when calculating the *undiversified VaR*, the portfolio's volatility is simply the weighted average of the individual standard deviations. This method assumes that all asset returns in the portfolio are perfectly positively correlated. Conversely, the *diversified VaR*, takes into account the correlation between the assets and uses the portfolio's standard deviation in the calculation. The standard deviation of a two-asset portfolio will be less than the weighted average of the individual standard deviations unless the returns of the two assets are perfectly positively correlated. So, in general, the diversified VaR will be lower than the undiversified VaR. In practice, financial institutions will calculate both diversified and undiversified VaR. Specifically, they use the diversified VaR measure to set trading limits, while the larger undiversified VaR measure is used to gauge an idea of the financial institution's risk exposure in the event of a significant correction or market crash. Undiversified VaR is more reflective of the financial institution's risk exposure because during a market dislocation liquidity dries up as market participants all attempt to rebalance their portfolios at once. If this occurs, the correlation between securities will increase and the portfolios' risk will increase, as all assets tend to move in the same direction.

As an illustration for calculating portfolio VaR, consider a portfolio of three U.S. Treasury strips displayed in the Bloomberg screen presented in Exhibit 8.3. The maturity dates and positions in each security are listed in the first two columns labelled "Security" and "Position," respectively. The market value of each security is listed in the fourth column labelled "Current Principal." Total market value of the portfolio is \$254,094.

EXHIBIT 8.3 **Bloomberg Screen Displaying the Portfolio of Treasury Strips**

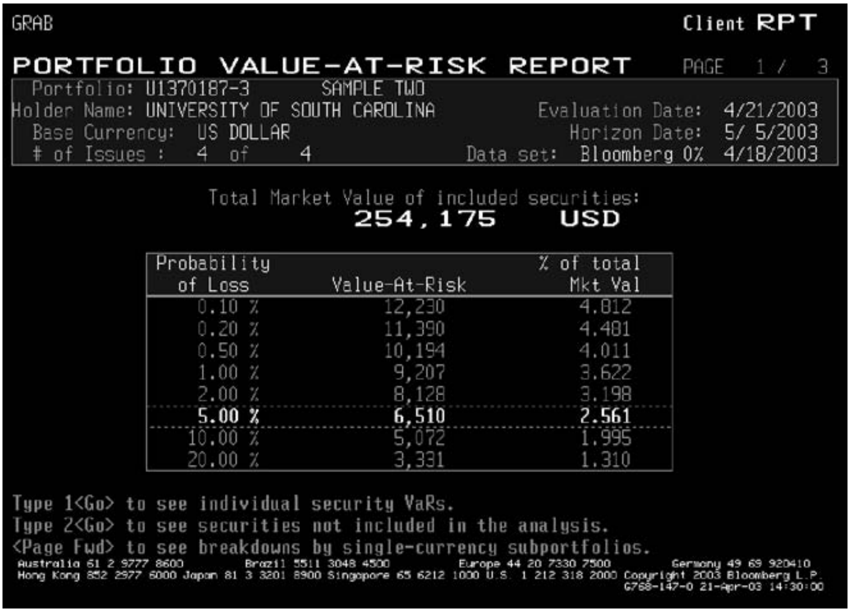


Source: Bloomberg Financial Markets

Let's employ Bloomberg's Portfolio Value-at-Risk function (PVAR) to compute the VaR for the portfolio of Treasury strips. Two important choices are the probability of loss and the horizon date. The probability of loss is percentage used to calculate the extreme loss tail of the distribution. The default is 5% just as we used in single asset example earlier. Conversely, the horizon date specifies the time period over which the loss is expected to occur. The default horizon date is two weeks hence which in this illustration is May 5, 2003.

Exhibit 8.4 presents Bloomberg's Portfolio Value-at-Risk Report. The VaR for the portfolio of the three Treasury strips is $6,510 for the specified 5% probability of loss. Specifically, the portfolio has a 5% chance of losing more than $6,510 of its market value between April 21, 2003 and May 5, 2003 (i.e., the horizon date). This number is the diversified VaR since the calculation computes the standard deviation of the portfolio using both historical volatilities and correlations. The undiversified VaR is just the sum of the individual securities" VaRs, which for this portfolio is $6,760. In this case, the diversification benefit of a portfolio of three U.S. Treasury strips is relatively small mainly because their volatilities tend to be highly correlated.

EXHIBIT 8.4 Bloomberg's Portfolio Value-at-Risk Report


GRAB Client RPT

PORTFOLIO VALUE-AT-RISK REPORT PAGE 1 / 3

Portfolio: U1370187-3 SAMPLE TWO
Holder Name: UNIVERSITY OF SOUTH CAROLINA Evaluation Date: 4/21/2003
Base Currency: US DOLLAR Horizon Date: 5/ 5/2003
\# of Issues : 4 of 4 Data set: Bloomberg 0% 4/18/2003

Total Market Value of included securities:
254,175 USD

Probability of Loss	Value-At-Risk	% of total Mkt Val
0.10 %	12,230	4.812
0.20 %	11,390	4.481
0.50 %	10,194	4.011
1.00 %	9,207	3.622
2.00 %	8,128	3.198
5.00 %	**6,510**	**2.561**
10.00 %	5,072	1.995
20.00 %	3,331	1.310

Type 1<Go> to see individual security VaRs.
Type 2<Go> to see securities not included in the analysis.
<Page Fwd> to see breakdowns by single-currency subportfolios.
Australia 61 2 9777 8600 Brazil 5511 3048 4500 Europe 44 20 7330 7500 Germany 49 69 920410
Hong Kong 852 2977 6000 Japan 81 3 3201 8900 Singapore 65 6212 1000 U.S. 1 212 318 2000 Copyright 2003 Bloomberg L.P.
G768-147-0 21-Apr-03 14:30:00

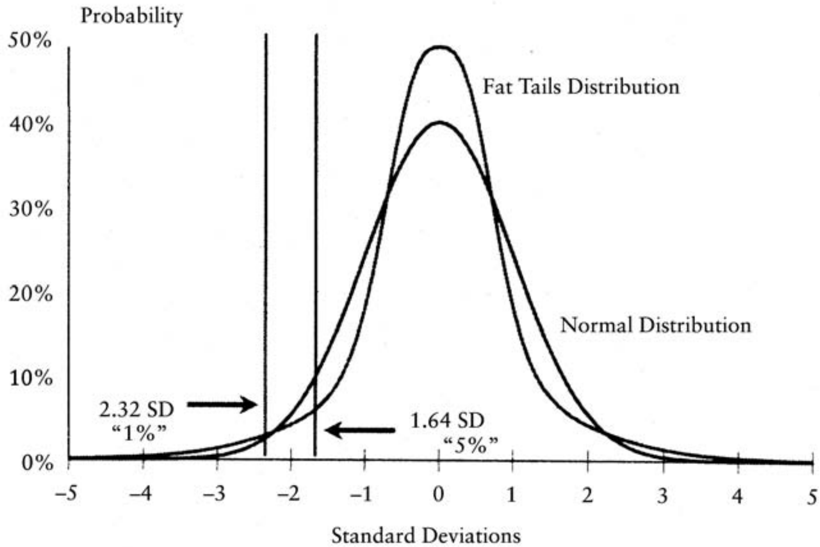
Source: Bloomberg Financial Markets

Advantages and Disadvantages of the Variance-Covariance Method

Michael Minnich states that primary advantages of the variance-covariance method are it is relatively easy to understand and implement.[1] As a result, it is the most commonly used method. This simplicity comes at a price as there are a number disadvantages to this method. First, the assumption of normality is problematic for the return distributions for several asset classes. Actual return distributions are such that large market moves occur more frequently than the normal distribution suggests. These distributions are described as having "fat tails." Exhibit 8.5 presents a comparison between a normal distribution versus a "fat tails" distribution. "Fat tails" introduces errors in the calculation of VaRs at higher confidence intervals. Second, the variance-covariance method does not capture the nonlinear payoff patterns of derivative securities (e.g., options) or securities with embedded options (e.g., callable corpo-

[1] Michael Minnich, "A Primer on Value at Risk," Chapter 3 in Frank J. Fabozzi (ed.), *Perspectives on Interest Rate Risk Management for Money Managers and Traders* (New Hope, PA: Frank J. Fabozzi Associates, 1998), pp. 39–50.

EXHIBIT 8.5 Normal versus "Fat Tails" Distribution



Source: Exhibit 5, p. 43, Michael Minnich, "A Primer on Value at Risk," Chapter 3 in Frank J. Fabozzi (ed.) *Perspectives on Interest Rate Risk Management for Money Managers and Traders* (New Hope, PA: Frank J. Fabozzi Associates, 1998), pp. 39–50.

rate bonds and mortgage-backed securities). Lastly, the variance-covariance method does not account for any time dependency of delta—the price sensitivity to the risk factor which for fixed-income securities is a measure like duration.

Historical Simulation Method

The historical simulation method for calculating VaR is an alternative approach to calculating VaR and avoids some of the assumptions of the variance-covariance method. Specifically the three main assumptions (normally distributed returns, constant correlations, constant deltas) are not needed for this method. When the historical simulation model is employed, potential losses are estimated using actual historical returns in the risk factors. Thus, rather than imposing a normal distribution, we permit nonnormal distributions of risk factor returns. This means that rare events and crashes can be included in the results. As the risk factor returns used for revaluing the portfolio are actual past movements, the correlations in the calculation are also actual past correlations. They capture the dynamic nature of correlation as well as scenarios when the usual correlation relationships break down.

The historical approach to value-at-risk is a relatively simple calculation, and it is also easy to implement and explain. To implement it, the user requires a database record of its past returns for the total portfolio; the required confidence interval is then applied to this record, to obtain a cut-off of the worst-case scenario. For example, to calculate the VaR at a 95% confidence level, the 5th percentile is value for the historical data is taken, and this is the VaR number. For a 99% confidence level measure, the 1% percentile is taken.

The advantage of the historical method is that it uses the actual market data that a financial institution has recorded and so produces a reasonably accurate figure. Its main strength is also its main weakness: It is reliant on actual historical data built up over a period of time. Generally, at least one year's data is required to make the calculation meaningful. Therefore it is not suitable for portfolios whose asset weightings frequently change, as another set of data would be necessary before a VaR number could be calculated.

Monte Carlo Simulation Method

The third method, Monte Carlo simulation, is more flexible than the previous two. As with historical simulation, Monte Carlo simulation allows the user to use actual historical distributions for risk factor returns rather than assuming normality. A large number of randomly generated simulations are run forward in time using volatility and correlation estimates chosen by the user. Each simulation will be different but in total the simulations will aggregate to the chosen statistical parameters (that is, historical distributions and volatility and correlation estimates). This method is more realistic than the previous two methods and therefore is more likely to estimate VaR more accurately. However its implementation requires powerful computers and there is also a trade-off in that the time required to perform calculations is longer.

ISSUES IN IMPLEMENTING VAR

There are a number of decisions to be prior to calculating VaR and simplifications that must be made to make calculating VaR tractable. Tanya Styblo Bender presents evidence that the choices that a user makes can have a drastic impact on the final result.[2]

[2] See Tanya Styblo Beder, "VAR: Seductive but Dangerous," *Financial Analysts Journal,* September–October 1995, pp. 12–24.

Time Horizon

Earlier we defined VaR as the expected loss of a portfolio over a specified time period for a set level of probability. The "specified time period" is the time horizon for which the VaR is calculated. The choice of time horizon depends on the portfolio's objectives as well as its liquidity.[3] For most fixed-income portfolios, the time horizon ranges from one day to two weeks. In principle, the time horizon for a portfolio's VaR calculation should represent the period of time required to unwind the portfolio, that is, sell off the assets in the portfolio. A 10-day holding period is recommended but would be unnecessary for a highly liquid portfolio of government bonds.

Confidence Intervals

The level of confidence at which the VaR is calculated will depend on the nature of the portfolio and what the VaR number is being used for. For financial institutions, the Basel Capital Accord stipulates a 99% confidence interval and a 10-day holding period if the VaR measure is to be used to calculate the regulatory capital requirement. However certain financial institutions prefer to use other confidence levels and holding periods; the decision on which level to use is a function of asset types in the portfolio, quality of market data available and the accuracy of the model itself, which will have been tested over time by the user.

For example, a financial institution may view a 99% confidence interval as providing no useful information, as it implies that there should only be two or three breaches of the VaR measure over the course of one year; that would leave no opportunity to test the accuracy of the model until a relatively long period of time had elapsed, in the meantime the financial institution would be unaware if the model was generating inaccurate numbers. A 95% confidence level implies the VaR level being exceeded around one day each month, if a year is assumed to contain 250 days. In the same way, there maybe occasions when a financial will wish to calculate VaR over a different holding period to that recommended by the Basel Committee.

Mapping

The cornerstone of variance-covariance methodologies for calculating VaR is the requirement for data on volatilities and correlations for assets in the portfolio. The Bloomberg and RiskMetrics™ datasets do not contain volatilities for every fixed-income security in the world and correlations between all possible pairs of security returns. This would result in an

[3] See Minnich, "A Primer on Value at Risk."

excessive amount of calculation. Instead, Bloomberg, for example, monitors several thousand primitive assets which represent specific sectors and maturities in the fixed-income market. Any security can be thought of a portfolio of one of more primitive assets. Accordingly, Bloomberg maps each security position into an equivalent position of primitive assets. The volatilities and correlations of the primitive assets are then used in the VaR calculation. RiskMetrics™ uses a similar approach.

Stress Testing

As noted, the VaR calculation relies several inputs/decisions made by the user in addition to the basic assumptions underlying each calculation method. It is important to understand what will happen should some of the calculation method's underlying assumptions break down. Stress testing is a process whereby a series of scenario analyses or simulations are carried out to investigate the effect of extreme market conditions on the VaR estimates calculated by a model. It is also an analysis of the effect of violating any of the basic assumptions behind VaR. If carried out efficiently stress testing will provide a clearer information on the potential exposures at risk due to significant market corrections.

One approach is to simulate extreme market moves over a range of different scenarios (e.g., Monte Carlo simulation). This method allows dealers to push the risk factors to greater limits; for example a 99% confidence interval captures events up to 2.33 standard deviations from the mean asset return level. A user can calculate the effect on the trading portfolio of a 10-standard-deviation move. Similarly one may want to change the correlation assumptions under which they typically work. For instance if markets all move down together, something that happened in Asian markets from the end of 1997 and emerging markets generally from July 1998 after the Russian bond technical default, losses will be greater than if some markets are offset by other negatively correlated markets.

For effective stress testing, a portfolio manager has to consider nonstandard situations. For financial institutions, the Basel policy group has recommended certain minimum standards in respect of specified market movements; the parameters chosen are considered large moves over a one day time horizon, including:

- Parallel yield curve shifts of 100 basis points up and down.
- Steepening and flattening of the yield curve (2-year to 10-year) by 25 basis points.
- Increase and decrease in 3-month yield volatilities by 20%.

These scenarios represent a starting point for a framework for routine stress testing.

KEY POINTS

1. Value-at-risk (VaR) is a statistical measure of the potential risk exposure of a portfolio of assets.
2. There are three different methods for calculating VaR: variance-covariance, historical simulation, and Monte Carlo simulation.
3. The variance-covariance method assumes that the returns on risk factors are normally distributed, the correlations between risk factors are constant and the delta of each security is constant.
4. The undiversified VaR of a portfolio is the sum of the individual asset VaRs.
5. The diversifed VaR takes into account the correlation between the assets in the portfolio.
6. The variance-covariance method is less effective when return distributions have “fat tails” and assets have nonlinear payoffs.
7. The historical simulation method for calculating VaR permits non-normal distributions of risk factor returns.
8. The Monte Carlo simulation method generates a large number of randomly generated simulations for possible returns in the future using volatility and correlation estimates chosen by the user.
9. There are several issues in implementing VaR: time horizon, confidence intervals, mapping , and stress testing.
10. The time horizon is the specified time period for which VaR is calculated.
11. The confidence interval specifies the probability of loss.
12. Mapping is the procedure used by Bloomberg and Riskmetrics to calculate the VaR of any security by viewing it a portfolio of one or more primitive assets.
13. Stress testing is the process whereby a series of simulations is carried out to investigate the impact of changing assumptions on the calculated VaR.

CHAPTER 9

Futures and Forward Rate Agreements

This chapter is the first of several chapters devoted to the derivative instruments employed by market participants to control their exposure to interest rate and credit risk. As the name implies, a derivative instrument is one that derives its value from some underlying variable or variables. The underlying variable could be the price of a financial asset, the level of an interest rate, or the spread between two interest rates. Indeed, the possibilities of variables underlying a derivative contract are virtually limitless. For example, there are derivative instruments whose payoffs depend on insurers' underwriting losses arising from natural catastrophes (e.g., hurricanes or earthquakes). The focus of this chapter is on interest rate futures and forward rate agreements. We will discuss forward contracts first as a way of introducing the topic and then proceed quickly to a discussion of interest rate futures contracts. Our discussion then turns to how futures are priced. In the last section of the chapter, we discuss forward rate agreements.

The objectives of this chapter are to:

1. Explain the basic features of interest rate futures and forward contracts.
2. Explain the risk and return characteristics of futures/forward contracts.
3. Eescribe the most popular interest rate futures contracts.
4. Explain the pricing of forward rate agreements.
5. Demonstrate how the theoretical price of a futures/forward contract is determined.
6. Explain the complications in extending the standard arbitrage pricing model to the valuation of several currently traded interest rate futures contracts.
7. Describe forward rate agreements.

FORWARD CONTRACTS

A *forward contract* is an over-the-counter agreement between two parties for the future delivery of the underlying at a specified price at the end of a designated time period. The designated date at which the parties must transact is called the *settlement* or *delivery date*. The party that assumes the long (short) position is obligated to buy (sell) the underlying at the specified price. The terms of the contract are the product of negotiation between the two parties. As such, a forward contract is specific to the two parties. Although we commonly refer to taking a long position as "buying a forward contract" and conversely taking a short position as "selling a forward contract," this is a misnomer. No money changes hands between the parties at the time the forward contract is established. Both sides are making a promise to engage in a transaction in the future according to terms negotiated upfront.

At the settlement date, the party with the long position pays the specified price called the *forward price* in exchange for delivery of the underlying from the party with the short position. The payoff of the forward contract for the long position on the settlement date is simply the difference between the price of the underlying minus the forward price. Conversely, the payoff of the forward contract for the short position on the settlement date is the difference between the forward price minus the price of the underlying. Clearly, a forward contract is a zero-sum game.

Now that we have introduced forward contracts, it is a short walk to futures contracts.

FUTURES CONTRACTS

A *futures contract* is a legal agreement between a buyer (seller) and an established exchange or its clearinghouse in which the buyer (seller) agrees to take (make) delivery of something at a specified price at the end of a designated time period. The price at which the parties agree to transact in the future is called the *futures price*. When a market participant takes a position by buying a futures contract, the individual is said to be in a *long futures position* or to be *long futures*. If, instead, the market participant's opening position is the sale of a futures contract, the investor is said to be in a *short position* or *short futures*.

As can be seen from the description, a futures contract is quite similar to a forward contract. They differ on four ways. First, futures contracts are standardized agreements as to the settlement date (or month) and quality of the deliverable. Moreover, because these contracts are standardized, they are traded on organized exchanges. In contrast, forward contracts are usually negotiated individually between buyer and seller and the secondary markets are often nonexistent or extremely thin. Second, an intermediary called a clearinghouse (whose function is discussed shortly) stands between the two counterparties to a futures contract and guarantees their performance. Both parties to a forward contract are subject to *counterparty risk*. Counterparty risk is the risk that the other party to the contract will fail to perform. Third, a futures contract is *marked-to-market* (discussed shortly) while a forward contract may or may not be marked-to-market. Last, although both a futures and forward contract set forth terms of delivery, futures contracts are not intended to be settled by delivery.

Role of the Clearinghouse

Associated with every futures exchange is a *clearinghouse*, which performs several functions. One of these functions is guaranteeing that the two parties to the transaction will perform. When a market participant takes a position in the futures market, the clearinghouse takes the opposite position and agrees to satisfy the terms set forth in the contract. Because of the clearinghouse, the user need not worry about the financial strength and integrity of the counterparty to the contract. After the initial execution of an order, the relationship between the two parties ends. The clearinghouse interposes itself as the buyer for every sale and the seller for every purchase. Thus, users are free to liquidate their positions without involving the other party in the original contract and without concern that the other party may default. This is the reason why we define a futures contract as an agreement between a party and a clearinghouse associated with an exchange. In addition to its guarantee function, the clearinghouse makes it simple for parties to a futures contract to unwind their positions prior to the settlement date.

Margin Requirements

When a position is established in a futures contract, each party must deposit a minimum dollar amount per contract as specified by the exchange in the terms of the contract. This amount, which is called the *initial margin*, is required as deposit by the exchange.[1]

The initial margin may be in the form of an interest-bearing security such as a Treasury bill. In some futures exchanges around the world, other forms of margin are accepted such as common stock, corporate

[1] Individual brokerage firms are free to set margin requirements above the minimum established by the exchange.

bonds or even letters of credit. As the price of the futures contract fluctuates, the value of the user's equity in the position changes. At the end of each trading day, the exchange determines the *settlement price* of the futures contract which is an average of the prices of the last few trades of the day. This price is used to mark-to-market the user's position, so that any gain or loss from the position is reflected in the investor's margin account.

Maintenance margin is the minimum level (specified by the exchange) to which a user's margin account may fall as a result of an unfavorable price change before the user is required to deposit additional margin. The additional margin deposited is called *variation margin* and it is an amount necessary to bring the margin in the account balance back to its initial margin level. Unlike initial margin, variation margin must be in cash, not interest-bearing instruments. If a party to a futures contract who is required to deposit variation margin fails to do so within 24 hours, the futures position is closed out. Conversely, any excess margin may be withdrawn by the user.

Although there are initial and maintenance margin requirements for buying securities on margin, the concept of margin differs for securities and futures. When securities are acquired on margin, the difference between the security's price and the initial margin is borrowed from the broker. The security purchased serves as collateral for the loan and the investor pays interest. For futures contracts, the initial margin, in effect, serves as a performance bond, an indication that the user will be able to satisfy the obligation of the contract. Normally, no money is borrowed.

EXCHANGE-TRADED INTEREST RATE FUTURES CONTRACTS

Interest rate futures contracts can be classified by the maturity of their underlying instrument. Short-term interest rate futures contracts have an underlying instrument that matures in one year or less. Examples of this type are futures contracts in which the underlying instrument is a 3-month U.S. Treasury bill or a 3-month Eurodollar certificate of deposit. The maturity of the underlying instrument of long-term futures contracts exceeds one year. Examples of this type are futures contracts in which the underlying is a coupon Treasury or an agency bond. In the section, we describe several futures contracts of each type.

Short-Term Interest Rate Futures Contracts

The more actively traded short-term interest futures contracts in the United States and the United Kingdom are described below.

U.S. Treasury Bill Futures

The Treasury bill futures market, which is traded on the International Monetary Market (IMM) of the Chicago Mercantile Exchange, is based on a 13-week (3-month) Treasury bill with a face value of $1 million. More specifically, the seller of a Treasury bill futures contract agrees to deliver to the buyer on the settlement date a Treasury bill with 13 weeks remaining to maturity and a face value of $1 million. The Treasury bill delivered could be a newly issued 13-week Treasury bill or a seasoned 26-week Treasury bill that has only 13 weeks remaining until maturity. The futures price is the price at which the Treasury bill will be sold by the short and purchased by the long. For example, a Treasury bill futures contract that settles in 3 months requires that 3 months from now the short deliver to the long $1 million face value of a Treasury bill with 13 weeks remaining to maturity.

The convention for quoting bids and offers in the secondary market is different for Treasury bills and Treasury coupon securities. Bids/offers on bills are quoted in a special way. Unlike bonds that pay coupon interest, Treasury bill values are quoted on a *bank discount basis*, not on a price basis. The yield on a bank discount basis is computed as follows:

$$Y_d = \frac{D}{F} \times \frac{360}{t}$$

where

Y_d = annualized yield on a bank discount basis (expressed as a decimal)
D = dollar discount, which is equal to the difference between the face value and the price
F = face value
t = number of days remaining to maturity

Given the yield on a bank discount basis, the price of a Treasury bill is found by first solving the formula for the dollar discount (D), as follows:

$$D = Y_d \times F \times (t/360)$$

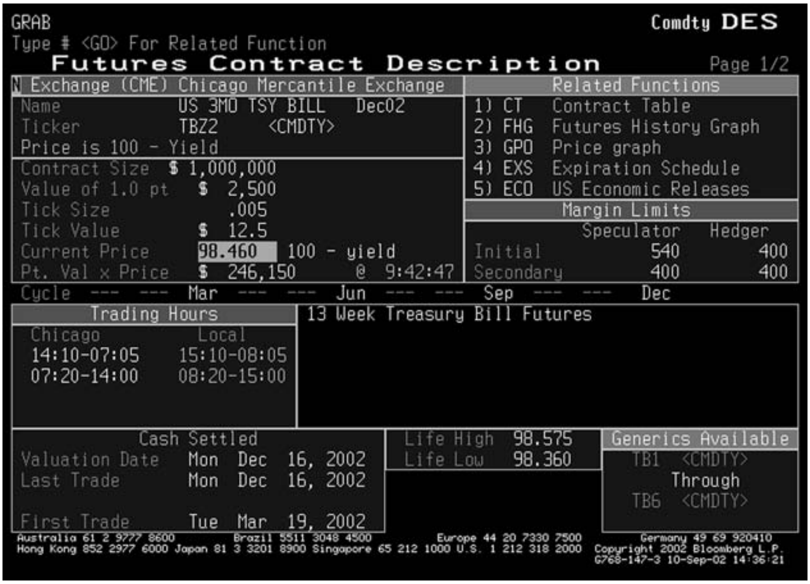
The price is then

$$\text{price} = F - D$$

In contrast, the Treasury bill futures contract is quoted not directly in terms of yield, but instead on an index basis that is related to the yield on a bank discount basis as follows:

$$\text{Index price} = 100 - (Y_d \times 100)$$

EXHIBIT 9.1 **Bloomberg Futures Contract Description Screen for a U.S. Treasury Bill Futures Contract**



Source: Bloomberg Financial Markets

For example, if Y_d is 1.54%, the index price is

$$100 - (0.0154 \times 100) = 98.460$$

Given the index price of the futures contract, the yield on a bank discount basis for the futures contract is determined as follows:

$$Y_d = \frac{100 - \text{Index price}}{100}$$

To illustrate how this works, let's use Bloomberg's Futures Contract Description screen presented in Exhibit 9.1. This 3-month U.S. Treasury bill futures contract began trading on March 19, 2002 and settles on December 16, 2002. On September 10, 2002, the index price was 98.460, which is labeled as "Current Price" and is located on the left-hand side of the screen. The yield on a bank discount basis for this Treasury bill futures contract is:

$$Y_d = \frac{100 - 98.460}{100} = 0.0154 \text{ or } 1.54\%$$

The invoice price that the buyer of a \$1 million face value 3-month Treasury bill must pay at settlement is found by first computing the dollar discount, as follows:

$$D = Y_d \times \$1{,}000{,}000 \times t/360$$

where t is either 90 or 91 days.

The number of days to maturity of a 3-month Treasury bill is usually 91 days or 13 weeks. The invoice price is then

$$\text{Invoice price} = \$1{,}000{,}000 - D$$

For example, if the index price is 98.460 (and a yield on a bank discount basis of 1.54%), the dollar discount for the 3-month Treasury bill to be delivered with 91 days to maturity is

$$D = 0.0154 \times \$1{,}000{,}000 \times 91/360 = \$3{,}892.778$$

The invoice price is

$$\text{Invoice price} = \$1{,}000{,}000 - \$3{,}892.778 = \$996{,}107.222$$

The minimum index price fluctuation or "tick" for this futures contract is 0.005. A change of 0.005 for the minimum index price translates into a change in the yield on a bank discount basis of one-half of a basis point (0.00005). A one-half basis point change results in a change in the invoice price as follows:

$$0.00005 \times \$1{,}000{,}000 \times t/360$$

For a 13-week Treasury bill with 91 days to maturity, the change in the dollar discount is:

$$0.00005 \times \$1{,}000{,}000 \times 91/360 = \$12.639$$

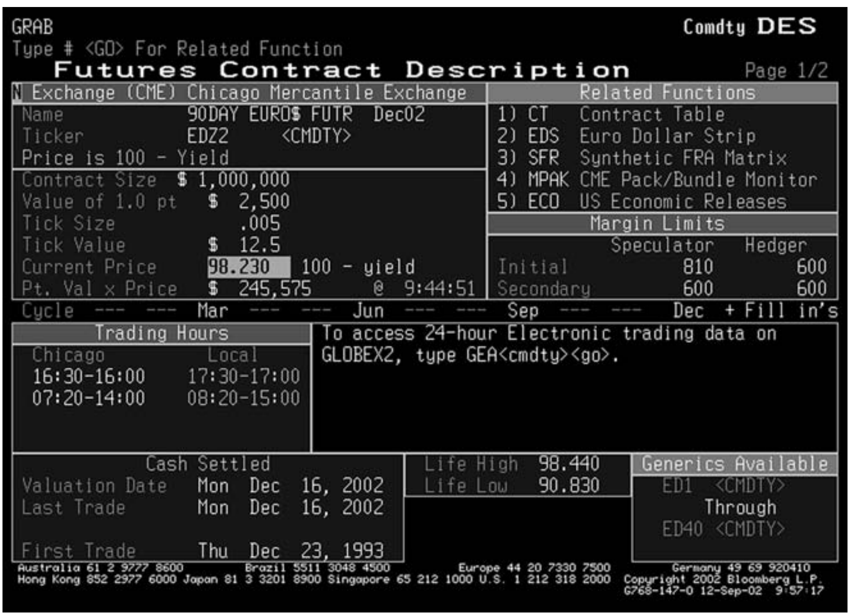
For a 13-week Treasury bill with 90 days to maturity, the change in the dollar discount would be \$12.50. Despite the fact that a 13-week Treasury bill usually has 91 days to maturity, market participants commonly refer to the value of a tick for this futures contract as \$12.50. As evidence of this, on the left side of Exhibit 9.1, the "Tick Value" is reported to be \$12.50.

Eurodollar CD Futures

Eurodollar CDs are U.S. dollar-denominated CDs issued primarily in London by U.S., Canadian, European, and Japanese banks. These CDs earn a fixed rate of interest related to dollar Libor. The term Libor comes from the London Interbank Offered Rate and is the interest rate at which one London bank offers funds to another London bank of acceptable credit quality in the form of a cash deposit. The rate is “fixed” by the British Bankers Association every business morning by the average of the rates supplied by member banks.

The 3-month (90 days) Eurodollar CD is the underlying instrument for the Eurodollar CD futures contract. The contracts are traded on the International Monetary Market of the Chicago Mercantile Exchange and the London International Financial Futures Exchange (LIFFE). Exhibit 9.2 presents the Bloomberg Futures Contract Description screen for the December 2002 contract. As with the Treasury bill futures contract, this contract has a $1 million face value and is traded on an index price basis. The index price basis in which the contract is quoted is equal to 100 minus the annualized futures Libor. For example, a Eurodollar CD futures price of 98.23 means a futures 3-month Libor of 1.77%.

EXHIBIT 9.2 **Bloomberg Futures Contract Description Screen for a Eurodollar CD Futures Contract**



Source: Bloomberg Financial Markets

EXHIBIT 9.3 Bloomberg Contract Table for a Eurodollar CD Futures Contract

GRAB Govt CTM
Enter # <GO> to scroll contracts. Run EXCH for realtime authorized exchanges
Session: D Contract Table
90DAY EURO$ FUTR Delayed monitoring enabled
Exchange Web Page Pricing Date: 9/12/02 Price Display: 2
Chicago Mercantile Exchange Delayed prices --LATEST AVAILABLE--- 2
Grey date = options trading

↓ Scroll		Last	2 Pct Chg	Time	Bid	1 Ask	4484182 OpenInt	571191 TotVol	Previous Close
1) EDU2	Sep02	98.1775d	unch	9:43	98.1775	98.1800	598348	51310	98.1775
2) EDV2	Oct02	98.200d	+.02%	9:00	98.190	98.200	45838	4491	98.185
3) EDX2	Nov02	98.230d	+.03%	8:40	98.210	98.215	10697	100	98.205
4) EDZ2	Dec02	98.230d	+.04%	9:44	98.225	98.230	730716	101741	98.195
5) EDF3	Jan03	98.190d	+.05%	9:22	98.180	98.195	3497	1	98.140
6) EDG3	Feb03				98.130	98.150	75	50	98.080
7) EDH3	Mar03	98.085df	+.06%	9:45	98.085	98.090	621177	144224	98.030
8) EDM3	Jun03	97.805d	+.08%	9:43	97.805	97.810	448751	118636	97.730
9) EDU3	Sep03	97.445d	+.08%	9:44	97.440	97.445	382442	47218	97.365
10) EDZ3	Dec03	97.090d	+.07%	9:42	97.090	97.095	320179	40553	97.020
11) EDH4	Mar04	96.775d	+.06%	9:37	96.780	96.785	203928	18704	96.720
12) EDM4	Jun04	96.525d	+.06%	9:41	96.530	96.540	163936	13015	96.470
13) EDU4	Sep04	96.310d	+.05%	9:36	96.320	96.320	133824	5481	96.260
14) EDZ4	Dec04	96.110d	+.06%	9:44	96.105		131564	5712	96.055
15) EDH5	Mar05	95.945d	+.05%	9:35	95.955		105660	3119	95.895
16) EDM5	Jun05	95.800d	+.06%	9:20	95.795		93155	4706	95.745
17) EDU5	Sep05	95.635d	+.04%	9:24	95.645		79061	2056	95.595

Australia 61 2 9777 8600 Brazil 5511 3048 4500 Europe 44 20 7330 7500 Germany 49 69 920410
Hong Kong 852 2977 6000 Japan 81 3 3201 8900 Singapore 65 212 1000 U.S. 1 212 318 2000 Copyright 2002 Bloomberg L.P.
G768-147-0 12-Sep-02 9:55:29

Source: Bloomberg Financial Markets

The minimum price fluctuation (tick) for this contract is 0.005 or ½ basis point. This means that the tick value for this contract is \$12.50, which is determined as follows:

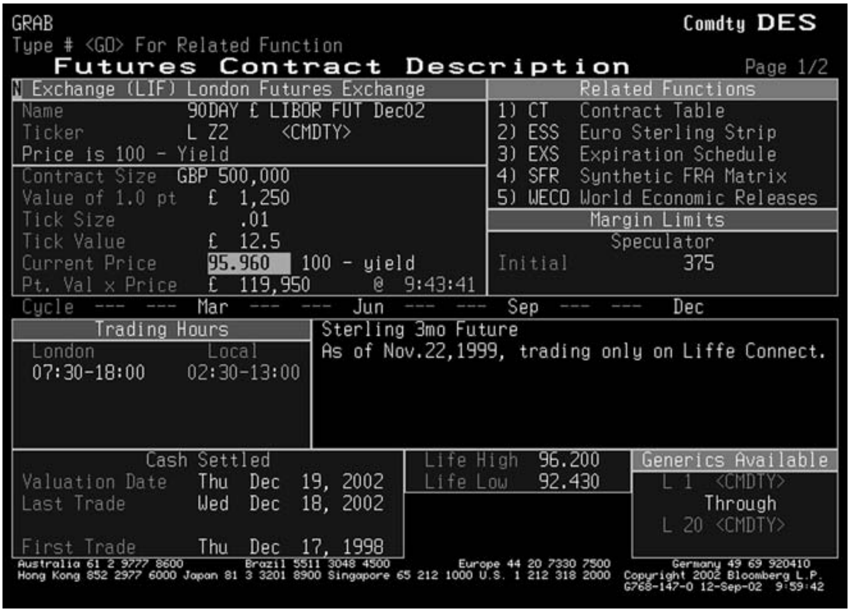
$$\text{Tick value} = \$1{,}000{,}000 \times (0.005 \times 90/360) = \$12.50$$

This expression appears in the lower right-hand corner of Exhibit 9.2.

The Eurodollar CD futures contract is a cash settlement contract. Specifically, the parties settle in cash for the value of a Eurodollar CD based on Libor at the settlement date. The Eurodollar CD futures contract is one of the most heavily traded futures contracts in the world. Exhibit 9.3 presents Bloomberg's Contract Table screen for the active 90-day Eurodollar CD futures contracts on September 12, 2002. Note the very large open interest for September 02, December 02, March 03, and June 03 contracts. Open interest is simply is the number of futures contracts established that have yet to be offset.

The Eurodollar CD futures contract is used frequently to trade the short end of the yield curve and many hedgers believe this contract to be the best hedging vehicle for a wide range of hedging situations.

EXHIBIT 9.4 **Bloomberg Futures Contract Description Screen for a 90-Day Sterling Libor Contract**

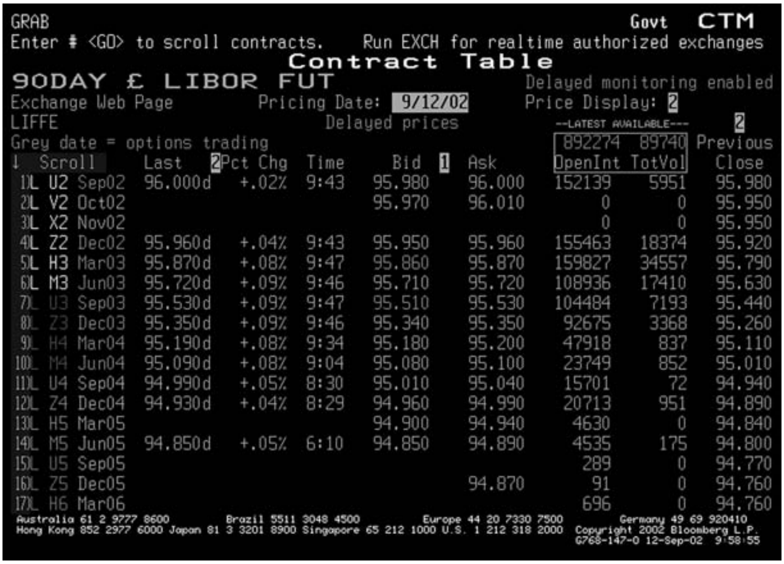


Source: Bloomberg Financial Markets

The 90-day sterling Libor interest rate futures contract trades on the main London futures exchange, LIFFE. The contract is structured similarly to the Eurodollar futures contract described above. The Bloomberg Futures Contract Description for the December 2002 contract is presented in Exhibit 9.4. Prices are quoted as 100 minus the interest rate and the delivery months are March, June, September, and December. The contract size is £500,000. A tick is 0.01 or one basis point and the tick value is £12.5. Exhibit 9.5 presents a Bloomberg Contract Table for the 90-day sterling Libor contract on September 12, 2002.

The LIFFE also trades short-term interest rate futures for other major currencies including euros, yen, and Swiss franc. For example, Exhibit 9.6 presents a Bloomberg Futures Contract Description screen for the December 2002 90-day Euro Euribor contract. Short-term interest rate contracts in other currencies are similar to the 90-day sterling Libor contract and trade on exchanges such as Deutsche Terminbourse in Frankfort and MATIF in Paris.

EXHIBIT 9.5 Bloomberg Contract Table for the 90-Day Sterling Libor Contracts



GRAB Govt CTM

Enter # <GO> to scroll contracts. Run EXCH for realtime authorized exchanges

Contract Table

90DAY £ LIBOR FUT Delayed monitoring enabled

Exchange Web Page Pricing Date: 9/12/02 Price Display: 2

LIFFE Delayed prices --LATEST AVAILABLE--- 2

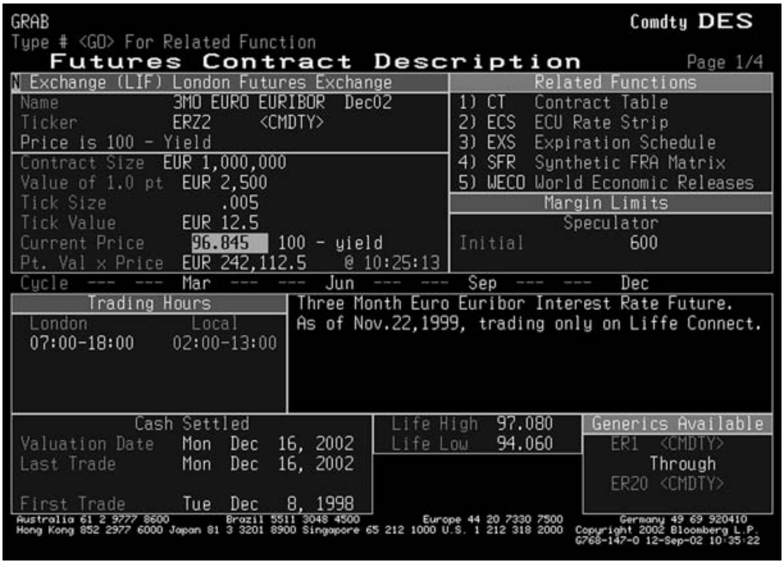
Grey date = options trading 892274 89740 Previous

↓ Scroll	Last	2 Pct Chg	Time	Bid	1 Ask	OpenInt	TotVol	Close
1)L U2 Sep02	96.000d	+.02%	9:43	95.980	96.000	152139	5951	95.980
2)L V2 Oct02				95.970	96.010	0	0	95.950
3)L X2 Nov02						0	0	95.950
4)L Z2 Dec02	95.960d	+.04%	9:43	95.950	95.960	155463	18374	95.920
5)L H3 Mar03	95.870d	+.08%	9:47	95.860	95.870	159827	34557	95.790
6)L M3 Jun03	95.720d	+.09%	9:46	95.710	95.720	108936	17410	95.630
7)L U3 Sep03	95.530d	+.09%	9:47	95.510	95.530	104484	7193	95.440
8)L Z3 Dec03	95.350d	+.09%	9:46	95.340	95.350	92675	3368	95.260
9)L H4 Mar04	95.190d	+.08%	9:34	95.180	95.200	47918	837	95.110
10)L M4 Jun04	95.090d	+.08%	9:04	95.080	95.100	23749	852	95.010
11)L U4 Sep04	94.990d	+.05%	8:30	95.010	95.040	15701	72	94.940
12)L Z4 Dec04	94.930d	+.04%	8:29	94.960	94.990	20713	951	94.890
13)L H5 Mar05				94.900	94.940	4630	0	94.840
14)L M5 Jun05	94.850d	+.05%	6:10	94.850	94.890	4535	175	94.800
15)L U5 Sep05						289	0	94.770
16)L Z5 Dec05					94.870	91	0	94.760
17)L H6 Mar06						696	0	94.760

Australia 61 2 9777 8600 Brazil 5511 3048 4500 Europe 44 20 7330 7500 Germany 49 69 920410
Hong Kong 852 2977 6000 Japan 81 3 3201 8900 Singapore 65 212 1000 U.S. 1 212 318 2000 Copyright 2002 Bloomberg L.P.
G768-147-0 12-Sep-02 9:58:55

Source: Bloomberg Financial Markets

EXHIBIT 9.6 Bloomberg Futures Contract Description Screen for the 90-Day Euro Euribor Contract



GRAB Comdty DES

Type # <GO> For Related Function

Futures Contract Description Page 1/4

N Exchange (LIF) London Futures Exchange

Name 3MO EURO EURIBOR Dec02

Ticker ERZ2 <CMDTY>

Price is 100 - Yield

Contract Size EUR 1,000,000

Value of 1.0 pt EUR 2,500

Tick Size .005

Tick Value EUR 12.5

Current Price 96.845 100 - yield

Pt. Val x Price EUR 242,112.5 @ 10:25:13

Related Functions

1) CT Contract Table

2) ECS ECU Rate Strip

3) EXS Expiration Schedule

4) SFR Synthetic FRA Matrix

5) WECO World Economic Releases

Margin Limits

Speculator

Initial 600

Cycle --- --- Mar --- --- Jun --- --- Sep --- --- Dec

Trading Hours

London Local

07:00-18:00 02:00-13:00

Three Month Euro Euribor Interest Rate Future.
As of Nov.22,1999, trading only on Liffe Connect.

Cash Settled

Valuation Date Mon Dec 16, 2002

Last Trade Mon Dec 16, 2002

First Trade Tue Dec 8, 1998

Life High 97.080

Life Low 94.060

Generics Available

ER1 <CMDTY>

Through

ER20 <CMDTY>

Australia 61 2 9777 8600 Brazil 5511 3048 4500 Europe 44 20 7330 7500 Germany 49 69 920410
Hong Kong 852 2977 6000 Japan 81 3 3201 8900 Singapore 65 212 1000 U.S. 1 212 318 2000 Copyright 2002 Bloomberg L.P.
G768-147-0 12-Sep-02 10:35:22

Source: Bloomberg Financial Markets

Fed Funds Futures Contract

Depository institutions are required to hold reserves to meet their reserve requirements. To meet these requirements, depository institutions hold reserves at their district Federal Reserve Bank. These reserves are called *federal funds*. Because no interest is earned on federal funds, a depository institution that maintains federal funds in excess of the amount required incurs an opportunity cost of the interest forgone on the excess reserves. Conversely, there are also depository institutions whose federal funds are short of the amount required. The federal funds market is where depository institutions buy and sell federal funds to address this imbalance. The interest rate at which federal funds are bought (borrowed) and sold (lent) is called the *federal funds rate*. Consequently, the federal funds rate is a benchmark short-term interest rate.

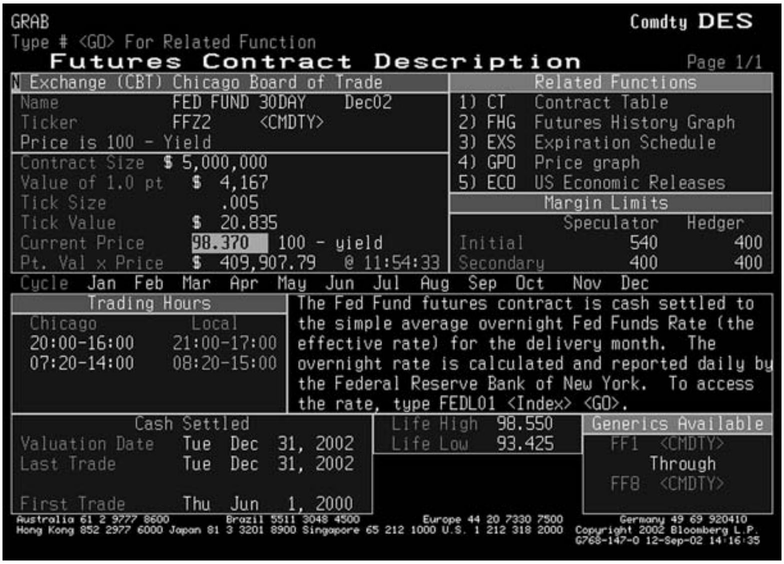
When the Federal Reserve formulates and executes monetary policy, the federal funds rate is a primary operating target. The Federal Open Market Committee (FOMC) sets a target level for the federal funds rate. Announcements of changes in monetary policy specify changes in the FOMC's target for this rate. Once the target is set, the Federal Reserve either adds or drains reserves from the banking system using open market operations so that the actual federal funds rate is, on average, equal to the target. The 30-day federal funds futures contract is designed for financial institutions and businesses who want to control their exposure to movements in the federal funds rate.

The federal funds futures contract began trading on the Chicago Board of Trade in October 1988. These contracts have a notional amount of $5 million and the contract can be written for the current month up to 24 months in the future. Underlying this contract is the simple average overnight federal funds rate (i.e., the effective rate) for the delivery month. As such, this contract is settled in cash on the last business day of the month. Exhibit 9.7 presents the Bloomberg Futures Contract Description screen for the December 2002 federal funds futures contract. Just as the other short-term interest rate futures contracts discussed above, prices are quoted on the basis of 100 minus the overnight federal funds rate for the delivery month. These contracts are marked to market using the effective daily federal funds rate as reported by the Federal Reserve Bank of New York. Exhibit 9.8 presents the Bloomberg Contract Table screen for the active federal funds futures contracts on September 12, 2002.

Long-Term Interest Rate Futures Contracts

The most actively traded long-term (greater than one year) interest rate futures contracts are described below.

EXHIBIT 9.7 Bloomberg Futures Contract Description Screen for the Federal Funds Futures Contract



GRAB Comdty DES
Type # <GO> For Related Function
Futures Contract Description Page 1/1

N Exchange (CBT) Chicago Board of Trade
Name FED FUND 30DAY Dec02
Ticker FFZ2 <CMDTY>
Price is 100 - Yield
Contract Size $ 5,000,000
Value of 1.0 pt $ 4,167
Tick Size .005
Tick Value $ 20.835
Current Price 98.370 100 - yield
Pt. Val x Price $ 409,907.79 @ 11:54:33

Related Functions
1) CT Contract Table
2) FHG Futures History Graph
3) EXS Expiration Schedule
4) GPO Price graph
5) ECO US Economic Releases

Margin Limits

	Speculator	Hedger
Initial	540	400
Secondary	400	400

Cycle Jan Feb Mar Apr May Jun Jul Aug Sep Oct Nov Dec

Trading Hours

Chicago	Local
20:00-16:00	21:00-17:00
07:20-14:00	08:20-15:00

The Fed Fund futures contract is cash settled to the simple average overnight Fed Funds Rate (the effective rate) for the delivery month. The overnight rate is calculated and reported daily by the Federal Reserve Bank of New York. To access the rate, type FEDL01 <Index> <GO>.

Cash Settled
Valuation Date Tue Dec 31, 2002
Last Trade Tue Dec 31, 2002
First Trade Thu Jun 1, 2000

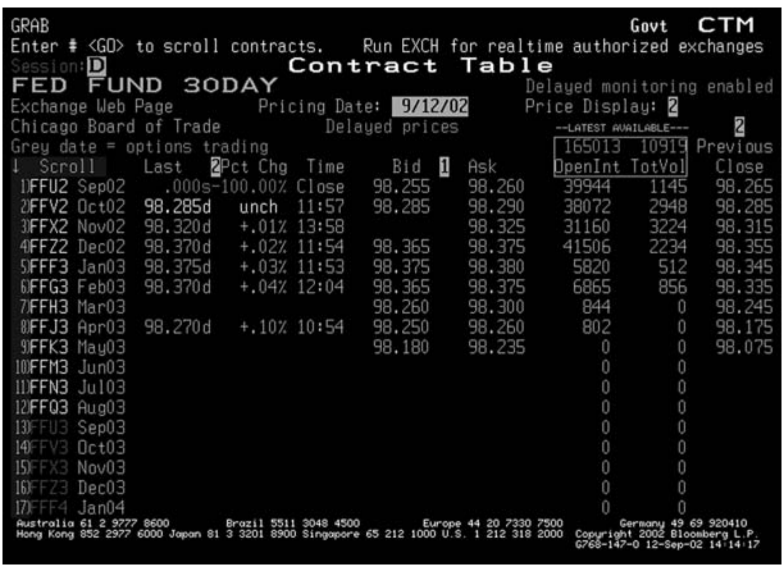
Life High 98.550
Life Low 93.425

Generics Available
FF1 <CMDTY>
Through
FF8 <CMDTY>

Australia 61 2 9777 8600 Brazil 5511 3048 4500 Europe 44 20 7330 7500 Germany 49 69 920410
Hong Kong 852 2977 6000 Japan 81 3 3201 8900 Singapore 65 212 1000 U.S. 1 212 318 2000 Copyright 2002 Bloomberg L.P.
G768-147-0 12-Sep-02 14:16:35

Source: Bloomberg Financial Markets

EXHIBIT 9.8 Bloomberg Contract Table for the Federal Funds Futures Contract



GRAB Govt CTM
Enter # <GO> to scroll contracts. Run EXCH for realtime authorized exchanges
Session: D Contract Table
FED FUND 30DAY Delayed monitoring enabled
Exchange Web Page Pricing Date: 9/12/02 Price Display: 2
Chicago Board of Trade Delayed prices --LATEST AVAILABLE--- 2
Grey date = options trading 165013 10919

↓ Scroll	Last	2 Pct Chg	Time	Bid	1 Ask	OpenInt	TotVol	Previous Close
1) FFU2 Sep02	.000s	-100.00%	Close	98.255	98.260	39944	1145	98.265
2) FFV2 Oct02	98.285d	unch	11:57	98.285	98.290	38072	2948	98.285
3) FFX2 Nov02	98.320d	+.01%	13:58		98.325	31160	3224	98.315
4) FFZ2 Dec02	98.370d	+.02%	11:54	98.365	98.375	41506	2234	98.355
5) FFF3 Jan03	98.375d	+.03%	11:53	98.375	98.380	5820	512	98.345
6) FFG3 Feb03	98.370d	+.04%	12:04	98.365	98.375	6865	856	98.335
7) FFH3 Mar03				98.260	98.300	844	0	98.245
8) FFJ3 Apr03	98.270d	+.10%	10:54	98.250	98.260	802	0	98.175
9) FFK3 May03				98.180	98.235	0	0	98.075
10) FFM3 Jun03						0	0	
11) FFN3 Jul03						0	0	
12) FFQ3 Aug03						0	0	
13) FFU3 Sep03						0	0	
14) FFV3 Oct03						0	0	
15) FFX3 Nov03						0	0	
16) FFZ3 Dec03						0	0	
17) FFF4 Jan04						0	0	

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G768-147-0 12-Sep-02 14:14:17

Source: Bloomberg Financial Markets

Treasury Bond Futures

The Treasury bond futures contract is traded on the Chicago Board of Trade (CBOT). The underlying instrument for this contract is $100,000 par value of a hypothetical 20-year coupon. This hypothetical bond's coupon rate is called the *notional coupon*. Currently, this notional coupon is 6%. Treasury futures contracts trade with March, June, September, and December settlement months.

The futures price is quoted in terms of par being 100. Published quotes have two parts namely the number of points (1% of par value) and the number of ticks ($\frac{1}{32}$ of 1% of par value). Thus, a quote for a Treasury bond futures contract of 97-16 means 97 and $\frac{16}{32}$ or 97.50. So, if a buyer and seller agree on a futures price of 97-16, this means simply that the buyer agrees to accept delivery of the hypothetical underlying Treasury bond and pay 97.50% of par value and the seller agrees to accept 97.50% of par value.[2] Since the par value of the bond underlying the futures contract is $100,000, the futures price that the buyer and seller agree to for this hypothetical bond is $97,500.

The minimum price fluctuation for the Treasury bond futures contract is $\frac{1}{32}$ of 1% as noted previously which is referred to as a 32*nd*. The dollar value of a 32*nd* for $100,000 par value (the par value for the underlying Treasury bond) is $31.25. This is true because each point (1% of the par value) is worth $1,000 and each point is comprised of 32 ticks. Thus, the minimum price fluctuation is $31.25 for this contract.

We have been referring to the underlying instrument as a hypothetical Treasury bond. While some interest rate futures contracts can only be settled in cash, the seller (the short) of a Treasury bond futures contract who chooses to make delivery rather than liquidate his/her position by buying back the contract prior to the settlement date must deliver some Treasury bond. This begs the question "which Treasury bond?" The CBOT allows the seller to deliver one of several Treasury bonds that the CBOT specifies are acceptable for delivery. These contracts have multiple deliverables to avoid having a single issue squeezed and to allow for varying schedules of new issues.[3] Exhibit 9.9 Panels A and B presents a Bloomberg screen (function DLV) that shows the 30 Treasury bond issues that the seller could have selected from to deliver to the buyer of the December 2002 futures contract. (In the illustrations of Treasury bond futures that follow, we will also be using the Septem-

[2] We will discuss the issue of accrued interest shortly.

[3] The term "squeeze" is used to describe a shortage of the supply of a particular security relative to the demand. A trader who is short a particular security is always concerned with the risk of being unable to obtain sufficient securities to cover their position.

ber 2002 futures contract.) The set of all bonds that meet the delivery requirements for a particular contract is called the *deliverable basket*. The CBOT makes its determination of the Treasury issues that are acceptable for delivery from all outstanding Treasury issues that have at least 15 years to maturity from the first day of the delivery month.[4] Moreover, all bonds delivered by the seller must be of the same issue.

It is important to keep in mind that while the underlying Treasury bond for this contract is a hypothetical issue and therefore cannot itself be delivered into the futures contract, the bond futures contract is not a cash settlement contract. The only way to close out a Treasury bond futures contract is to either initiate an offsetting futures position or to deliver a Treasury issue from the deliverable basket.

EXHIBIT 9.9 **Bloomberg Cheapest to Deliver Screen for a Treasury Bond Futures Contract**
Panel A: Deliverable Basket for the December 2002 Contract

GRAB Comdty DLV
Hit {NUMBER} <GO> to view Historical Basis/Repo
Cheapest to Deliver Trade 9/16/02 Dlv 12/31/02 p.1 / 2
US LONG BOND(CBT) Dec02 **USZ2** 112-14 Set 9/17/02 Cheapest IRP= 1.50
105 Days Act/360

PRICES AS DECIMALS? N
Order DR re-sort? Y

	(Mid) Price	Source	Conv. Yield	C.Factor	(32NDS) Gross Basis	Implied Repo%	Actual Repo%	(32NDS) Net Basis
MASTER:							1.67	
1) T 8 3/4 08/15/20	147-16	BGN	4.774	1.2954	59.2	1.50	1.67	2.4
2) T 8 1/8 08/15/21	140-20	BGN	4.825	1.2355	54.7	1.48	1.67	2.5
3) T 7 7/8 02/15/21	137-03+	BGN	4.813	1.2047	53.0	1.47	1.67	2.6
4) T 8 1/2 02/15/20	143-30+	BGN	4.758	1.2641	58.3	1.43	1.67	3.2
5) T 8 1/8 05/15/21	140-15	BGN	4.813	1.2336	56.5	1.38	1.67	3.8
6) T 8 11/15/21	139-09	BGN	4.832	1.2232	55.9	1.35	1.67	4.2
7) T 8 1/8 08/15/19	139-00+	BGN	4.744	1.2206	56.8	1.33	1.67	4.4
8) T 8 3/4 05/15/20	147-10	BGN	4.759	1.2929	62.1	1.32	1.67	4.8
9) T 8 7/8 02/15/19	147-11+	BGN	4.701	1.2931	62.9	1.31	1.67	5.0
10) T 7 1/4 08/15/22	130-01+	BGN	4.874	1.1426	50.4	1.29	1.67	4.6
11) T 7 1/8 02/15/23	128-21	BGN	4.890	1.1300	51.3	1.14	1.67	6.4
12) T 9 11/15/18	148-18+	BGN	4.681	1.3028	67.0	1.13	1.67	7.6
13) T 7 5/8 11/15/22	135-05	BGN	4.867	1.1864	56.3	1.09	1.67	7.4
14) T 9 1/8 05/15/18	149-15	BGN	4.644	1.3092	72.5	.81	1.67	12.0
15) T 6 1/4 08/15/23	117-09	BGN	4.917	1.0293	49.6	.68	1.67	10.9
16) T 7 5/8 02/15/25	136-19	BGN	4.913	1.1971	63.8	.45	1.67	15.6

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G274-147-0 14-Sep-02 14:21:17

Source: Bloomberg Financial Markets

[4] For settlement purposes, a given issue's term to maturity is calculated in complete three month increments. For example, 15 years and 5 months would result in a maturity of 15 years and 1 quarter.

EXHIBIT 9.9 (Continued)
Panel B: Deliverable Basket for the December 2002 Contract continued

GRAB Comdty DLV
Hit (NUMBER) <GO> to view Historical Basis/Repo
Cheapest to Deliver Trade 9/16/02 Dlv 12/31/02 p.2 / 2
US LONG BOND(CBT) Dec02 **USZ2** 112-14 Set 9/17/02 Cheapest IRP= 1.50
105 Days Act/360
PRICES AS DECIMALS? N
Order DR re-sort? Y

	(Mid) Price	Source	Conv. Yield	C.Factor	(32NDS) Gross Basis	Implied Repo%	Actual Repo%	(32NDS) Net Basis
MASTER:							1.67	
17) T 7 1/2 11/15/24	134-27+	BGN	4.903	1.1808	67.1	.15	1.67	19.3
18) T 6 7/8 08/15/25	126-14+	BGN	4.934	1.1073	62.4	.03	1.67	19.5
19) T 6 3/4 08/15/26	125-09	BGN	4.937	1.0938	73.5	-1.01	1.67	31.5
20) T 6 5/8 02/15/27	123-20+	BGN	4.946	1.0790	74.3	-1.19	1.67	33.1
21) T 6 1/2 11/15/26	121-26	BGN	4.943	1.0627	74.4	-1.28	1.67	33.7
22) T 6 02/15/26	114-19+	BGN	4.940	1.0000	69.5	-1.37	1.67	32.7
23) T 6 3/8 08/15/27	120-08+	BGN	4.950	1.0478	78.5	-1.80	1.67	39.1
24) T 6 1/8 11/15/27	116-26	BGN	4.949	1.0159	82.8	-2.41	1.67	44.8
25) T 5 1/2 08/15/28	108-00	BGN	4.948	.9351	91.5	-4.08	1.67	58.2
26) T 6 1/8 08/15/29	117-24+	BGN	4.926	1.0165	111.1	-5.00	1.67	73.7
27) T 5 1/4 11/15/28	104-15+	BGN	4.942	.9022	97.4	-5.00	1.67	65.5
28) T 5 1/4 02/15/29	104-21	BGN	4.932	.9019	104.0	-5.71	1.67	72.4
29) T 6 1/4 05/15/30	120-01	BGN	4.917	1.0332	123.5	-5.86	1.67	84.9
30) T 5 3/8 02/15/31	109-12	BGN	4.769	.9157	205.3	-15.24	1.67	173.4

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Hong Kong 852 2977 6000 Japan 81 3 3201 8900 Singapore 65 212 1000 U.S. 1 212 318 2000 Copyright 2002 Bloomberg L.P.
G274-147-0 14-Sep-02 14:22:24

Source: Bloomberg Financial Markets

Conversion Factors The delivery process for the Treasury bond futures contract is innovative and has served as a model for government bond futures contracts traded on various exchanges throughout the world. On the settlement date, the seller of the futures contract (the short) is required to deliver the buyer (the long) \$100,000 par value of a 6% 20-year Treasury bond. As noted, no such bond exists, so the seller must choose a bond from the deliverable basket to deliver to the long. Suppose the seller selects a 5% coupon, 20-year Treasury bond to settle the futures contract. Since the coupon of this bond is less than the notional coupon of 6%, this would be unacceptable to the buyer who contracted to receive a 6% coupon, 20-year bond with a par value of \$100,000. Alternatively, suppose the seller is compelled to deliver a 7% coupon, 20-year bond. Since the coupon of this bond is greater than the notional coupon of 6%, the seller would find this unacceptable. In summary, how do we adjust for the fact that bonds in the deliverable basket have coupons and maturities that differ from the notional coupon of 6%?

To make delivery equitable to both parties, the CBOT uses conversion factors for adjusting the price of each Treasury issue that can be delivered to satisfy the Treasury bond futures contract. Within the deliverable basket, conversion factors are designed to make each bond approximately equally cheap to deliver if the yield curve were flat at 6%. The conversion factor is determined by the CBOT before a contract with a specific settlement date begins trading using the following formula:

$$CF = \frac{1}{1.03^{K/6}}\left[\frac{C}{2} + \frac{C}{0.06}\left(1 - \frac{1}{1.03^{2N}}\right) + \frac{1}{1.03^{2N}}\right]$$

where

CF = conversion factor
N = complete years to maturity as of the settlement month
C = annual coupon rate (in decimal form)
K = number of months that the maturity exceeds N (rounded down to complete quarters)

For example, if the maturity of a Treasury bond from the deliverable basket is 24 years and 4.5 months, K is 3 since the 4.5 months is rounded down to complete quarters, or 3 months. Further, if the maturity is 24 years and 11 months, K is 9.

The convention of rounding down to the nearest complete quarter adds a slight distortion into the calculation of the conversion factors. To see this, recall Treasury futures contracts have expiration months of March, June, September, and December. Also note that all Treasury bonds mature on February 15, May 15, August 15 or November 15.[5] Since conversion factors are computed as of the first day of the delivery month, bonds that mature on say, August 15 are treated as if they mature on June 1 (the first delivery day of the June contract.) The Treasury's maturity is artificially shortened by 2½ months so that there is 2½ months of "pull to par" built into the conversion factors.[6] As a result, for Treasury bonds with coupon rates below 6%, the conversion factors will be slightly higher than they should be. Conversely, for issues with coupon rates above 6%, the conversion factors will be slightly lower than they should be.

Exhibit 9.9, Panels A and B, show the conversion factors for each Treasury bond in the deliverable basket for the December 2002 bond futures contract. These conversion factors are located in the column labeled "C. Factor." The conversion factor is constant throughout the life of the futures contract.

[5] Virtually all 10-year Treasury notes mature on the same dates. The only exceptions are the July 15, 2006 and the October 15, 2006 notes, which were issued when the U.S. Treasury briefly issued six 10-year notes a year. However, their maturities have shortened to the point that they are no longer deliverable into the 10-year note contract discussed shortly.

[6] "Pull to par" is the name given to the predictable price increase of discount bonds and the price decrease of premium bonds with the passage of time at constant yields.

Given the conversion factor for an issue and the futures price, the adjusted price is found by multiplying the conversion factor by the futures price. The adjusted price is called the *converted price*.

The price that the buyer must pay the seller when a Treasury bond is delivered is called the *invoice price*. Intuitively, the invoice price should be the futures settlement price plus accrued interest. However, as just noted, the seller can choose any Treasury issue from the deliverable basket. To make delivery fair to both parties, the invoice price must be adjusted using the conversion factor of the actual Treasury issue delivered. The invoice price is:

$$\begin{aligned}\text{Invoice price} = {} & \text{Contract size} \times \text{Futures settlement price} \\ & \times \text{Conversion factor} + \text{Accrued interest}\end{aligned}$$

Suppose the that settlement price of the September 2002 Treasury bond futures contract is 113-30 and the issue selected by short to deliver is the 8.125% coupon bond that matures on 8/15/21. The futures contract settlement price of 113-30 means 113.9375% of par value or 1.139375 times par value. The conversion factor for this issue is 1.2371. Since the contract size is \$100,000, the invoice price the buyer pays the seller is:

$$\begin{aligned}& \$100{,}000 \times 1.139375 \times 1.2371 + \text{Accrued interest} \\ = {} & \$140{,}584.50 + \text{Accrued interest}\end{aligned}$$

Cheapest-to-Deliver Issue In selecting the issue to be delivered, the short will select from all the deliverable issues the one that will give the largest rate of return from a cash-and-carry trade. A *cash-and-carry trade* is one in which a cash bond that is acceptable for delivery is purchased with borrowed funds and simultaneously the Treasury bond futures contract is sold. The bond purchased can be delivered to satisfy the short futures position. Thus, by buying the Treasury issue that is acceptable for delivery and selling the futures, an investor has effectively sold the bond at the delivery price (i.e., the converted price).

A rate of return can be calculated for this trade. This rate of return is referred to as the *implied repo rate* and is determined by

1. The price plus accrued interest at which the Treasury issue could be purchased.
2. The converted price plus the accrued interest that will be received upon delivery of that Treasury bond issue to satisfy the short futures position.
3. The coupon payments that will be received between today and the date the issue is delivered to satisfy the futures contract.

4. The reinvestment income that will be realized on the coupon payments between the time the interim coupon payment is received and the date that the issue is delivered to satisfy the Treasury bond futures contract.

The first three elements are known. The last element will depend on the reinvestment rate that can be earned. While the reinvestment rate is unknown, typically this is a small part of the rate of return and not much is lost by assuming that the implied repo rate can be predicted with certainty.

The general formula for the implied repo rate is as follows:

$$\text{Implied repo rate} = \frac{\text{Dollar return}}{\text{Cost of the investment}} \times \frac{360}{\text{Days}_1}$$

where Days_1 is equal to the number of days until settlement of the futures contract. Below we will explain the other components in the formula for the implied repo rate.

Let's begin with the dollar return. The *dollar return* for an issue is the difference between the *proceeds received* and the *cost of the investment*. The proceeds received are equal to the proceeds received at the settlement date of the futures contract and any interim coupon payment plus interest from reinvesting the interim coupon payment. The proceeds received at the settlement date include the converted price (i.e., futures settlement price multiplied by the conversion factor for the issue) and the accrued interest received from delivery of the issue. That is,

$$\begin{aligned}\text{Proceeds received} = {} & \text{Converted price} + \text{Accrued interest received} \\ & + \text{Interim coupon payment} \\ & + \text{Interest from reinvesting the interim coupon payment}\end{aligned}$$

As noted earlier, all of the elements are known except the interest from reinvesting the interim coupon payment. This amount is estimated by assuming that the coupon payment can be reinvested at the term repo rate. The repo rate is not only a borrowing rate for an investor who wants to borrow in the repo market but also the rate at which an investor can invest proceeds on a short-term basis. For how long is the reinvestment of the interim coupon payment? It is the number of days from when the interim coupon payment is received and the actual delivery date to satisfy the futures contract. The reinvestment income is then computed as follows:

$$\begin{aligned}&\text{Interest from reinvesting the interim coupon payment} \\ &= \text{Interim coupon} \times \text{Term repo rate} \times (\text{Days}_2/360)\end{aligned}$$

where Days_2 is the number of days between when the interim coupon payment is received and the actual delivery date of the futures contract.

The reason for dividing Days_2 by 360 is that the ratio represents the number of days the interim coupon is reinvested as a percentage of the number of days in a year as measured in the money market.

The cost of the investment is the amount paid to purchase the issue. This cost is equal to the purchase price plus accrued interest paid. That is,

$$\text{Cost of the investment} = \text{Purchase price} + \text{Accrued interest paid}$$

Thus, the dollar return for the numerator of the formula for the implied repo rate is equal to

$$\text{Dollar return} = \text{Proceeds received} - \text{Cost of the investment}$$

The dollar return is then divided by the cost of the investment.[7]

So, now we know how to compute the numerator and the denominator in the formula for the implied repo rate. The second ratio in the formula for the implied repo rate simply involves annualizing the return using a convention in the money market for the number of days. (The money market convention is to use a 360-day year.) Since the investment resulting from the cash-and-carry trade is a synthetic money market instrument, 360 days are used.

Let's compute the implied repo rate for a hypothetical issue that may be delivered to satisfy a hypothetical Treasury bond futures contract. Assume the following for the deliverable issue and the futures contract:

Futures contract:
Futures price = 96
Days to futures delivery date (Days_1) = 82 days

Deliverable issue:
Price of issue = 107
Accrued interest paid = \$3.8904
Coupon rate = 10%
Days remaining before interim coupon paid = 40 days
Interim coupon = \$5
Number of days between when the interim coupon payment is received and the actual delivery date of the futures contract (days_2) = 42
Conversion factor = 1.1111
Accrued interest received at futures settlement date = 1.1507

[7] Actually, the cost of the investment should be adjusted because the amount that the investor ties up in the investment is reduced if there is an interim coupon payment. We ignore this adjustment here.

Other information:
82-day term repo rate = 3.8%

Let's begin with the proceeds received. We need to compute the converted price and the interest from reinvesting the interim coupon payment. The converted price is:

$$\begin{aligned}\text{Converted price} &= \text{Futures price} \times \text{Conversion factor} \\ &= 96 \times 1.1111 = 106.6656\end{aligned}$$

The interest from reinvesting the interim coupon payment depends on the term repo rate. The term repo rate is assumed to be 3.8%. Therefore,

$$\begin{aligned}&\text{Interest from reinvesting the interim coupon payment} \\ &= \$5 \times 0.038 \times \left(\frac{42}{360}\right) = 0.0222\end{aligned}$$

To summarize:

Converted price	=	106.6656
Accrued interest received	=	1.1507
Interim coupon payment	=	5.0000
Interest from reinvesting the interim coupon payment	=	0.0222
Proceeds received	=	112.8385

The cost of the investment is the purchase price for the issue plus the accrued interest paid, as shown below:

$$\text{Cost of the investment} = 107 + 3.8904 = 110.8904$$

The implied repo rate is then:

$$\text{Implied repo rate} = \frac{112.8385 - 110.8904}{110.8904} \times \frac{360}{82} = 0.0771 = 7.71\%$$

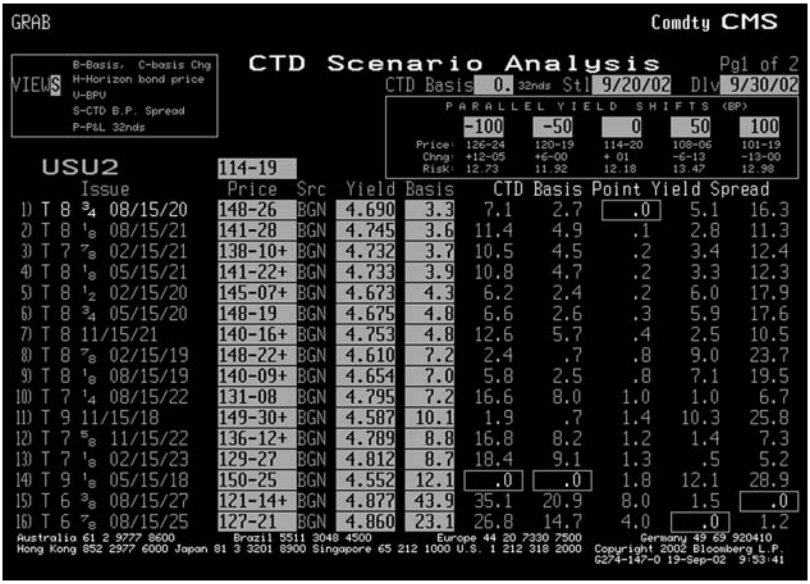
Once the implied repo rate is calculated for each bond in the deliverable basket, the issue selected will be the one that has the highest implied repo rate (i.e., the issue that gives the maximum return in a cash-and-carry trade).[8] The issue with the highest return is referred to as the *cheapest-to-deliver issue*. This issue plays a key role in the pricing of a Treasury futures contract. Exhibit 9.9 shows the implied repo rates for each issue in the deliverable basket for the December 2002 Treasury

[8] The implied repo rate can be negative.

bond futures contract. The column is labeled "Implied Repo%" and the bonds are ranked in order of descending implied repo rates. For this contract, the highest implied repo rate is 1.50% for the 8¾% of 8/15/20. Accordingly, this issue is the cheapest-to-deliver.

While a particular Treasury bond may be the cheapest-to-deliver today, changes in interest rates, for example, may cause some other issue to be the cheapest to deliver at a future date. A sensitivity analysis can be performed to determine how a change in yield affects the cheapest to deliver bond. Exhibit 9.10 presents Bloomberg's CTD Scenario Analysis screen for issues deliverable into the September 2002 Treasury bond futures contract. For settlement on September 20, 2002, the 8¾% of 8/15/20 was the cheapest-to-deliver issue. However, as the screen indicates, for a parallel shifts in the yield curve of –100, –50, +50, and +100 basis points, the cheapest-to-deliver issue changes. The five columns on the right-hand side of the screen indicate the basis point spread between each issue and the cheapest-to-deliver issue for a given parallel yield curve shift. The "new" cheapest-to-deliver issue given the yield change is indicated with a rectangular box. Exhibit 9.11 shows Bloomberg's Historical Cheapest Graph for the September 2002 bond futures contract for the period June 19, 2002 to September 19, 2002. Note how much the cheapest-to-deliver issue changes over time.

EXHIBIT 9.10 Bloomberg Cheapest to Deliver Scenario Analysis Screen


GRAB   Comdty CMS

VIEWS: B-Basis, C-basis Chg; H-Horizon bond price; V-BPV; S-CTD B.P. Spread; P-P&L 32nds

CTD Scenario Analysis   Pg1 of 2

CTD Basis 0. 32nds Stl 9/20/02 Dlv 9/30/02

PARALLEL YIELD SHIFTS (BP)

	-100	-50	0	50	100
Price	126-24	120-19	114-20	108-06	101-19
Chng	+12-05	+6-00	+ 01	-6-13	-13-00
Risk	12.73	11.92	12.18	13.47	12.98

USU2 114-19

Issue	Price	Src	Yield	Basis	CTD Basis Point Yield Spread				
1) T 8 ¾ 08/15/20	148-26	BGN	4.690	3.3	7.1	2.7	$\boxed{.0}$	5.1	16.3
2) T 8 ⅛ 08/15/21	141-28	BGN	4.745	3.6	11.4	4.9	.1	2.8	11.3
3) T 7 ⅞ 02/15/21	138-10+	BGN	4.732	3.7	10.5	4.5	.2	3.4	12.4
4) T 8 ⅛ 05/15/21	141-22+	BGN	4.733	3.9	10.8	4.7	.2	3.3	12.3
5) T 8 ½ 02/15/20	145-07+	BGN	4.673	4.3	6.2	2.4	.2	6.0	17.9
6) T 8 ¾ 05/15/20	148-19	BGN	4.675	4.8	6.6	2.6	.3	5.9	17.6
7) T 8 11/15/21	140-16+	BGN	4.753	4.8	12.6	5.7	.4	2.5	10.5
8) T 8 ⅞ 02/15/19	148-22+	BGN	4.610	7.2	2.4	.7	.8	9.0	23.7
9) T 8 ⅛ 08/15/19	140-09+	BGN	4.654	7.0	5.8	2.5	.8	7.1	19.5
10) T 7 ¼ 08/15/22	131-08	BGN	4.795	7.2	16.6	8.0	1.0	1.0	6.7
11) T 9 11/15/18	149-30+	BGN	4.587	10.1	1.9	.7	1.4	10.3	25.8
12) T 7 ⅝ 11/15/22	136-12+	BGN	4.789	8.8	16.8	8.2	1.2	1.4	7.3
13) T 7 ⅛ 02/15/23	129-27	BGN	4.812	8.7	18.4	9.1	1.3	.5	5.2
14) T 9 ⅛ 05/15/18	150-25	BGN	4.552	12.1	$\boxed{.0}$	$\boxed{.0}$	1.8	12.1	28.9
15) T 6 ⅜ 08/15/27	121-14+	BGN	4.877	43.9	35.1	20.9	8.0	1.5	$\boxed{.0}$
16) T 6 ⅞ 08/15/25	127-21	BGN	4.860	23.1	26.8	14.7	4.0	$\boxed{.0}$	1.2

Australia 61 2 9777 8600 Brazil 5511 3048 4500 Europe 44 20 7330 7500 Germany 49 69 920410
Hong Kong 852 2977 6000 Japan 81 3 3201 8900 Singapore 65 212 1000 U.S. 1 212 318 2000 Copyright 2002 Bloomberg L.P.
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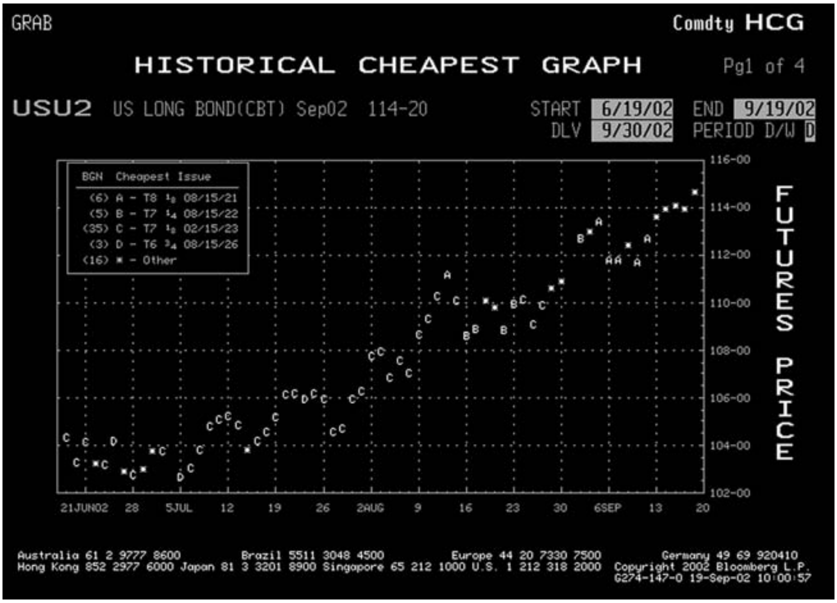
Source: Bloomberg Financial Markets

EXHIBIT 9.11 Bloomberg Historical Cheapest to Deliver Graph



Source: Bloomberg Financial Markets

Other Delivery Options In addition to the choice of which acceptable Treasury issue to deliver—sometimes referred to as the *quality option* or *swap option*—the short has at least two more options granted under CBOT delivery guidelines. The short is permitted to decide when in the delivery month delivery actually will take place. This is called the *timing option*. The other option is the right of the short to give notice of intent to deliver up to 8:00 p.m. Chicago time after the closing of the exchange (3:15 p.m. Chicago time) on the date when the futures settlement price has been fixed. This option is referred to as the *wild card option*. The quality option, the timing option, and the wild card option (in sum referred to as the *delivery options*), mean that the long position can never be sure which Treasury bond issue will be delivered or when it will be delivered. These three delivery options are summarized below:

Delivery Option	Description
Quality or swap option	Choice of which acceptable Treasury issue to deliver
Timing option	Choice of when in delivery month to deliver
Wild card option	Choice to deliver after the closing price of the futures contract is determined

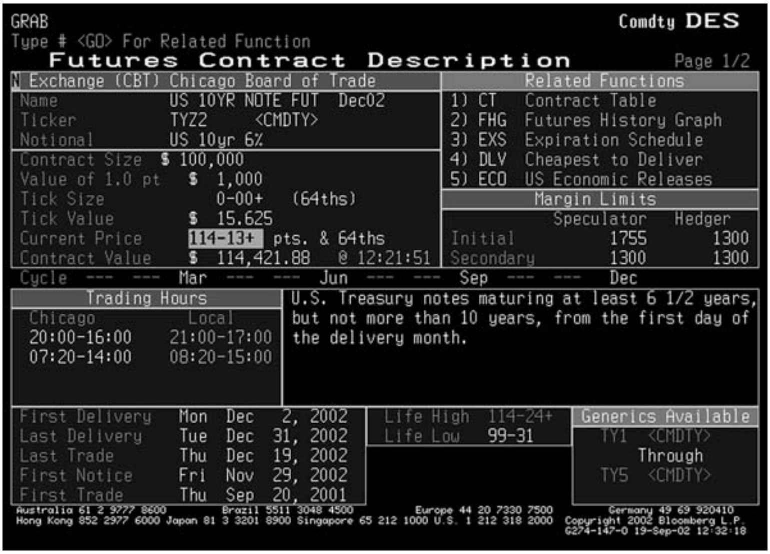
Delivery Procedure For a short who wants to deliver, the delivery procedure involves three days. The first day is the *position day.* On this day, the short notifies the CBOT that it intends to deliver. The short has until 8:00 p.m. Central Standard Time to do so. The second day is the *notice day.* On this day, the short specifies which particular issue will be delivered. The short has until 2:00 p.m. Central Standard Time to make this declaration. (On the last possible notice day in the delivery month, the short has until 3:00 p.m.) The CBOT then selects the long to whom delivery will be made. This is the long position that has been outstanding for the longest period of time. The long is then notified by 4:00 p.m. that delivery will be made. The third day is the *delivery day.* By 10:00 a.m. on this day the short must have in its account the Treasury issue that it specified on the notice day and by 1:00 p.m. must deliver that bond to the long that was assigned by the CBOT to accept delivery. The long pays the short the invoice price upon receipt of the bond.

Treasury Note Futures

There are three Treasury note futures contracts: 10-year, 5-year, and 2-year. All three contracts are modeled after the Treasury bond futures contract and are traded on the CBOT. These note contracts have increased in importance since the Treasury announced on October 31, 2001 that it was suspending issuance of the 30-year bond. As of Fall 2002, the open interest of both the 10-year and 5-year Treasury notes contracts exceeded the open interest of the Treasury bond contract. The underlying instrument for the 10-year Treasury note contract is $100,000 par value of a hypothetical 10-year 6% Treasury note. There are several acceptable issues that may be delivered by the short. An issue is acceptable if the maturity is not less than 6.5 years and not greater than 10 years from the first day of the delivery month. The delivery options granted to the short position and the minimum price fluctuation are the same as for the Treasury bond futures contract. Exhibit 9.12 shows the Bloomberg Futures Contract Description screen for the 10-year note contract.

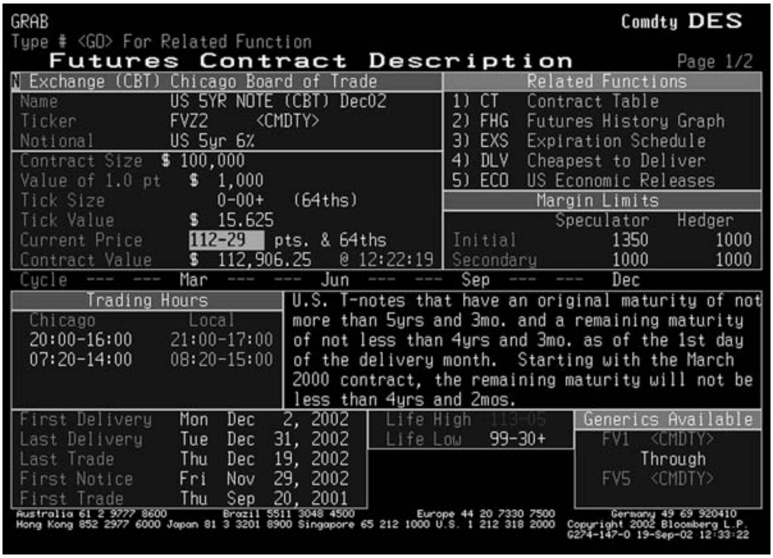
For the 5-year Treasury note futures contract, the underlying instrument is $100,000 par value of a 6% notional coupon Treasury note. An issue in the deliverable basket must satisfy the following conditions: (1) an original maturity of not more than five years and three months; (2) a remaining maturity of not more than five years and three months; and (3) a remaining maturity not less than four years and two months. The minimum price fluctuation for this contract is $\frac{1}{64}$ of 1% of par. The dollar value of a 64*th* for a $100,000 par value is $15.625 ($100,000/6,400) and is therefore the minimum price fluctuation. Exhibit 9.13 shows the Bloomberg Futures Contract Description screen for the 5-year Treasury note contract.

EXHIBIT 9.12 Bloomberg Futures Contract Description Screen for a 10-Year Treasury Note Futures Contract



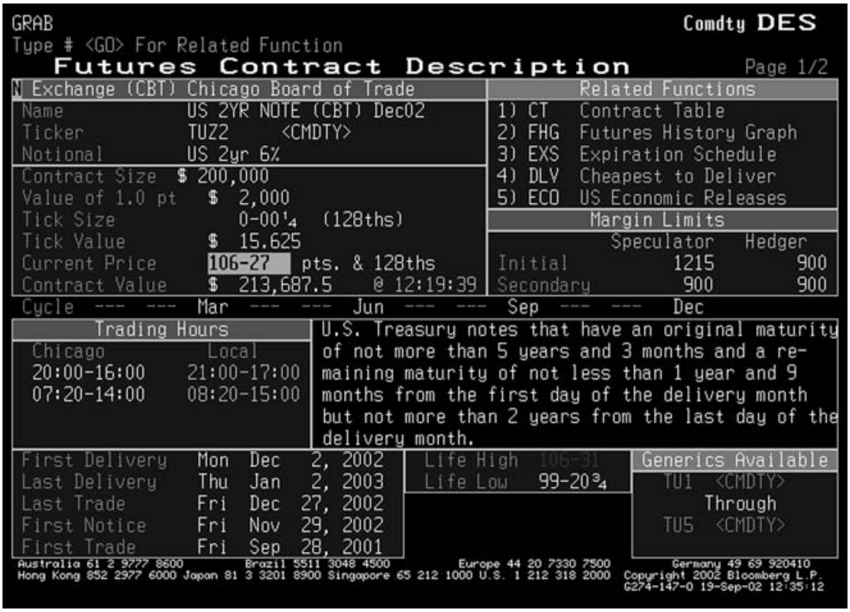
Source: Bloomberg Financial Markets

EXHIBIT 9.13 Bloomberg Futures Contract Description Screen for a 5-Year Treasury Note Futures Contract



Source: Bloomberg Financial Markets

EXHIBIT 9.14 **Bloomberg Futures Contract Description Screen for a 2-Year Treasury Note Futures Contract**



Source: Bloomberg Financial Markets

The underlying instrument for the 2-year Treasury note futures contract is $200,000 par value of a 6% notional coupon Treasury note. Issues acceptable for delivery must have a remaining maturity of not more than two years and not less than one year and nine months. Moreover, the original maturity of the note in the deliverable basket cannot be more than five years and three months. The minimum price fluctuation for this contract is 1⁄128 of 1% of par value. The dollar value of a 128*th* for a $200,000 par value is $15.625 ($100,000/12,800) and is therefore the minimum price fluctuation. Exhibit 9.14 shows the Bloomberg Futures Contract Description screen for the 2-year Treasury note contract.

Agency Note Futures

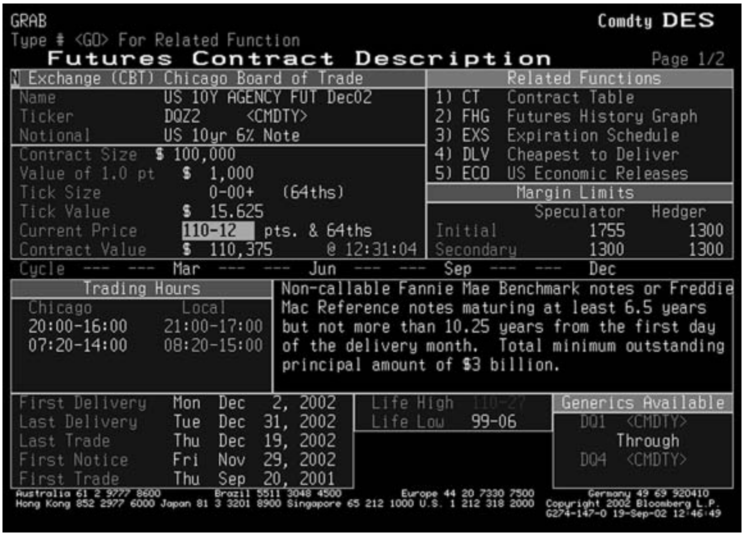
As will be explained in **Chapter XX**, portfolio managers use Treasury futures contracts to control their exposure to interest rate risk. However, in general, interest rate risk has two dimensions: the risk of changes in the level of Treasury yields and the risk of changes in the spread in the yield between non-Treasury securities and comparable maturity Treasuries. The latter risk, called *spread risk*, has increased significantly since the summer of 1998 when the Russian debt default roiled the financial markets. At the same time, there has been a reduction in the issuance of long-term Treasury securities. For example, since October 31, 2001, the Treasury no longer issues 30-year bonds. Two government sponsored enterprises, Fannie Mae and Freddie Mac, have stepped in to issue debenture securities that they hope will become the benchmark interest rates in the financial market.

As a result of the greater spread risk and increasingly role of agency securities, the CBOT and the Chicago Mercantile Exchange (CME) began trading in 2000 futures contracts in which the underlying instrument is a Fannie Mae or Freddie Mac agency debenture security.

The underlying instrument for the CBOT 10-year Agency note futures contract is a Freddie Mac Reference Note or a Fannie Mae Benchmark Note having a par value of $100,000 and a notional coupon of 6%. As with the Treasury futures contracts, there are several issues which can be delivered to settle the contract. For an issue to be deliverable it must be (1) a noncallable Freddie Mac Reference Note or a Fannie Mae Benchmark Note maturing at least 6.5 years but not more than 10.25 years (original maturity) from the first day of the delivery month, (2) have a minimum principal outstanding amount of at least $3 billion, and (3) pay semiannual fixed coupons. The tick size is $\frac{1}{64}$ of 1% of par value ($15.625). Exhibit 9.15 presents the Bloomberg Futures Contract Description screen for the December 2002 10-year agency futures contract. The contract delivery months are March, June, September, and December. Like Treasury futures, there is a conversion factor associated with each issue in the deliverable basket. Exhibit 9.16 shows the Bloomberg Cheapest to Deliver screen of the deliverable basket for the December 2002 contract. Note that the conversion factors and the implied repo rates perform the same functions as for the Treasury futures contracts.

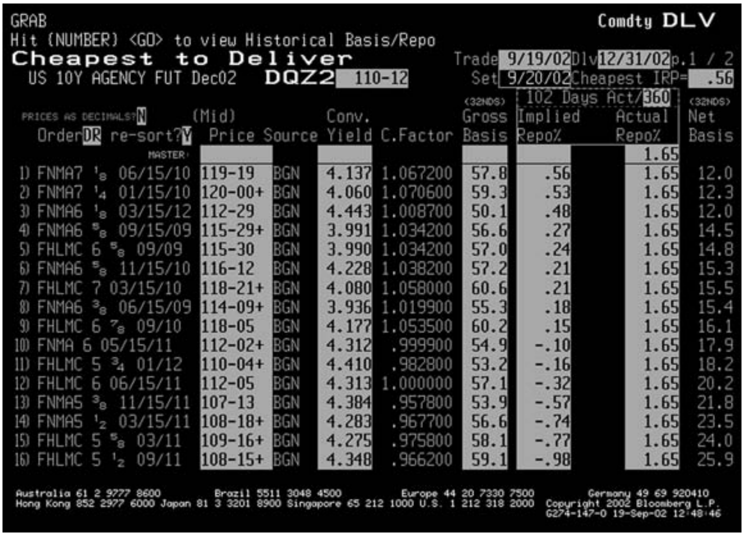
The 10-year Agency note futures contract of the CME is similar to that of the CBOT but has a notional coupon of 6.5% instead of 6%. For an issue to be deliverable, the CME requires that the original maturity is 10 years and which does not mature for a period of at least 6.5 years from the date of delivery. Finally, both the CBOT and the CME have a 5-year Agency note futures contract. Again, the CBOT's underlying is a 6% notional coupon and the CME's contract has a 6.5% notional coupon.

EXHIBIT 9.15 Bloomberg Futures Contract Description Screen for a 10-Year Agency Futures Contract



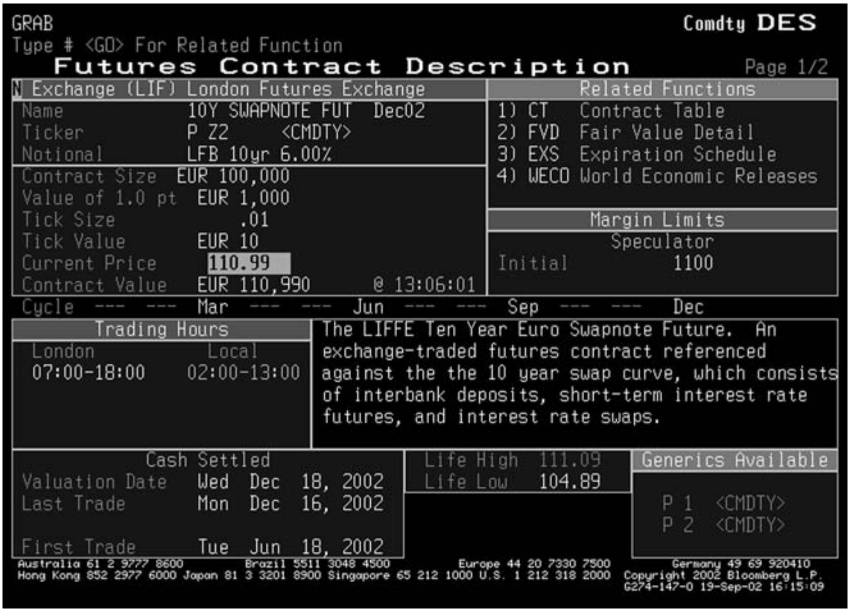
Source: Bloomberg Financial Markets

EXHIBIT 9.16 Bloomberg Cheapest to Deliver Screen for the December 2002 10-Year Agency Futures Contract



Source: Bloomberg Financial Markets

EXHIBIT 9.17 **Bloomberg Futures Contract Description Screen for a 10-Year Swap Contract**



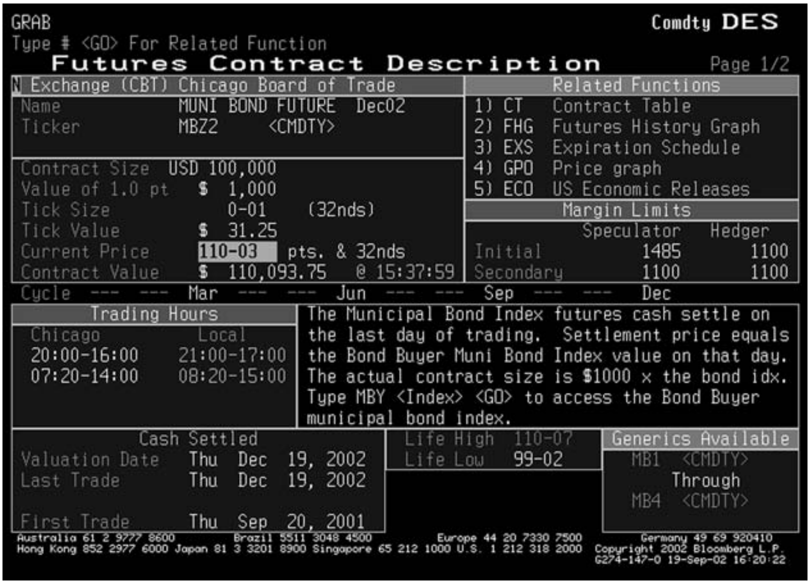
Source: Bloomberg Financial Markets

Swap Futures Contracts

The CBOT introduced a swap futures contract in late October 2001. The underlying instrument is the notional price of the fixed-rate side of a 10-year interest rate swap that has a notional principal equal to \$100,000 and that exchanges semiannual interest payments at a fixed annual rate of 6% for floating interest rate payments based on 3-month Libor.[9] Interest rate swaps are discussed in **Chapter XX**. Exhibit 9.17 shows the Bloomberg Futures Contract Description for the December 2002 10-year swap futures contract. This swap futures contract is cash-settled with a settlement price determined by the ISDA benchmark 10-year swap rate on the last day of trading before the contract expires. This benchmark rate is published with a one-day lag in the Federal Reserve Board's statistical release H.15. Contracts have settlement months of March, June, September, and December just like the other CBOT interest rate futures contracts that we have discussed.

[9] The CBOT has a 5-year swap futures contract.

EXHIBIT 9.18 **Bloomberg Futures Contract Description Screen for a Municipal Bond Futures Contract**



Source: Bloomberg Financial Markets

The London International Financial Futures Exchange (LIFFE) introduced the first swap futures contract called Swapnote® which is referenced to the euro interest rate swap curve. Swapnotes are available in 2-, 5-, and 10-year maturities. The CME also lists a swap futures contract with maturities of 2, 5, and 10 years that is similar to those listed on the CBOT.

Bond Buyer's Municipal Bond Index Futures Contract

The CBOT's municipal bond index futures contract is based on the value of the Bond Buyer Index (BBI) which consists of 40 municipal bonds. Unlike the Treasury bond futures contract, where the underlying instrument to be delivered is $100,000 par value of a hypothetical 6% 20-year Treasury bond, the municipal bond index futures contract does not specify a par amount of the underlying index to be delivered. Instead, the dollar value of a futures contract is equal to the product of the futures price and $1,000. The settlement price on the last day of trading is equal to the product of the Bond Buyer Index value and $1,000. Since delivery on all 40 bonds in the index would be extremely costly, the contract is settled in cash. Exhibit 9.18 shows the Bloomberg Futures Contract

Description screen for the December 2002 Municipal Bond Index futures contract. The settlement months are March, June, September, and December and delivery can take place anytime during the month.

FUTURES PRICING

In the section, we discuss a model of how futures are priced.

Theoretical Futures/Forward Price

To understand how futures contracts are valued, consider the following example. Suppose that a 12% 20-year bond is selling at par. Also suppose that this bond is the deliverable for a futures contract that settles in three months. If the current 3-month interest rate at which funds can be loaned or borrowed is 8% per year, what should be the price of this futures contract?

Suppose the price of the futures contract is 107. Consider the following strategy:

Sell the futures contract at 107.
Purchase the bond for 100.
Borrow 100 for 3 months at 8% per year.

The borrowed funds are used to purchase the bond, resulting in no initial cash outlay for this strategy. Three months from now, the bond must be delivered to settle the futures contract and the loan must be repaid. These trades will produce the following cash flows:

From settlement of the futures contract:		
Flat price of bond	=	107
Accrued interest (12% for 3 months)	=	3
Total proceeds	=	110
From the loan:		
Repayment of principal of loan	=	100
Interest on loan (8% for 3 months)	=	2
Total outlay	=	102
Profit = Total proceeds – Total outlay	=	8

This strategy will guarantee a profit of 8. Moreover, the profit is generated with no initial outlay because the funds used to purchase the bond are borrowed. The profit will be realized *regardless of the futures price* *at the settlement date*. Obviously, in a well-functioning market, arbitrageurs would buy the bond and sell the futures, forcing the futures price down and bidding up the bond price so as to eliminate this profit.

In contrast, suppose that the futures price is 92 instead of 107. Consider the following strategy:

Buy the futures contract at 92.
Sell (short) the bond for 100.
Invest (lend) 100 for 3 months at 8% per year.

Once again, there is no initial cash outlay. Three months from now a bond will be purchased to settle the long position in the futures contract. That bond will then be used to cover the short position (i.e., to cover the short sale in the cash market). The outcome in three months would be as follows:

From settlement of the futures contract:		
Flat price of bond	=	92
Accrued interest (12% for 3 months)	=	3
Total outlay	=	95
From the loan:		
Principal received from maturing investment	=	100
Interest earned (8% for 3 months)	=	2
Total proceeds	=	102
Profit = Total proceeds – Total outlay	=	7

The 7 profit is a pure arbitrage profit. It requires no initial cash outlay and will be realized regardless of the futures price at the settlement date.

However, there is a futures price that will eliminate the arbitrage profit. There will be no arbitrage if the futures price is 99. Let's look at what would happen if the two previous strategies are followed and the futures price is 99. First, consider the following strategy:

Sell the futures contract at 99.
Purchase the bond for 100.
Borrow 100 for 3 months at 8% per year.

In three months, the outcome would be as follows:

From settlement of the futures contract:		
Flat price of bond	=	99
Accrued interest (12% for 3 months)	=	3
Total proceeds	=	102

From the loan:		
Repayment of principal of loan	=	100
Interest on loan (8% for 3 months)	=	2
Total outlay	=	102
Profit = Total proceeds – Total outlay	=	0

There is no arbitrage profit in this case. Next consider the following strategy:

Buy the futures contract at 99.
Sell (short) the bond for 100.
Invest (lend) 100 for 3 months at 8% per year.

The outcome in three months would be as follows:

From settlement of the futures contract:		
Flat price of bond	=	99
Accrued interest (12% for 3 months)	=	3
Total outlay	=	102
From the loan:		
Principal received from maturing investment	=	100
Interest earned (8% for 3 months)	=	2
Total proceeds	=	102
Total proceeds – Total outlay = Profit	=	0

Thus neither strategy results in a profit. Hence the futures price of 99 is the theoretical price, because any higher or lower futures price will permit arbitrage profits.

Theoretical Futures Price Based on Arbitrage Model

Considering the arbitrage arguments just presented, the theoretical futures price can be determined on the basis of the following information:

1. The price of the bond in the cash market.
2. The coupon rate on the bond. In our example, the coupon rate is 12% per year.
3. The interest rate for borrowing and lending until the settlement date. The borrowing and lending rate is referred to as the financing rate. In our example, the financing rate is 8% per year.

We will let

r = annualized financing rate (%)
c = annualized current yield, or annual coupon rate divided by the cash market price (%)
P = cash market price
F = futures price
t = time, in years, to the futures delivery date

and then consider the following strategy that is initiated on a coupon date:

Sell the futures contract at F.
Purchase the bond for P.
Borrow P until the settlement date at r.

The outcome at the settlement date is

From settlement of the futures contract:	
Flat price of bond	= F
Accrued interest	= ctP
Total proceeds	= $F + ctP$
From the loan:	
Repayment of principal of loan	= P
Interest on loan	= rtP
Total outlay	= $P + rtP$

The profit will equal:

$$\text{Profit} = \text{Total proceeds} - \text{Total outlay}$$

$$\text{Profit} = F + ctP - (P + rtP)$$

In equilibrium the theoretical futures price occurs where the profit from this trade is zero. Thus to have equilibrium, the following must hold:

$$0 = F + ctP - (P + rtP)$$

Solving for the theoretical futures price, we have

$$F = P + Pt(r - c) \tag{9.1}$$

Alternatively, consider the following strategy:

Buy the futures contract at F.

Sell (short) the bond for P.
Invest (lend) P at r until the settlement date.

The outcome at the settlement date would be

From settlement of the futures contract:

Flat price of bond	=	F
Accrued interest	=	ctP
Total outlay	=	$F + ctP$

From the loan:

Proceeds received from maturing of investment	=	P
Interest earned	=	rtP
Total proceeds	=	$P + rtP$

The profit will equal:

$$\text{Profit} = \text{Total proceeds} - \text{Total outlay}$$

$$\text{Profit} = P + rtP - (F + ctP)$$

Setting the profit equal to zero so that there will be no arbitrage profit and solving for the futures price, we obtain the same equation for the futures price as equation (9.1).

Let's apply equation (9.1) to our previous example in which

$r = 0.08$
$c = 0.12$
$P = 100$
$t = 0.25$

Then the theoretical futures price is

$$F = 100 + 100 \times 0.25(0.08 - 0.12) = 100 - 1 = 99$$

The theoretical price may be at a premium to the cash market price (higher than the cash market price) or at a discount from the cash market price (lower than the cash market price), depending on $(r - c)$. The term $r - c$ is called the *net financing cost* because it adjusts the financing rate for the coupon interest earned. The net financing cost is more commonly called the *cost of carry*, or simply *carry*. *Positive carry* means that the current yield earned is greater than the financing cost; *negative carry* means that the financing cost exceeds the current yield. The relationships can be expressed as follows:

Carry	Theoretical Futures Price
Positive ($c < r$)	Will sell at a discount to cash price ($F < P$)
Negative ($c > r$)	Will sell at a premium to cash price ($F > P$)
Zero ($c = r$)	Will be equal to cash price ($F = P$)

In the case of interest rate futures, carry (the relationship between the short-term financing rate and the current yield on the bond) depends on the shape of the yield curve. When the yield curve is upward sloping, the short-term financing rate will be less than the current yield on the bond, resulting in positive carry. The theoretical futures price will then sell at a discount to the cash price for the bond. The opposite will hold true when the yield curve is inverted.

Adjustments to the Theoretical Pricing Model

Several assumptions were made to derive the theoretical futures price using the arbitrage argument. First, no interim cash flows due to variation margin or coupon interest payments were assumed in the model. However, we know that interim cash flows can occur for both of these reasons. Consider first variation margin. If interest rates rise, the short position in futures will receive margin as the futures price decreases; the margin can then be reinvested at a higher interest rate. If interest rates fall, there will be variation margin that must be financed by the short position; however, because interest rates have declined, financing will be possible at a lower cost. The same is true for a forward contract that is marked to market. Thus, whichever way rates move, those who are short futures or forwards that are marked to market gain relative to those who are short forwards that are not marked to market. Conversely, those who are long futures or forwards that are not marked to market lose relative to those who are long forwards that are marked to market. These facts account for the difference between futures prices and forward prices for nonmarked-to-market contracts.

Incorporating interim coupon payments into the pricing model is not difficult. However, the value of the coupon payments at the settlement date will depend on the interest rate at which they can be reinvested. The shorter the maturity of the contract and the lower the coupon rate, the less important the reinvestment income is in determining the theoretical futures price.

The second assumption in deriving the theoretical futures price is that the borrowing and lending rates are equal. Typically, however, the borrowing rate is higher than the lending rate. As a result, there is not one theoretical futures price but rather there are lower and upper boundaries for the theoretical futures price.

The third assumption made to derive equation (9.1) is that only one instrument is deliverable. But as explained earlier, the futures contract on Treasury bonds and Treasury notes are designed to allow the short the choice of delivering one of a number of deliverable issues (the quality or swap option). Because there may be more than one deliverable, market participants track the price of each deliverable bond and determine which bond is the cheapest to deliver. The theoretical futures price will then trade in relation to the cheapest-to-deliver issue.

There is the risk that while an issue may be the cheapest to deliver at the time a position in the futures contract is taken, it may not be the cheapest to deliver after that time. A change in the cheapest to deliver can dramatically alter the futures price. Because the swap option is an option granted by the long to the short, the long will want to pay less for the futures contract than indicated by equation (9.1). Therefore, as a result of the quality option, the theoretical futures price as given by equation (9.1) must be adjusted as follows:

$$F = P + Pt(r - c) - \text{Value of quality option} \tag{9.2}$$

Market participants have employed theoretical models in attempting to estimate the fair value of the quality option.

Finally, in deriving equation (9.1) a known delivery date is assumed. For Treasury bond and note futures contracts, the short has a timing and wild card option, so the long does not know when the securities will be delivered. The effect of the timing and wild card options on the theoretical futures price is the same as with the quality option. These delivery options result in a theoretical futures price that is lower than the one suggested in equation (9.1), as shown below:

$$\begin{aligned} F = P &+ Pt(r - c) - \text{Value of quality option} \\ &- \text{Value of timing option} - \text{Value of wild card option} \end{aligned} \tag{9.3}$$

or alternatively,

$$F = P + Pt(r - c) - \text{Delivery options} \tag{9.4}$$

Market participants attempt to value the delivery options in order to apply equation (9.4).

FORWARD RATE AGREEMENTS

A *forward rate agreement* (FRA) is an over-the-counter derivative instrument that trades as part of the money market. In essence, an FRA is a forward-starting loan, but with no exchange of principal, so the cash exchanged between the counterparties depend only on the difference in interest rates. While the FRA market is truly global, most business is transacted in London. Trading in FRAs began in the early 1980s and the market now is large and liquid. According to the British Bankers Association, turnover in London exceeds $5 billion each day.

In effect an FRA is a forward dated loan, transacted at a fixed rate, but with no exchange of principal—only the interest applicable on the notional amount between the rate agreed to when the contract is established and the actual rate prevailing at the time of settlement changes hands. For this reason, FRAs are off-balance sheet instruments. By trading today at an interest rate that is effective at some point in the future, FRAs enable banks and corporations to hedge forward interest rate exposure.

FRA Basics

An FRA is an agreement to borrow or lend a notional cash sum for a period of time lasting up to 12 months, starting at any point over the next 12 months, at an agreed rate of interest (the FRA rate). The "buyer" of a FRA is borrowing a notional sum of money while the "seller" is lending this cash sum. Note how this differs from all other money market instruments. In the cash market, the party buying a CD, Treasury bill, or bidding for bond in the repo market, is the lender of funds. In the FRA market, to "buy" is to "borrow." Of course, we use the term "notional" because with an FRA no borrowing or lending of cash actually takes place. The notional sum is simply the amount on which the interest payment is calculated (i.e., a scale factor).

Accordingly, when a FRA is traded, the buyer is borrowing (and the seller is lending) a specified notional sum at a fixed rate of interest for a specified period, the "loan" to commence at an agreed date in the future. The buyer is the notional borrower, and so if there is a rise in interest rates between the date that the FRA is traded and the date that the FRA comes into effect, she will be protected. If there is a fall in interest rates, the buyer must pay the difference between the rate at which the FRA was traded and the actual rate, as a percentage of the notional sum.

The buyer may be using the FRA to hedge an actual exposure, that is an actual borrowing of money, or simply speculating on a rise in interest rates. The counterparty to the transaction, the seller of the FRA, is the notional lender of funds, and has fixed the rate for lending funds. If there is a fall in interest rates, the seller will gain, and if there is a rise in rates, the seller will pay. Again, the seller may have an actual loan of cash to hedge or is acting as a speculator.

In FRA trading, only the payment that arises because of the difference in interest rates changes hands. There is no exchange of cash at the time of the trade. The cash payment that does arise is the difference in interest rates between that at which the FRA was traded and the actual rate prevailing when the FRA matures, as a percentage of the notional amount. FRAs are traded by both banks and corporations. The FRA market is liquid in all major currencies and rates are readily quoted on screens by both banks and brokers. Dealing is over the telephone or over a dealing system such as Reuters.

The terminology quoting FRAs refers to the borrowing time period and the time at which the FRA comes into effect (or matures). Hence if a buyer of a FRA wished to hedge against a rise in rates to cover a 3-month loan starting in three months' time, she would transact a "3-against-6 month" FRA, or more usually denoted as a 3×6 or 3v6 FRA. This is referred to in the market as a "threes-sixes" FRA, and means a 3-month loan beginning in three months' time. So correspondingly, a "ones-fours" FRA (1v4) is a 3-month loan in one month's time, and a "three-nines" FRA (3v9) is a 6-month loan in three months' time.

As an illustration, suppose a corporation anticipates it will need to borrow in six months time for a 6-month period. It can borrow today at 6-month Libor plus 50 basis points. Assume that 6-month Libor rates are 4.0425% but the corporation's treasurer expects rates to go up to about 4.50% over the next several weeks. If the treasurer's suspicion is correct, the corporation will be forced to borrow at higher rates unless some sort of hedge is put in place to protect the borrowing requirement. The treasurer elects to buy a 6v12 FRA to cover the 6-month period beginning six months from now. A bank quotes 4.3105% for the FRA, which the corporation buys for a £1,000,000 notional principal. Suppose that six months from now, 6-month Libor has indeed backed-up to 4.50%, so the treasurer must borrow funds at 5% (Libor plus the 50 basis point spread). However, offsetting this rise in rates, the corporation will receive a settlement amount which will be the difference between the rate at which the FRA was bought (4.3105%) and today's 6-month Libor rate (4.50%) as a percentage of the notional principal of £1,000,000. This payment will compensate for some of the increased borrowing costs.

FRA Mechanics

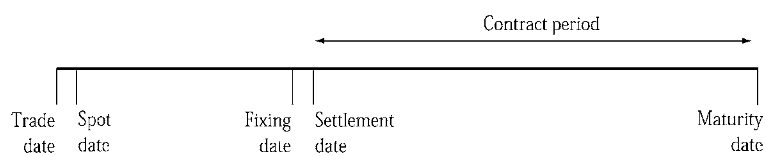
In virtually every market, FRAs trade under a set of terms and conventions that are identical. The British Bankers Association (BBA) has compiled standard legal documentation to cover FRA trading. The following standard terms are used in the market:

- *Notional sum:* The amount for which the FRA is traded.
- *Trade date:* The date on which the FRA is transacted.
- *Settlement date:* The date on which the notional loan or deposit of funds becomes effective, that is, is said to begin. This date is used, in conjunction with the notional sum, for calculation purposes only as no actual loan or deposit takes place.
- *Fixing date:* This is the date on which the reference rate is determined, that is, the rate to which the FRA rate is compared.
- *Maturity date:* The date on which the notional loan or deposit expires.
- *Contract period:* The time between the settlement date and maturity date.
- *FRA rate:* The interest rate at which the FRA is traded.
- *Reference rate:* This is the rate used as part of the calculation of the settlement amount, usually the Libor rate on the fixing date for the contract period in question.
- *Settlement sum:* The amount calculated as the difference between the FRA rate and the reference rate as a percentage of the notional sum, paid by one party to the other on the settlement date.

These key dates are illustrated in Exhibit 9.19.

The spot date is usually two business days after the trade date, however it can by agreement be sooner or later than this. The settlement date will be the time period after the spot date referred to by the FRA terms: for example a 1×4 FRA will have a settlement date one calendar month after the spot date. The fixing date is usually two business days before the settlement date. The settlement sum is paid on the settlement date, and as it refers to an amount over a period of time that is paid up front (i.e., at the start of the contract period), the calculated sum is a discounted present value. This is because a normal payment of interest on a loan/deposit is paid at the end of the time period to which it relates; because an FRA makes this payment at the start of the relevant period, the settlement amount is a discounted present value sum. With most FRA trades, the reference rate is the level of Libor on the fixing date.

EXHIBIT 9.19 **Key Dates in a FRA Trade**



The settlement sum is calculated after the fixing date, for payment on the settlement date. We can illustrate this with a hypothetical example. Consider a case where a corporation has bought £1 million notional sum of a 1×4 FRA, and transacted at 5.75%, and that the market rate is 6.50% on the fixing date. The contract period is 90 days. In the cash market the extra interest charge that the corporate would pay is a simple interest calculation, and is:

$$\begin{aligned}\text{Extra interest charge} &= \frac{6.50 - 5.75}{100} \times £1{,}000{,}000 \times (91/365)\\ &= £1{,}869.86\end{aligned}$$

Note that in the U.S. money market, a 360-day year is assumed rather than the 365 day year used in the UK money market.

This extra interest that the corporation is facing would be payable with the interest payment for the loan, which (as it is a money market loan) is paid when the loan matures. Under a FRA then, the settlement sum payable should, if it was paid on the same day as the cash market interest charge, be exactly equal to this. This would make it a perfect hedge. As we noted above though, FRA settlement value is paid at the start of the contract period, that is, the beginning of the underlying loan and not the end. Therefore, the settlement sum has to be adjusted to account for this, and the amount of the adjustment is the value of the interest that would be earned if the unadjusted cash value were invested for the contract period in the money market. The settlement value is given by the following expression:

$$\text{Settlement value} = \frac{(r_{\text{ref}} - r_{\text{FRA}}) \times M \times (n/B)}{1 + [r_{\text{ref}} \times (n/B)]}$$

where

r_{ref} = the reference interest fixing rate
r_{FRA} = the FRA rate or contract rate
M = the notional value sum
n = the number of days in the contract period
B = the day-count basis (360 or 365)

The expression for the settlement value above simply calculates the extra interest payable in the cash market, resulting from the difference between the two interest rates, and then discounts the amount because it is payable at the start of the period and not, as would happen in the cash market, at the end of the period.

In our hypothetical illustration, as the fixing rate is higher than the contract rate, the buyer of the FRA receives the settlement sum from the seller. This payment compensates the buyer for the higher borrowing costs that they would have to pay in the cash market. If the fixing rate had been lower than 5.75%, the buyer would pay the difference to the seller, because the cash market rates will mean that they are subject to a lower interest rate in the cash market. What the FRA has done is hedge the interest rate exposure, so that whatever happens in the market, the buyer will pay 5.75% on its borrowing.

A market maker in FRAs is trading short-term interest rates. The settlement sum is the value of the FRA. The concept is exactly as with trading short-term interest-rate futures; a trader who buys a FRA is running a long position, so that if on the fixing date the reference rate is greater than the contract rate then the settlement sum is positive and the trader realizes a profit. What has happened is that the trader, by buying the FRA, "borrowed" money at the FRA rate, which subsequently rose. This is a gain, exactly like a short position in an interest rate futures contract, where if the price goes down (that is, interest rates go up), the trader realizes a gain. Conversely, a "short" position in a FRA which is accomplished by selling a FRA realizes a gain if on the fixing date the reference rate is less than the FRA rate.

FRA Pricing

FRAs are forward rate instruments and are priced using standard forward rate principles.[10] Consider an investor who has two alternatives, either a 6-month investment at 5% or a 1-year investment at 6%. If the investor wishes to invest for six months and then rollover the investment for a further six months, what rate is required for the rollover period such that the final return equals the 6% available from the 1-year investment? If we view a FRA rate as the break-even forward rate between the two periods, we simply solve for this forward rate and that is our approximate FRA rate.

In practice, FRAs are priced off the exchange-traded short-term interest rate futures for that currency. For this reason, the contract rates (FRA rates) for FRAs are possibly the most liquid and transparent of any nonexchange-traded derivative instrument. To illustrate the pricing of FRAs, we will assume that

- The FRAs start today, January 1 of year 1 (FRA settlement date).
- The reference rate is Libor.
- Today 3-month Libor is 4.05%.

[10] For a discussion of these principles, see Frank J. Fabozzi and Steven V. Mann, *Introduction to Fixed-Income Analytics* (New Hope, PA: Frank J. Fabozzi Associates, 2001).

EXHIBIT 9.20 **Calculating the Implied Forward Rates**

(1)	(2)	(3)	(4)	(5)	(6)	(7)
Quarter Starts	Quarter Ends	Number of Days in Quarter	Current 3-Month Libor	Eurodollar CD Futures Price	Forward Rate	Period Forward Rate
Jan 1 year 1	Mar 1 year 1	90	4.05%	—	—	1.0125%
Apr 1 year 1	June 30 year1	91	—	95.85	4.15%	1.0490%
July 1 year 1	Sept 30 year 1	92	—	95.45	4.55%	1.1628%
Oct 1 year 1	Dec 31 year 1	92	—	95.28	4.72%	1.2062%

Exhibit 9.20 presents the information that we will utilize in the FRA pricing. We will in an analogous manner as when we determined the future floating-rate payments in a swap contract in the next chapter. Shown in Column (1) is when the quarter begins and in Column (2) when the quarter ends in year 1. Column (3) lists the number of days in each quarter. Column (4) shows the current value of 3-month Libor. Column (5) contains the prices of 3-month Eurodollar CD futures contracts used to determine the implied 3-Libor forward rates in Column (6). Lastly, Column (7) contains the forward rate for the period that we will refer to as the *period forward rate*. The period forward rate is computed using the following formula:

$$\text{Period forward rate} = \text{Annual forward rate} \times (\text{Days in period}/360)$$

For example, the annual forward rate for the second quarter is 4.15%. The period forward rate for quarter 2 is:

$$\text{Period forward rate} = 4.15\% \times (91/360) = 1.0490\%$$

Using the information presented above, let's illustrate the pricing of a 3v9 FRA. Simply put, using the forward rates implied by the Eurodollar CD futures contracts, we are asking what is the annualized implied 6-month Libor forward rate three months hence. Accordingly, the 3v9 FRA price is calculated as follows:

$$[(1.010490)(1.011628) - 1](360/183) = 0.043751 = 4.3751\%$$

A couple of points should be noted here. First, in the U.S. money markets an Actual/360, day count convention is used but in the U.K. the day count convention is Actual/365. Second, in the calculation, the 183 days is the length of the 6-month period beginning three months from now.

EXHIBIT 9.21 **FRA Rates for Various Maturities and Currencies**
Panel A: U.S. Dollar FRAs


GRAB M-Mkt TTFR

15:34 TULLETT & TOKYO PAGE 1 / 1

	USD FRAs	Bid	Ask	Time		USD FRAs	Bid	Ask	Time
1)	1 X 4	1.92	1.95	15:09	16)	12 X 18	2.53	2.56	15:19
2)	2 X 5	1.84	1.87	15:09	17)	18 X 24	3.16	3.19	15:09
3)	3 X 6	1.66	1.69	15:23	18)	1 X 10	2.03	2.06	15:01
4)	4 X 7	1.70	1.73	15:23	19)	2 X 11	2.01	2.04	15:07
5)	5 X 8	1.68	1.71	15:23	20)	3 X 12	1.81	1.84	15:23
6)	6 X 9	1.71	1.74	15:23	21)	6 X 18	2.23	2.26	15:33
7)	7 X 10	1.83	1.86	15:23	22)	12 X 24	2.85	2.88	15:09
8)	8 X 11	1.92	1.95	15:03					
9)	9 X 12	2.00	2.03	15:03					
10)	1 X 7	1.97	2.00	15:01					
11)	2 X 8	1.95	1.98	15:23					
12)	3 X 9	1.71	1.74	15:23					
13)	4 X 10	1.76	1.79	15:23					
14)	5 X 11	1.82	1.85	15:23					
15)	6 X 12	1.86	1.89	15:23					

Australia 61 2 9777 8600 Brazil 5511 3048 4500 Europe 44 20 7330 7500 Germany 49 69 920410
Hong Kong 852 2977 6000 Japan 81 3 3201 8900 Singapore 65 212 1000 U.S. 1 212 318 2000 Copyright 2002 Bloomberg L.P.
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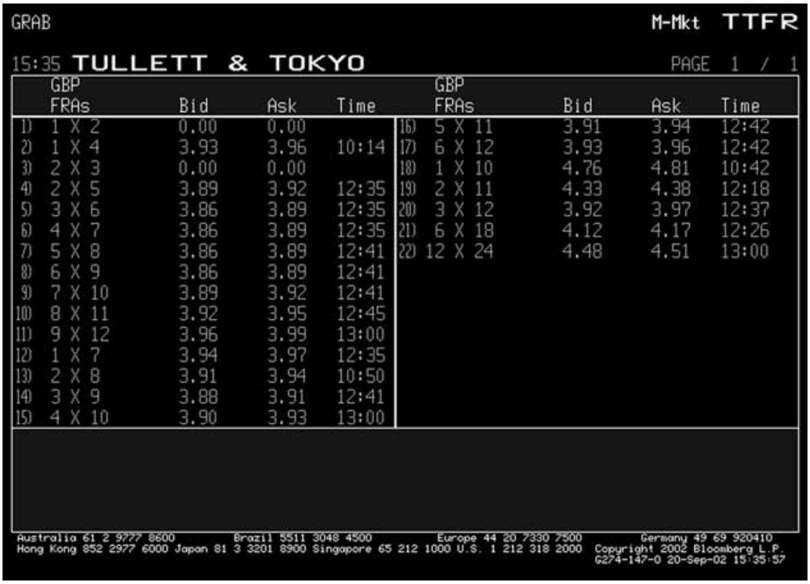
Source: Bloomberg Financial Markets

By the same reasoning, we can price a 3v12 FRA. In this illustration, we are calculating the implied 9-month forward rate (annualized) three months hence. The price of a 3v12 is calculate as follows:

$$[(1.010490)(1.011628)(1.012062) - 1](360/275)$$
$$= 0.045256 = 4.5256\%$$

Exhibit 9.21, Panels A, B, and C present three Bloomberg screens of bid/ask rates for FRAs for various maturities and currencies. These data are supplied to Bloomberg by Tullett and Tokyo Forex International. The currencies are U.S. dollars, pound sterling, and euros, respectively.

EXHIBIT 9.21 (Continued)
Panel B: Pound Sterling FRAs



	GBP FRAs	Bid	Ask	Time		GBP FRAs	Bid	Ask	Time
1)	1 X 2	0.00	0.00		16)	5 X 11	3.91	3.94	12:42
2)	1 X 4	3.93	3.96	10:14	17)	6 X 12	3.93	3.96	12:42
3)	2 X 3	0.00	0.00		18)	1 X 10	4.76	4.81	10:42
4)	2 X 5	3.89	3.92	12:35	19)	2 X 11	4.33	4.38	12:18
5)	3 X 6	3.86	3.89	12:35	20)	3 X 12	3.92	3.97	12:37
6)	4 X 7	3.86	3.89	12:35	21)	6 X 18	4.12	4.17	12:26
7)	5 X 8	3.86	3.89	12:41	22)	12 X 24	4.48	4.51	13:00
8)	6 X 9	3.86	3.89	12:41					
9)	7 X 10	3.89	3.92	12:41					
10)	8 X 11	3.92	3.95	12:45					
11)	9 X 12	3.96	3.99	13:00					
12)	1 X 7	3.94	3.97	12:35					
13)	2 X 8	3.91	3.94	10:50					
14)	3 X 9	3.88	3.91	12:41					
15)	4 X 10	3.90	3.93	13:00					

Panel C: Euro FRAs

	EUR FRAS	Bid	Ask	Time		EUR FRAS	Bid	Ask	Time
1)	1 X 4	3.2080	3.2230	12:44	16)	12 X 18	3.3090	3.3240	13:00
2)	2 X 5	3.1290	3.1440	12:44	17)	18 X 24	3.6800	3.6950	13:00
3)	3 X 6	3.0700	3.0850	12:44	18)	1 X 10	3.1310	3.1460	12:59
4)	4 X 7	3.0240	3.0390	13:00	19)	2 X 11	3.0850	3.1000	12:59
5)	5 X 8	2.9960	3.0110	13:00	20)	3 X 12	3.0520	3.0670	13:00
6)	6 X 9	2.9810	2.9960	13:00	21)	6 X 18	3.1870	3.2020	13:00
7)	7 X 10	2.9990	3.0140	13:00	22)	12 X 24	3.5240	3.5390	13:00
8)	8 X 11	3.0140	3.0290	12:59					
9)	9 X 12	3.0380	3.0530	12:59					
10)	1 X 7	3.1660	3.1810	13:00					
11)	2 X 8	3.0940	3.1090	13:00					
12)	3 X 9	3.0420	3.0570	13:00					
13)	4 X 10	3.0230	3.0380	13:00					
14)	5 X 11	3.0180	3.0330	13:00					
15)	6 X 12	3.0230	3.0380	13:00					

Source: Bloomberg Financial Markets

KEY POINTS

1. A forward contract is an agreement for the future delivery of something at a specified price at the end of a designated period of time but differs from a futures contract in that it is nonstandardized and does not trade on an organized exchange.

2. Parties to a forward contract are exposed to counterparty risk which is the risk that the counterparty will not satisfy its contractual obligations.

3. A futures contract is an agreement between a buyer (seller) and an established exchange or its clearinghouse in which the buyer (seller) agrees to take (make) delivery of something at a specified price at the end of a designated period of time.

4. The parties to a futures contract are required to satisfy margin requirements.

5. An investor who takes a long futures position realizes a gain when the futures price increases; an investor who takes a short futures position realizes a loss when the futures price decreases.

6. The Treasury bill futures contract is based on a 3-month (13 week) Treasury bill with a face value of $1 million.

7. The Eurodollar CD futures contract is a cash settlement contract whose underlying is a 3-month Eurodollar CD and is one of the most heavily traded futures contracts in the world.

8. The Federal Funds futures contract is a cash settlement contract whose underlying is the average overnight federal funds for the delivery month.

9. For the Treasury bond futures contract, the underlying instrument is $100,000 par value of a hypothetical 20-year, 6% coupon Treasury bond.

10. Conversion factors are used to adjust the invoice price of a Treasury bond futures contract to make delivery equitable to both parties.

11. The short in a Treasury bond futures contract has several delivery options: quality option (or swap option), timing option, and wild card option.

12. The cheapest-to-deliver issue is the issue in the deliverable basket that has the largest implied repo rate.

13. The 2-year, 5-year, and 10-year Treasury note futures contracts are modeled after the Treasury bond futures contract.

14. The underlying instrument for an 10-year Agency note futures contract is a Freddie Mac Reference Note or a Fannie Mae Benchmark Note having a par value of $100,000 and a notional coupon of 6%.

15. The underlying instrument for a swap futures contract is the notional price of the fixed-rate side of a 10-year interest rate swap that has a notional principal equal to $100,000 and that exchanges semi-annual interest payments at a fixed annual rate of 6% for floating interest rate payments based on 3-month Libor.

16. The municipal bond futures contract is based on the value of the Bond Buyer Index.

17. The theoretical price of a futures contract is equal to the cash or spot price plus the cost of carry.

18. The cost of carry is equal to the cost of financing the position less the cash yield on the underlying security.

19. The shape of the yield curve affects the cost of carry.

20. The standard arbitrage model must be modified to take into consideration the nuances of particular futures contracts.

21. For a Treasury bond futures contract, the delivery options granted to the short reduce the theoretical futures price below the theoretical futures price suggested by the standard arbitrage model.

22. A forward rate agreement is an over-the-counter derivative instrument which is essentially a forward-starting loan, but with no exchange of principal, so the cash exchanged between the counterparties depend only on the difference in interest rates.

23. The elements of an FRA are the FRA rate, reference rate, notional amount, contract period, and settlement date.

24. The buyer of an FRA is agreeing to pay the FRA rate and the seller of the FRA is agreeing to receive the FRA rate.

25. The amount that must be exchanged at the settlement date is the present value of the interest differential.

26. In contrast to an interest rate futures contract, the buyer of an FRA benefits if the reference rate increases and the seller benefits if the reference rate decreases.

CHAPTER 10

Interest Rate Swaps and Swaptions

Swaps and swaptions are also used extensively by market participants to control interest rate risk. These derivative instruments are the focus of this chapter.

The most prevalent swap contract is an *interest rate swap*. An interest rate swap contract provides a vehicle for market participants to transform the nature of cash flows and the interest rate exposure of a portfolio or balance sheet. In this chapter, we explain how to analyze interest rate swaps. We will describe a generic interest rate swap, the parties to a swap, the risk and return of a swap, and the economic interpretation of a swap. Then we look at how to compute the floating-rate payments and calculate the present value of these payments. Next we will see how to calculate the fixed-rate payments given the swap rate. Before we look at how to calculate the value of a swap, we will see how to calculate the swap rate. Given the swap rate, we will then see how the value of a swap is determined after the inception of a swap. We will also discuss other types of swaps as well as options on swaps called *swaptions*. Swaptions are used ever more frequently as a tool for investors to control their interest rate risk. These instruments are described in the latter part of the chapter.

The objectives of this chapter are to:
1. Explain what a generic interest rate swap is.
2. Explain how a swap should be interpreted.
3. Explain swap terminology, conventions, and market quotes.
4. Demonstrate how the swap rate is determined.
5. Demonstrate how the value of a swap is determined.
6. Explain the primary determinants of swap spreads.
7. Describe several types of non-generic swaps.
8. Explain what a swaption is.
9. Explain the important elements of swaption valuation.

DESCRIPTION OF AN INTEREST RATE SWAP

In an *interest rate swap*, two parties (called *counterparties*) agree to exchange periodic interest payments. The dollar amount of the interest payments exchanged is based on some predetermined dollar principal, which is called the *notional amount*. The dollar amount each counterparty pays to the other is the agreed-upon periodic interest rate times the notional amount. The only dollars that are exchanged between the parties are the interest payments, not the notional amount. Accordingly, the notional principal serves only as a scale factor to translate an interest rate into a cash flow. In the most common type of swap, one party agrees to pay the other party fixed interest payments at designated dates for the life of the contract. This party is referred to as the *fixed-rate payer*. The other party, who agrees to make interest rate payments that float with some reference rate, is referred to as the *floating-rate payer*.

The reference rates that have been used for the floating rate in an interest rate swap are various money market rates: Treasury bill rate, the London interbank offered rate, commercial paper rate, bankers acceptances rate, certificates of deposit rate, the federal funds rate, and the prime rate. The most common is the London interbank offered rate (Libor). Libor is the rate at which prime banks offer to pay on Eurodollar deposits available to other prime banks for a given maturity. There is not just one rate but a rate for different maturities. For example, there is a 1-month Libor, 3-month Libor, and 6-month Libor.

To illustrate an interest rate swap, suppose that for the next five years party X agrees to pay party Y 10% per year, while party Y agrees to pay party X 6-month Libor (the reference rate). Party X is a fixed-rate payer/floating-rate receiver, while party Y is a floating-rate payer/fixed-rate receiver. Assume that the notional amount is $50 million, and that payments are exchanged every six months for the next five years. This means that every six months, party X (the fixed-rate payer/floating-rate receiver) will pay party Y $2.5 million (10% times $50 million divided by 2). The amount that party Y (the floating-rate payer/fixed-rate receiver) will pay party X will be 6-month Libor times $50 million divided by 2. If 6-month Libor is 7%, party Y will pay party X $1.75 million (7% times $50 million divided by 2). Note that we divide by two because one-half year's interest is being paid.

Interest rate swaps are over-the-counter instruments. This means that they are not traded on an exchange. An institutional investor wishing to enter into a swap transaction can do so through either a securities firm or a commercial bank that transacts in swaps.[1] These entities can do one of the following. First, they can arrange or broker a swap between two parties that want to enter into an interest rate swap. In this case, the securities firm or commercial bank is acting in a brokerage capacity.

The second way in which a securities firm or commercial bank can get an institutional investor into a swap position is by taking the other side of the swap. This means that the securities firm or the commercial bank is a dealer rather than a broker in the transaction. Acting as a dealer, the securities firm or the commercial bank must hedge its swap position in the same way that it hedges its position in other securities. Also it means that the swap dealer is the counterparty to the transaction.

The risks that the two parties take on when they enter into a swap is that the other party will fail to fulfill its obligations as set forth in the swap agreement. That is, each party faces default risk. The default risk in a swap agreement is called *counterparty risk*. In any agreement between two parties that must perform according to the terms of a contract, counterparty risk is the risk that the other party will default. With futures and exchange-traded options the counterparty risk is the risk that the clearinghouse will default. Market participants view this risk as small. In contrast, counterparty risk in a swap can be significant.

Because of counterparty risk, not all securities firms and commercial banks can be swap dealers. Several securities firms have established subsidiaries that are separately capitalized so that they have a high credit rating which permit them to enter into swap transactions as a dealer.

Thus, it is imperative to keep in mind that any party who enters into a swap is subject to counterparty risk.

INTERPRETING A SWAP POSITION

There are two ways that a swap position can be interpreted: (1) a package of forward/futures contracts and (2) a package of cash flows from buying and selling cash market instruments.

Package of Forward Contracts

Consider the hypothetical interest rate swap used earlier to illustrate a swap. Let's look at party X's position. Party X has agreed to pay 10% and receive 6-month Libor. More specifically, assuming a $50 million

[1] Do not get confused here about the role of commercial banks. A bank can use a swap in its asset/liability management. Or a bank can transact (buy and sell) swaps to clients to generate fee income. It is in the latter sense that we are discussing the role of a commercial bank in the swap market here.

notional amount, X has agreed to buy a commodity called "6-month Libor" for $2.5 million. This is effectively a 6-month forward contract where X agrees to pay $2.5 million in exchange for delivery of 6-month Libor. The fixed-rate payer is effectively long a 6-month forward contract on 6-month Libor. The floating-rate payer is effectively short a 6-month forward contract on 6-month Libor. There is therefore an implicit forward contract corresponding to each exchange date.

Consequently, interest rate swaps can be viewed as a package of more basic interest rate derivative instruments—forwards. The pricing of an interest rate swap will then depend on the price of a package of forward contracts with the same settlement dates in which the underlying for the forward contract is the same reference rate.

While an interest rate swap may be nothing more than a package of forward contracts, it is not a redundant contract for several reasons. First, maturities for forward or futures contracts do not extend out as far as those of an interest rate swap; an interest rate swap with a term of 15 years or longer can be obtained. Second, an interest rate swap is a more transactionally efficient instrument. By this we mean that in one transaction an entity can effectively establish a payoff equivalent to a package of forward contracts. The forward contracts would each have to be negotiated separately. Third, the interest rate swap market has grown in liquidity since its establishment in 1981; interest rate swaps now provide more liquidity than forward contracts, particularly long-dated (i.e., long-term) forward contracts.

Package of Cash Market Instruments

To understand why a swap can also be interpreted as a package of cash market instruments, consider an investor who enters into the transaction below:

- Buy $50 million par value of a 5-year floating-rate bond that pays 6-month Libor every six months.
- Finance the purchase by borrowing $50 million for five years at a 10% annual interest rate paid every six months.

The cash flows for this transaction are set forth in Exhibit 10.1. The second column of the exhibit shows the cash flows from purchasing the 5-year floating-rate bond. There is a $50 million cash outlay and then ten cash inflows. The amount of the cash inflows is uncertain because they depend on future levels of 6-month Libor. The next column shows the cash flows from borrowing $50 million on a fixed-rate basis. The last column shows the net cash flows from the entire transaction. As the last

EXHIBIT 10.1 **Cash Flows for the Purchase of a 5-Year Floating-Rate Bond Financed by Borrowing on a Fixed-Rate Basis**

Transaction:

- Purchase for \$50 million a 5-year floating-rate bond:
  Floating rate = Libor, semiannual pay
- Borrow \$50 million for five years:
  Fixed rate = 10%, semiannual payments

Six-Month	Cash Flow (in Millions of Dollars) From:		
Period	Floating-Rate Bond [a]	Borrowing Cost	Net
0	–\$50	+\$50.0	\$0
1	+ $(\text{Libor}_1/2) \times 50$	–2.5	+ $(\text{Libor}_1/2) \times 50 - 2.5$
2	+ $(\text{Libor}_2/2) \times 50$	–2.5	+ $(\text{Libor}_2/2) \times 50 - 2.5$
3	+ $(\text{Libor}_3/2) \times 50$	–2.5	+ $(\text{Libor}_3/2) \times 50 - 2.5$
4	+ $(\text{Libor}_4/2) \times 50$	–2.5	+ $(\text{Libor}_4/2) \times 50 - 2.5$
5	+ $(\text{Libor}_5/2) \times 50$	–2.5	+ $(\text{Libor}_5/2) \times 50 - 2.5$
6	+ $(\text{Libor}_6/2) \times 50$	–2.5	+ $(\text{Libor}_6/2) \times 50 - 2.5$
7	+ $(\text{Libor}_7/2) \times 50$	–2.5	+ $(\text{Libor}_7/2) \times 50 - 2.5$
8	+ $(\text{Libor}_8/2) \times 50$	–2.5	+ $(\text{Libor}_8/2) \times 50 - 2.5$
9	+ $(\text{Libor}_9/2) \times 50$	–2.5	+ $(\text{Libor}_9/2) \times 50 - 2.5$
10	+ $(\text{Libor}_{10}/2) \times 50 + 50$	–52.5	+ $(\text{Libor}_{10}/2) \times 50 - 2.5$

[a] The subscript for Libor indicates the 6-month Libor as per the terms of the floating-rate bond at time *t*.

column indicates, there is no initial cash flow (the cash inflow and cash outlay offset each other). In all ten 6-month periods, the net position results in a cash inflow of Libor and a cash outlay of \$2.5 million. This net position, however, is identical to the position of a fixed-rate payer/floating-rate receiver.

It can be seen from the net cash flow in Exhibit 10.1 that a fixed-rate payer has a cash market position that is equivalent to a long position in a floating-rate bond and a short position in a fixed-rate bond—the short position being the equivalent of borrowing by issuing a fixed-rate bond.

What about the position of a floating-rate payer? It can be easily demonstrated that the position of a floating-rate payer is equivalent to purchasing a fixed-rate bond and financing that purchase at a floating-rate, where the floating rate is the reference rate for the swap. That is, the position of a floating-rate payer is equivalent to a long position in a fixed-rate bond and a short position in a floating-rate bond.

TERMINOLOGY, CONVENTIONS, AND MARKET QUOTES

Here we review some of the terminology used in the swaps market and explain how swaps are quoted. The *trade date* for a swap is the date on which the swap is transacted. The terms of the trade include the fixed interest rate, the maturity, the notional amount of the swap, and the payment bases of both legs of the swap. The date from which floating interest payments are determined is the *reset* or *setting date*, which may also be the trade date. In the same way as for FRAs (discussed in the previous chapter), the rate is fixed two business days before the interest period begins. The second (and subsequent) reset date will be two business days before the beginning of the second (and subsequent) swap periods. The *effective date* is the date from which interest on the swap is calculated, and this is typically two business days after the trade date. In a *forward-start swap* the effective date will be at some point in the future, specified in the swap terms. The floating-interest rate for each period is fixed at the start of the period, so that the interest payment amount is known in advance by both parties (the fixed rate is known of course, throughout the swap by both parties).

While our illustrations assume that the timing of the cash flows for both the fixed-rate payer and floating-rate payer will be the same, this is rarely the case in a swap. An agreement may call for the fixed-rate payer to make payments annually but the floating-rate payer to make payments more frequently (semiannually or quarterly). Also, the way in which interest accrues on each leg of the transaction differs. Normally, the fixed interest payments are paid on the basis of a 30/360 day count which is described in Chapter 2. Floating-rate payments for dollar and euro-denominated swaps use an Actual/360 day count similar to other money market instruments in those currencies. Sterling-denominated swaps use an Actual/365 day count.

Accordingly, the fixed interest payments will differ slightly owing to the differences in the lengths of successive coupon periods. The floating payments will differ owing to day counts as well as movements in the reference rate.

The terminology used to describe the position of a party in the swap markets combines cash market jargon and futures market jargon, given that a swap position can be interpreted either as a position in a package of cash market instruments or a package of futures/forward positions. As we have said, the counterparty to an interest rate swap is either a fixed-rate payer or floating-rate payer.

The fixed-rate payer receives floating-rate interest and is said to be "long" or to have "bought" the swap. The long side has conceptually purchased a floating-rate note (because it receives floating-rate interest) and issued a fixed coupon bond (because it pays out fixed interest at periodic intervals). In essence, the fixed-rate payer is borrowing at fixed-rate and investing in a floating-rate asset. The floating-rate payer is said to be "short" or to have "sold" the swap. The short side has conceptually purchased a coupon bond (because it receives fixed-rate interest) and issued a floating-rate note (because it pays floating-rate interest). A floating-rate payer is borrowing at floating rate and investing in a fixed rate asset.

The convention that has evolved for quoting swaps levels is that a swap dealer sets the floating rate equal to the reference rate and then quotes the fixed rate that will apply. To illustrate this convention, consider the following 10-year swap terms available from a dealer:

- *Floating-rate payer:*
  Pay floating rate of 3-month Libor quarterly.
  Receive fixed rate of 8.75% semiannually.
- *Fixed-rate payer:*
  Pay fixed rate of 8.85% semiannually.
  Receive floating rate of 3-month Libor quarterly.

The offer price that the dealer would quote the fixed-rate payer would be to pay 8.85% and receive Libor "flat." (The word *flat* means with no spread.) The bid price that the dealer would quote the floating-rate payer would be to pay Libor flat and receive 8.75%. The bid offer spread is 10 basis points.

In order to solidify our intuition, it is useful to think of the swap market as a market where two counterparties trade the floating reference rate in a series of exchanges for a fixed price. In effect, the swap market is a market to buy and sell Libor. So, buying a swap (pay fixed/receive floating) can be thought of as buying Libor on each reset date for the fixed rate agreed to on the trade date. Conversely, selling a swap (receive fixed/pay floating) is effectively selling Libor on each reset date for a fixed rate agreed to on the trade date. In this framework, a dealer's bid-offer spread can be easily interpreted. Using the numbers presented above, the bid price of 8.75% is the price the dealer will pay to the counterparty to receive 3-month Libor. In other words, buy Libor at the bid. Similarly, the offer price of 8.85% is the price the dealer receives from the counterparty in exchange for 3-month Libor. In other words, sell Libor at the offer.

The fixed rate is some spread above the Treasury yield curve with the same term to maturity as the swap. In our illustration, suppose that the 10-year Treasury yield is 8.35%. Then the offer price that the dealer would quote to the fixed-rate payer is the 10-year Treasury rate plus 50 basis points versus receiving Libor flat. For the floating-rate payer, the bid price quoted would be Libor flat versus the 10-year Treasury rate plus 40 basis points. The dealer would quote such a swap as 40-50, meaning that the dealer is willing to enter into a swap to receive Libor and pay a fixed rate equal to the 10-year Treasury rate plus 40 basis points; and he or she would be willing to enter into a swap to pay Libor and receive a fixed rate equal to the 10-year Treasury rate plus 50 basis points. The difference between the Treasury rate paid and received is the bid-offer spread.[2]

VALUING INTEREST RATE SWAPS

In an interest rate swap, the counterparties agree to exchange periodic interest payments. The dollar amount of the interest payments exchanged is based on the notional principal. In the most common type of swap, there is a fixed-rate payer and a fixed-rate receiver. The convention for quoting swap rates is that a swap dealer sets the floating rate equal to the reference rate and then quotes the fixed rate that will apply.

Computing the Payments for a Swap

In the previous section we described in general terms the payments by the fixed-rate payer and fixed-rate receiver but we did not give any details. That is, we explained that if the swap rate is 6% and the notional amount is $100 million, then the fixed-rate payment will be $6 million for the year and the payment is then adjusted based on the frequency of settlement. So, if settlement is semiannual, the payment is $3 million. If it is quarterly, it is $1.5 million. Similarly, the floating-rate payment would be found by multiplying the reference rate by the notional amount and then scaling based on the frequency of settlement.

It was useful to illustrate the basic features of an interest rate swap with simple calculations for the payments such as described above and then explain how the parties to a swap either benefit or hurt when interest rates change. However, we will show how to value a swap in this section. To value a swap, it is necessary to determine both the present value of the fixed-rate payments and the present value of the floating-rate payments. The difference between these two present values is the

[2] A question that commonly arises is why is the fixed rate of a swap quoted as a fixed spread above a Treasury rate when Treasury rates are not used directly in swap valuation? Because of the timing difference between the quote and settlement, quoting the fixed-rate side as a spread above a Treasury rate allows the swap dealer to hedge against changing interest rates.

value of a swap. As will be explained below, whether the value is positive (i.e., an asset) or negative (i.e., a liability) will depend on the party.

At the inception of the swap, the terms of the swap will be such that the present value of the floating-rate payments is equal to the present value of the fixed-rate payments. That is, the value of the swap is equal to zero at its inception. This is the fundamental principle in determining the swap rate (i.e., the fixed rate that the fixed-rate payer will make).

Here is a roadmap of the presentation. First we will look at how to compute the floating-rate payments. We will see how the future values of the reference rate are determined to obtain the floating rate for the period. From the future values of the reference rate we will then see how to compute the floating-rate payments taking into account the number of days in the payment period. Next we will see how to calculate the fixed-rate payments given the swap rate. Before we look at how to calculate the value of a swap, we will see how to calculate the swap rate. This will require an explanation of how the present value of any cash flow in an interest rate swap is computed. Given the floating-rate payments and the present value of the floating-rate payments, the swap rate can be determined by using the principle that the swap rate is the fixed rate that will make the present value of the fixed-rate payments equal to the present value of the floating-rate payments. Finally, we will see how the value of swap is determined after the inception of a swap.

Calculating the Floating-Rate Payments

For the first floating-rate payment, the amount is known. For all subsequent payments, the floating-rate payment depends on the value of the reference rate when the floating rate is determined. To illustrate the issues associated with calculating the floating-rate payment, we will assume that

- A swap starts today, January 1 of year 1(swap settlement date).
- The floating-rate payments are made quarterly based on "actual/360."
- The reference rate is 3-month Libor.
- The notional amount of the swap is $100 million.
- The term of the swap is three years.

The quarterly floating-rate payments are based on an "actual/360" day count convention. Recall that this convention means that 360 days are assumed in a year and that in computing the interest for the quarter, the actual number of days in the quarter is used. The floating-rate payment is set at the beginning of the quarter but paid at the end of the quarter—that is, the floating-rate payments are made in arrears.

Suppose that today 3-month Libor is 4.05%. Let's look at what the fixed-rate payer will receive on March 31 of year 1—the date when the first quarterly swap payment is made. There is no uncertainty about what the floating-rate payment will be. In general, the floating-rate payment is determined as follows:

$$\text{Notional amount} \times (\text{3-month LIBOR}) \times \frac{\text{No. of days in period}}{360}$$

In our illustration, assuming a nonleap year, the number of days from January 1 of year 1 to March 31 of year 1 (the first quarter) is 90. If 3-month Libor is 4.05%, then the fixed-rate payer will receive a floating-rate payment on March 31 of year 1 equal to

$$\$100{,}000{,}000 \times 0.0405 \times \frac{90}{360} = \$1{,}012{,}500$$

Now the difficulty is in determining the floating-rate payment after the first quarterly payment. That is, for the 3-year swap there will be 12 quarterly floating-rate payments. So, while the first quarterly payment is known, the next 11 are not. However, there is a way to hedge the next 11 floating-rate payments by using a futures contract. Specifically, the futures contract used to hedge the future floating-rate payments in a swap whose reference rate is 3-month Libor is the Eurodollar CD futures contract.

Determining Future Floating-Rate Payments

Now let's determine the future floating-rate payments. These payments can be locked in over the life of the swap using the Eurodollar CD futures contract. We will show how these floating-rate payments are computed using this contract.

We will begin with the next quarterly payment—from April 1 of year 1 to June 30 of year 1. This quarter has 91 days. The floating-rate payment will be determined by 3-month Libor on April 1 of year 1 and paid on June 30 of year 1. Where might the fixed-rate payer look today (January 1 of year 1) to project what 3-month Libor will be on April 1 of year 1? One possibility is the Eurodollar CD futures market. There is a 3-month Eurodollar CD futures contract for settlement on June 30 of year 1. That futures contract will express the market's expectation of 3-month Libor on April 1 of year 1. For example, if the futures price for the 3-month Eurodollar CD futures contract that settles on June 30 of year 1 is 95.85, then as explained above, the 3-month Eurodollar futures rate is 4.15%. We will refer to that rate for 3-month Libor as the "forward rate." Therefore, if the fixed-rate payer bought 100 of these 3-month Eurodollar CD futures contracts on January 1 of year 1 (the inception of the swap) that settle on June 30 of year 1, then the payment that will be locked in for the quarter (April 1 to June 30 of year 1) is

$$\$100{,}000{,}000 \times 0.0415 \times \frac{91}{360} = \$1{,}049{,}028$$

Note that each futures contract is for $1 million and hence 100 contracts have a notional amount of $100 million.

Similarly, the Eurodollar CD futures contract can be used to lock in a floating-rate payment for each of the next 10 quarters.[3] Once again, it is important to emphasize that the reference rate at the beginning of period t determines the floating-rate that will be paid for the period. However, the floating-rate payment is not made until the end of period t.

Exhibit 10.2 shows this for the 3-year swap. Shown in Column (1) is when the quarter begins and in Column (2) when the quarter ends. The payment will be received at the end of the first quarter (March 31 of year 1) and is $1,012,500. That is the known floating-rate payment as explained earlier. It is the only payment that is known. The information used to compute the first payment is in Column (4), which shows the current 3-month Libor (4.05%). The payment is shown in the last column, Column (8).

Notice that Column (7) numbers the quarters from 1 through 12. Look at the heading for Column (7). It identifies each quarter in terms of the end of the quarter. This is important because we will eventually be discounting the payments (cash flows). We must take care to understand when each payment is to be exchanged in order to properly discount. So, for the first payment of $1,012,500 it is going to be received at the end of quarter 1. When we refer to the time period for any payment, the reference is to the end of quarter. So, the fifth payment of $1,225,000 would be identified as the payment for period 5, where period 5 means that it will be exchanged at the end of the fifth quarter.

[3] The Chicago Mercantile Exchange offers prepackaged series of Eurodollar CD futures contracts that expire on consecutive dates called *bundles*. Specifically, a bundle is the simultaneous sale or purchase of one of each of a consecutive series of Eurodollar CD futures contracts. So, rather than construct the same positions with individual contracts, a series of contracts can be sold or purchased in a single transaction.

EXHIBIT 10.2 Floating-Rate Payments Based on Initial Libor and Eurodollar CD Futures

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Quarter starts	Quarter ends	Number of days in quarter	Current 3-month Libor	Eurodollar CD futures price	Forward rate	Period = End of quarter	Floating-rate payment at end of quarter
Jan 1 year 1	Mar 31 year 1	90	4.05%		—	1	1,012,500
Apr 1 year 1	June 30 year 1	91		95.85	4.15%	2	1,049,028
July 1 year 1	Sept 30 year 1	92		95.45	4.55%	3	1,162,778
Oct 1 year 1	Dec 31 year 1	92		95.28	4.72%	4	1,206,222
Jan 1 year 2	Mar 31 year 2	90		95.10	4.90%	5	1,225,000
Apr 1 year 2	June 30 year 2	91		94.97	5.03%	6	1,271,472
July 1 year 2	Sept 30 year 2	92		94.85	5.15%	7	1,316,111
Oct 1 year 2	Dec 31 year 2	92		94.75	5.25%	8	1,341,667
Jan 1 year 3	Mar 31 year 3	90		94.60	5.40%	9	1,350,000
Apr 1 year 3	June 30 year 3	91		94.50	5.50%	10	1,390,278
July 1 year 3	Sept 30 year 3	92		94.35	5.65%	11	1,443,889
Oct 1 year 3	Dec 31 year 3	92		94.24	5.76%	12	1,472,000

Calculating the Fixed-Rate Payments

The swap will specify the frequency of settlement for the fixed-rate payments. The frequency need not be the same as the floating-rate payments. For example, in the 3-year swap we have been using to illustrate the calculation of the floating-rate payments, the frequency is quarterly. The frequency of the fixed-rate payments could be semiannual rather than quarterly.

In our illustration we will assume that the frequency of settlement is quarterly for the fixed-rate payments, the same as with the floating-rate payments. The day count convention is the same as for the floating-rate payment, "actual/360." The equation for determining the dollar amount of the fixed-rate payment for the period is

$$\text{Notional amount} \times (\text{Swap rate}) \times \frac{\text{No. of days in period}}{360}$$

It is the same equation as for determining the floating-rate payment except that the swap rate is used instead of the reference rate (3-month Libor in our illustration).

For example, suppose that the swap rate is 4.98% and the quarter has 90 days. Then the fixed-rate payment for the quarter is

$$\$100{,}000{,}000 \times 0.0498 \times \frac{90}{360} = \$1{,}245{,}000$$

If there are 92 days in a quarter, the fixed-rate payment for the quarter is

$$\$100{,}000{,}000 \times 0.0498 \times \frac{92}{360} = \$1{,}272,\ 667$$

Note that the rate is fixed for each quarter but the dollar amount of the payment depends on the number of days in the period.

Exhibit 10.3 shows the fixed-rate payments based on different assumed values for the swap rate. The first three columns of the exhibit show the same information as in Exhibit 10.2—the beginning and end of the quarter and the number of days in the quarter. Column (4) simply uses the notation for the period. That is, period 1 means the end of the first quarter, period 2 means the end of the second quarter, and so on. The other columns of the exhibit show the payments for each assumed swap rate.

EXHIBIT 10.3 Fixed-Rate Payments for Several Assumed Swap Rates

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Quarter starts	Quarter ends	Number of days in quarter	Period = End of quarter	Fixed-rate payment if swap rate is assumed to be				
				4.9800%	4.9873%	4.9874%	4.9875%	4.9880%
Jan 1 year 1	Mar 31 year 1	90	1	1,245,000	1,246,825	1,246,850	1,246,875	1,247,000
Apr 1 year 1	June 30 year 1	91	2	1,258,833	1,260,679	1,260,704	1,260,729	1,260,856
July 1 year 1	Sept 30 year 1	92	3	1,272,667	1,274,532	1,274,558	1,274,583	1,274,711
Oct 1 year 1	Dec 31 year 1	92	4	1,272,667	1,274,532	1,274,558	1,274,583	1,274,711
Jan 1 year 2	Mar 31 year 2	90	5	1,245,000	1,246,825	1,246,850	1,246,875	1,247,000
Apr 1 year 2	June 30 year 2	91	6	1,258,833	1,260,679	1,260,704	1,260,729	1,260,856
July 1 year 2	Sept 30 year 2	92	7	1,272,667	1,274,532	1,274,558	1,274,583	1,274,711
Oct 1 year 2	Dec 31 year 2	92	8	1,272,667	1,274,532	1,274,558	1,274,583	1,274,711
Jan 1 year 3	Mar 31 year 3	90	9	1,245,000	1,246,825	1,246,850	1,246,875	1,247,000
Apr 1 year 3	June 30 year 3	91	10	1,258,833	1,260,679	1,260,704	1,260,729	1,260,856
July 1 year 3	Sept 30 year 3	92	11	1,272,667	1,274,532	1,274,558	1,274,583	1,274,711
Oct 1 year 3	Dec 31 year 3	92	12	1,272,667	1,274,532	1,274,558	1,274,583	1,274,711

Calculation of the Swap Rate

Now that we know how to calculate the payments for the fixed-rate and floating-rate sides of a swap where the reference rate is 3-month Libor given (1) the current value for 3-month Libor, (2) the expected 3-month Libor from the Eurodollar CD futures contract, and (3) the assumed swap rate, we can demonstrate how to compute the swap rate.

At the initiation of an interest rate swap, the counterparties are agreeing to exchange future payments and no upfront payments are made by either party. This means that the swap terms must be such that the present value of the payments to be made by the counterparties must be at least equal to the present value of the payments that will be received. In fact, to eliminate arbitrage opportunities, the present value of the payments made by a party will be equal to the present value of the payments received by that same party. *The equivalence (or no arbitrage) of the present value of the payments is the key principle in calculating the swap rate.*

Since we will have to calculate the present value of the payments, let's show how this is done.

Calculating the Present Value of the Floating-Rate Payments

As explained earlier, we must be careful about how we compute the present value of payments. In particular, we must carefully specify (1) the timing of the payment and (2) the interest rates that should be used to discount the payments. We have already addressed the first issue. In constructing the exhibit for the payments, we indicated that the payments are at the end of the quarter. So, we denoted the time periods with respect to the end of the quarter.

Now let's turn to the interest rates that should be used for discounting. First, every cash flow should be discounted at its own discount rate using a spot rate. So, if we discounted a cash flow of \$1 using the spot rate for period t, the present value would be

$$\text{Present value of \$1 to be received in period } t = \frac{\$1}{(1+\text{Spot rate for period } t)^t}$$

Second, forward rates are derived from spot rates so that if we discounted a cash flow using forward rates rather than spot rates, we would come up with the same value. That is, the present value of \$1 to be received in period t can be rewritten as:

$$\text{Present value of \$1 to be received in period } t = \frac{\$1}{(1+\text{Forward rate for period 1})(1+\text{Forward rate for period 2})\cdots(1+\text{Forward rate for period } t)}$$

We will refer to the present value of $1 to be received in period t as the *forward discount factor*. In our calculations involving swaps, we will compute the forward discount factor for a period using the forward rates. These are the same forward rates that are used to compute the floating-rate payments—those obtained from the Eurodollar CD futures contract. We must make just one more adjustment. We must adjust the forward rates used in the formula for the number of days in the period (i.e., the quarter in our illustrations) in the same way that we made this adjustment to obtain the payments. Specifically, the forward rate for a period, which we will refer to as the period forward rate, is computed using the following equation:

$$\text{Period forward rate} = \text{Annual forward rate} \times \left(\frac{\text{Days in period}}{360}\right)$$

For example, look at Exhibit 10.2. The annual forward rate for period 4 is 4.72%. The period forward rate for period 4 is

$$\text{Period forward rate} = 4.72\% \times \left(\frac{92}{360}\right) = 1.2062\%$$

Column (5) in Exhibit 10.4 shows the annual forward rate for all 12 periods (reproduced from Exhibit 10.3) and Column (6) shows the period forward rate for all 12 periods. Note that the period forward rate for period 1 is 4.05%, the known rate for 3-month Libor.

Also shown in Exhibit 10.4 is the forward discount factor for all 12 periods. These values are shown in the last column. Let's show how the forward discount factor is computed for periods 1, 2, and 3. For period 1, the forward discount factor is

$$\text{Forward discount factor} = \frac{\$1}{(1.010125)} = 0.98997649$$

For period 2,

$$\text{Forward discount factor} = \frac{\$1}{(1.010125)(1.010490)} = 0.97969917$$

EXHIBIT 10.4 Calculating the Forward Discount Factor

(1) Quarter starts	(2) Quarter ends	(3) Number of days in quarter	(4) Period = End of quarter	(5) Forward rate	(6) Period forward rate	(7) Forward discount factor
Jan 1 year 1	Mar 31 year 1	90	1	4.05%	1.0125%	0.98997649
Apr 1 year 1	June 30 year 1	91	2	4.15%	1.0490%	0.97969917
July 1 year 1	Sept 30 year 1	92	3	4.55%	1.1628%	0.96843839
Oct 1 year 1	Dec 31 year 1	92	4	4.72%	1.2062%	0.95689609
Jan 1 year 2	Mar 31 year 2	90	5	4.90%	1.2250%	0.94531597
Apr 1 year 2	June 30 year 2	91	6	5.03%	1.2715%	0.93344745
July 1 year 2	Sept 30 year 2	92	7	5.15%	1.3161%	0.92132183
Oct 1 year 2	Dec 31 year 2	92	8	5.25%	1.3417%	0.90912441
Jan 1 year 3	Mar 31 year 3	90	9	5.40%	1.3500%	0.89701471
Apr 1 year 3	June 30 year 3	91	10	5.50%	1.3903%	0.88471472
July 1 year 3	Sept 30 year 3	92	11	5.65%	1.4439%	0.87212224
Oct 1 year 3	Dec 31 year 3	92	12	5.76%	1.4720%	0.85947083

For period 3,

$$\begin{aligned}\text{Forward discount factor} &= \frac{\$1}{(1.010125)(1.010490)(1.011628)}\\ &= 0.96843839\end{aligned}$$

Given the floating-rate payment for a period and the forward discount factor for the period, the present value of the payment can be computed. For example, from Exhibit 10.2 we see that the floating-rate payment for period 4 is \$1,206,222. From Exhibit 10.4, the forward discount factor for period 4 is 0.95689609. Therefore, the present value of the payment is:

$$\begin{aligned}\text{Present value of period 4 payment} &= \$1{,}206{,}222 \times 0.95689609\\ &= \$1{,}154{,}229\end{aligned}$$

Exhibit 10.5 shows the present value for each payment. The total present value of the 12 floating-rate payments is \$14,052,917. Thus, the present value of the payments that the fixed-rate payer will receive is \$14,052,917 and the present value of the payments that the fixed-rate receiver will make is \$14,052,917.

EXHIBIT 10.5 Present Value of the Floating-Rate Payments

(1)	(2)	(3)	(4)	(5)	(6)
Quarter starts	Quarter ends	Period = End of quarter	Forward discount factor	Floating-rate payment at end of quarter	PV of floating-rate payment
Jan 1 year 1	Mar 31 year 1	1	0.98997649	1,012,500	1,002,351
Apr 1 year 1	June 30 year 1	2	0.97969917	1,049,028	1,027,732
July 1 year 1	Sept 30 year 1	3	0.96843839	1,162,778	1,126,079
Oct 1 year 1	Dec 31 year 1	4	0.95689609	1,206,222	1,154,229
Jan 1 year 2	Mar 31 year 2	5	0.94531597	1,225,000	1,158,012
Apr 1 year 2	June 30 year 2	6	0.93344745	1,271,472	1,186,852
July 1 year 2	Sept 30 year 2	7	0.92132183	1,316,111	1,212,562
Oct 1 year 2	Dec 31 year 2	8	0.90912441	1,341,667	1,219,742
Jan 1 year 3	Mar 31 year 3	9	0.89701471	1,350,000	1,210,970
Apr 1 year 3	June 30 year 3	10	0.88471472	1,390,278	1,229,999
July 1 year 3	Sept 30 year 3	11	0.87212224	1,443,889	1,259,248
Oct 1 year 3	Dec 31 year 3	12	0.85947083	1,472,000	1,265,141
				Total	14,052,917

Determination of the Swap Rate

The fixed-rate payer will require that the present value of the fixed-rate payments that must be made based on the swap rate not exceed the \$14,052,917 payments to be received from the floating-rate payments. The fixed-rate receiver will require that the present value of the fixed-rate payments to be received is at least as great as the \$14,052,917 that must be paid. This means that both parties will require a present value for the fixed-rate payments to be \$14,052,917. If that is the case, the present value of the fixed-rate payments is equal to the present value of the floating-rate payments and therefore the value of the swap is zero for both parties at the inception of the swap. The interest rates that should be used to compute the present value of the fixed-rate payments are the same interest rates as those used to discount the floating-rate payments.

To show how to compute the swap rate, we begin with the basic relationship for no arbitrage to exist:

$$PV \text{ of floating-rate payments} = PV \text{ of fixed-rate payments}$$

We know the value for the left-hand side of the equation.

If we let

$$SR = \text{swap rate}$$

and

$$\text{Days}_t = \text{Number of days in the payment period } t$$

Then the fixed-rate payment for period t is equal to

$$\text{Notional amount} \times SR \times \frac{\text{Days}_t}{360}$$

The present value of the fixed-rate payment for period t is found by multiplying the previous expression by the forward discount factor. If we let FDF_t denote the forward discount factor for period t, then the present value of the fixed-rate payment for period t is equal to:

$$\text{Notional amount} \times SR \times \frac{\text{Days}_t}{360} \times FDF_t$$

We can now sum up the present value of the fixed-rate payment for each period to get the present value of the floating-rate payments. Using the Greek symbol sigma, Σ, to denote summation and letting N be the number of periods in the swap, then the present value of the fixed-rate payments can be expressed as

$$\sum_{t=1}^{N} \text{Notional amount} \times SR \times \frac{\text{Days}_t}{360} \times FDF_t$$

This can also be expressed as

$$SR \sum_{t=1}^{N} \text{Notional amount} \times \frac{\text{Days}_t}{360} \times FDF_t$$

The condition for no arbitrage is that the present value of the fixed-rate payments as given by the expression above is equal to the present value of the floating-rate payments. That is,

$$SR \sum_{t=1}^{N} \text{Notional amount} \times \frac{\text{Days}_t}{360} \times FDF_t = PV \text{ of floating-rate payments}$$

Solving for the swap rate

$$SR = \frac{PV \text{ of floating-rate payments}}{\sum_{t=1}^{N} \text{Notional amount} \times \frac{\text{Days}_t}{360} \times FDF_t}$$

All of the values needed to compute the swap rate are known.

Let's apply the formula to determine the swap rate for our 3-year swap. Exhibit 10.6 shows the calculation of the denominator of the formula. The forward discount factor for each period shown in Column (5) is obtained from Column (4) of Exhibit 10.5. The sum of the last column in Exhibit 10.6 shows that the denominator of the swap rate formula is $281,764,282. We know from Exhibit 10.5 that the present value of the floating-rate payments is $14,052,917. Therefore, the swap rate is

$$SR = \frac{\$14{,}052{,}917}{\$281{,}764{,}282} = 0.049875 = 4.9875\%$$

Given the swap rate, the *swap spread* can be determined. For example, since this is a 3-year swap, the convention is to use the 3-year on-the-run Treasury rate as the benchmark. If the yield on that issue is 4.5875%, the swap spread is 40 basis points (4.9875% – 4.5875%).

The calculation of the swap rate for all swaps follows the same principle: equating the present value of the fixed-rate payments to that of the floating-rate payments.

Valuing a Swap

Once the swap transaction is completed, changes in market interest rates will change the payments of the floating-rate side of the swap. The value of an interest rate swap is the difference between the present value of the payments of the two sides of the swap. The 3-month Libor forward rates from the current Eurodollar CD futures contracts are used to (1) calculate the floating-rate payments and (2) determine the discount factors at which to calculate the present value of the payments.

To illustrate this, consider the 3-year swap used to demonstrate how to calculate the swap rate. Suppose that one year later, interest rates change as shown in Columns (4) and (6) in Exhibit 10.7. In Column (4) shows the current 3-month Libor. In Column (5) are the Eurodollar CD futures price for each period. These rates are used to compute the forward rates in Column (6). Note that the interest rates have increased

EXHIBIT 10.6 Calculating the Denominator for the Swap Rate Formula

(1)	(2)	(3)	(4)	(5)	(6)	(7)
Quarter starts	Quarter ends	Number of days in quarter	Period = End of quarter	Forward discount factor	Days/360	Forward discount factor × Days/360 × notional
Jan 1 year 1	Mar 31 year 1	90	1	0.98997649	0.25000000	24,749,412
Apr 1 year 1	June 30 year 1	91	2	0.97969917	0.25277778	24,764,618
July 1 year 1	Sept 30 year 1	92	3	0.96843839	0.25555556	24,748,981
Oct 1 year 1	Dec 31 year 1	92	4	0.95689609	0.25555556	24,454,011
Jan 1 year 2	Mar 31 year 2	90	5	0.94531597	0.25000000	23,632,899
Apr 1 year 2	June 30 year 2	91	6	0.93344745	0.25277778	23,595,477
July 1 year 2	Sept 30 year 2	92	7	0.92132183	0.25555556	23,544,891
Oct 1 year 2	Dec 31 year 2	92	8	0.90912441	0.25555556	23,233,179
Jan 1 year 3	Mar 31 year 3	90	9	0.89701471	0.25000000	22,425,368
Apr 1 year 3	June 30 year 3	91	10	0.88471472	0.25277778	22,363,622
July 1 year 3	Sept 30 year 3	92	11	0.87212224	0.25555556	22,287,568
Oct 1 year 3	Dec 31 year 3	92	12	0.85947083	0.25555556	21,964,255
					Total	281,764,282

EXHIBIT 10.7 Rates and Floating-Rate Payments One Year Later if Rates Increase

(1) Quarter starts	(2) Quarter ends	(3) Number of days in quarter	(4) Current 3-month Libor	(5) Eurodollar futures price	(6) Forward rate	(7) Period = End of quarter	(8) Floating-rate payments at end of quarter
Jan 1 year 2	Mar 31 year 2	90	5.25%			1	1,312,500
Apr 1 year 2	June 30 year 2	91		94.27	5.73%	2	1,448,417
July 1 year 2	Sept 30 year 2	92		94.22	5.78%	3	1,477,111
Oct 1 year 2	Dec 31 year 2	92		94.00	6.00%	4	1,533,333
Jan 1 year 3	Mar 31 year 3	90		93.85	6.15%	5	1,537,500
Apr 1 year 3	June 30 year 3	91		93.75	6.25%	6	1,579,861
July 1 year 3	Sept 30 year 3	92		93.54	6.46%	7	1,650,889
Oct 1 year 3	Dec 31 year 3	92		93.25	6.75%	8	1,725,000

one year later since the rates in Exhibit 10.7 are greater than those in Exhibit 10.2. As in Exhibit 10.2, the current 3-month Libor and the forward rates are used to compute the floating-rate payments. These payments are shown in Column (8) of Exhibit 10.7.

In Exhibit 10.8, the forward discount factor is computed for each period. The calculation is the same as in Exhibit 10.4 to obtain the forward discount factor for each period. The forward discount factor for each period is shown in the last column of Exhibit 10.8.

In Exhibit 10.9 the forward discount factor (from Exhibit 10.8) and the floating-rate payments (from Exhibit 10.7) are shown. The fixed-rate payments need not be recomputed. They are the payments shown in Column (8) of Exhibit 10.3. These are fixed-rate payments for the swap rate of 4.9875% and are reproduced in Exhibit 10.9. Now the two payment streams must be discounted using the new forward discount factors. As shown at the bottom of Exhibit 10.9, the two present values are as follows:

Present value of floating-rate payments	\$11,459,495
Present value of fixed-rate payments	\$9,473,390

The two present values are not equal and therefore for one party the value of the swap increased and for the other party the value of the swap decreased. Let's look at which party gained and which party lost.

The fixed-rate payer will receive the floating-rate payments. And these payments have a present value of \$11,459,495. The present value of the payments that must be made by the fixed-rate payer is \$9,473,390. Thus, the swap has a positive value for the fixed-rate payer equal to the difference in the two present values of \$1,986,105. This is the value of the swap to the fixed-rate payer. Notice, consistent with what we said earlier, when interest rates increase (as they did in our illustration), the fixed-rate payer benefits because the value of the swap increases.

In contrast, the fixed-rate receiver must make payments with a present value of \$11,459,495 but will only receive fixed-rate payments with a present value equal to \$9,473,390. Thus, the value of the swap for the fixed-rate receiver is –\$1,986,105. Again, as explained earlier, the fixed-rate receiver is adversely affected by a rise in interest rates because it results in a decline in the value of a swap.

The same valuation principle applies to more complicated swaps. For example, there are swaps whose notional amount changes in a predetermined way over the life of the swap. These include amortizing swaps, accreting swaps, and roller coaster swaps. Once the payments are specified, the present value is calculated as described above by simply adjusting the

EXHIBIT 10.8 Period Forward Rates and Forward Discount Factors One Year Later if Rates Increase

(1)	(2)	(3)	(4)	(5)	(6)	(7)
Quarter starts	Quarter ends	Number of days in quarter	Period = End of quarter	Forward rate	Period forward rate	Forward discount factor
Jan 1 year 2	Mar 31 year 2	90	1	5.25%	1.3125%	0.98704503
Apr 1 year 2	June 30 year 2	91	2	5.73%	1.4484%	0.97295263
July 1 year 2	Sept 30 year 2	92	3	5.78%	1.4771%	0.95879023
Oct 1 year 2	Dec 31 year 2	92	4	6.00%	1.5333%	0.94431080
Jan 1 year 3	Mar 31 year 3	90	5	6.15%	1.5375%	0.93001186
Apr 1 year 3	June 30 year 3	91	6	6.25%	1.5799%	0.91554749
July 1 year 3	Sept 30 year 3	92	7	6.46%	1.6509%	0.90067829
Oct 1 year 3	Dec 31 year 3	92	8	6.75%	1.7250%	0.88540505

EXHIBIT 10.9 Valuing the Swap One Year Later if Rates Increase

(1)	(2)	(3)	(4)	(5)	(6)	(7)
Quarter starts	Quarter ends	Forward discount factor	Floating cash flow at end of quarter	PV of floating cash flow	Fixed cash flow at end of quarter	PV of fixed cash flow
Jan 1 year 2	Mar 31 year 2	0.98704503	1,312,500	1,295,497	1,246,875	1,230,722
Apr 1 year 2	June 30 year 2	0.97295263	1,448,417	1,409,241	1,260,729	1,226,630
July 1 year 2	Sept 30 year 2	0.95879023	1,477,111	1,416,240	1,274,583	1,222,058
Oct 1 year 2	Dec 31 year 2	0.94431080	1,533,333	1,447,943	1,274,583	1,203,603
Jan 1 year 3	Mar 31 year 3	0.93001186	1,537,500	1,429,893	1,246,875	1,159,609
Apr 1 year 3	June 30 year 3	0.91554749	1,579,861	1,446,438	1,260,729	1,154,257
July 1 year 3	Sept 30 year 3	0.90067829	1,650,889	1,486,920	1,274,583	1,147,990
Oct 1 year 3	Dec 31 year 3	0.88540505	1,725,000	1,527,324	1,274,583	1,128,523
			Total	11,459,495		9,473,390

Summary	Fixed-rate payer	Fixed-rate receiver
PV of payments received	11,459,495	9,473,390
PV of payments made	9,473,390	11,459,495
Value of swap	1,986,105	–1,986,105

payment amounts by the changing notional amounts—the methodology does *not* change.

PRIMARY DETERMINANTS OF SWAP SPREADS

As we have seen, interest rate swaps are valued using no-arbitrage relationships relative to instruments (funding or investment vehicles) that produce the same cash flows under the same circumstances. Earlier we provided two interpretations of a swap: (1) a package of futures/forward contracts and (2) a package of cash market instruments. The swap spread is defined as the difference between the swap's fixed rate and the rate on a Treasury whose maturity matches the swap's tenor.

Exhibit 10.10 displays a Bloomberg screen with generic interest rate swap rates (in percent) and swap spreads (in basis points) for various maturities out to 30 years on January 7, 2003. Recall, the bid price is the fixed rate that the broker/dealer is willing to pay in order to receive a floating rate. Conversely, the ask price is the fixed rate the broker/dealer wants to receive in order to pay a floating rate. Current swap rates and spreads for a number of countries can be obtained on

EXHIBIT 10.10 **Swap Rates and Spreads for Various Maturities**

GRAB Equity IRSB

13:18 US SWAP RATES Page 1 / 2

94<GO> View News.

SECURITY	Bid	Ask	CHANGE
US Semi 30/360			
3) 2 YR	2.0510	2.0910	-.0540
4) 3 YR	2.6000	2.6300	-.0525
5) 4 YR	3.0400	3.0700	-.0500
6) 5 YR	3.4100	3.4400	-.0500
7) 6 YR	3.6950	3.7350	-.0405
8) 7 YR	3.9300	3.9700	-.0455
9) 8 YR	4.1220	4.1620	-.0445
10) 9 YR	4.2800	4.3200	-.0460
11) 10 YR	4.4260	4.4660	-.0490
12) 15 YR	4.9590	4.9990	-.0365
13) 20 YR	5.1900	5.2300	-.0410
14) 30 YR	5.2870	5.3270	-.0345

SECURITY	Bid	Ask	CHANGE
US SPREADS			
23) 2YR	30.50	32.50	-.50
24) 3YR	44.00	46.00	-.50
25) 4YR	47.00	49.00	-.50
26) 5YR	42.50	44.50	-.75
27) 6YR	51.00	53.00	-.50
28) 7YR	53.25	55.25	-1.25
29) 8YR	52.50	54.50	-1.00
30) 9YR	49.50	51.50	-1.00
31) 10YR	42.00	44.00	-1.00
32) 15YR	72.00	74.00	--
33) 20YR	72.00	74.00	-.25
34) 30YR	34.75	36.75	-.25

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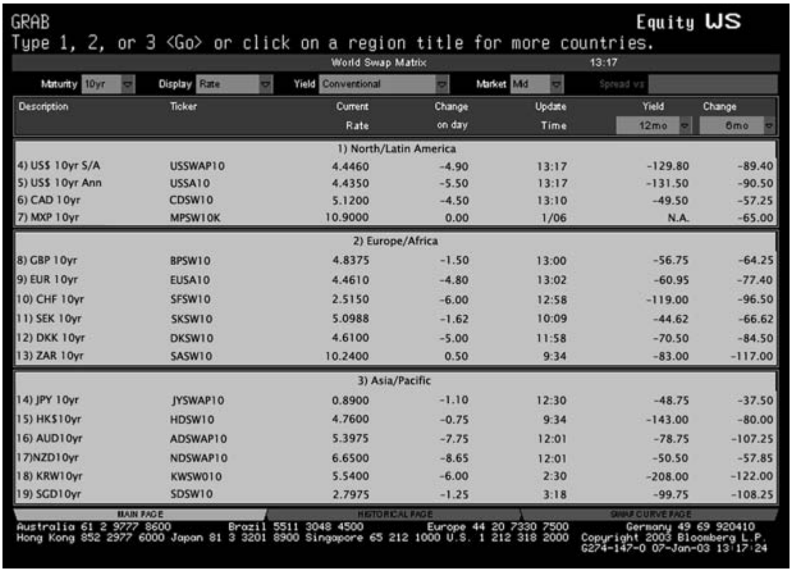
Change on day IYC4 I48<GO> Change on Month IYC6 I48<GO>

Page for ANN Rates

Australia 61 2 9777 8600 Brazil 5511 3048 4500 Europe 44 20 7330 7500 Germany 49 69 920410
Hong Kong 852 2977 6000 Japan 81 3 3201 8900 Singapore 65 212 1000 U.S. 1 212 318 2000 Copyright 2003 Bloomberg L.P.
G274-147-0 07-Jan-03 13:18:54

Source: Bloomberg Financial Markets

EXHIBIT 10.11 Bloomberg's World Swap Screen



GRAB Equity WS
Type 1, 2, or 3 <Go> or click on a region title for more countries.
World Swap Matrix 13:17
Maturity 10yr Display Rate Yield Conventional Market Mid Spread vs

Description	Ticker	Current Rate	Change on day	Update Time	Yield 12mo	Change 6mo
1) North/Latin America						
4) US$ 10yr S/A	USSWAP10	4.4460	-4.90	13:17	-129.80	-89.40
5) US$ 10yr Ann	USSA10	4.4350	-5.50	13:17	-131.50	-90.50
6) CAD 10yr	CDSW10	5.1200	-4.50	13:10	-49.50	-57.25
7) MXP 10yr	MPSW10K	10.9000	0.00	1/06	N.A.	-65.00
2) Europe/Africa						
8) GBP 10yr	BPSW10	4.8375	-1.50	13:00	-56.75	-64.25
9) EUR 10yr	EUSA10	4.4610	-4.80	13:02	-60.95	-77.40
10) CHF 10yr	SFSW10	2.5150	-6.00	12:58	-119.00	-96.50
11) SEK 10yr	SKSW10	5.0988	-1.62	10:09	-44.62	-66.62
12) DKK 10yr	DKSW10	4.6100	-5.00	11:58	-70.50	-84.50
13) ZAR 10yr	SASW10	10.2400	0.50	9:34	-83.00	-117.00
3) Asia/Pacific						
14) JPY 10yr	JYSWAP10	0.8900	-1.10	12:30	-48.75	-37.50
15) HK$10yr	HDSW10	4.7600	-0.75	9:34	-143.00	-80.00
16) AUD10yr	ADSWAP10	5.3975	-7.75	12:01	-78.75	-107.25
17)NZD10yr	NDSWAP10	6.6500	-8.65	12:01	-50.50	-57.85
18) KRW10yr	KWSW010	5.5400	-6.00	2:30	-208.00	-122.00
19) SGD10yr	SDSW10	2.7975	-1.25	3:18	-99.75	-108.25

MAIN PAGE HISTORICAL PAGE SWAP CURVE PAGE
Australia 61 2 9777 8600 Brazil 5511 3048 4500 Europe 44 20 7330 7500 Germany 49 69 920410
Hong Kong 852 2977 6000 Japan 81 3 3201 8900 Singapore 65 212 1000 U.S. 1 212 318 2000 Copyright 2003 Bloomberg L.P.
G274-147-0 07-Jan-03 13:17:24

Source: Bloomberg Financial Markets

Bloomberg with the function IRSB. Exhibit 10.11 presents Bloomberg's World Swap screen which presents swap rates for various countries around the world. In this screen, the tenor of the swaps in this screen is ten years as can be seen in the box labeled "Maturity" in the upper left-hand corner. Among the other choices available, a user can choose to display swap spreads rather than rates. Exhibit 10.12 is a time series plot obtained from Bloomberg for daily values of the 5-year swap spread (in basis points) for the period January 7, 2002 to January 7, 2003. This plot can be obtained using the function USSP5 Index GP.

The swap spread is determined by the same factors that drive the spread over Treasuries on instruments that replicate a swap's cash flows i.e., produce a similar return or funding profile. As discussed below, the swap spread's key determinant for swaps with tenors (i.e., maturities) of five years or less is the cost of hedging in the Eurodollar CD futures market.[4] Although listed contracts exist with delivery dates out to 10 years, the liquidity of the Eurodollar CD futures market diminishes con-

[4] Naturally, this presupposes the reference rate used for the floating-rate cash flows is Libor. Furthermore, part of swap spread is attributable simply to the fact that Libor for a given maturity is higher than the rate on a comparable-maturity U.S. Treasury.

EXHIBIT 10.12 Time Series of the 5-Year Swap Spread

Source: Bloomberg Financial Markets

siderably after about five years. For longer tenor swaps, the swap spread is largely driven by credit spreads in the corporate bond market.[5] Specifically, longer-dated swaps are priced relative to rates paid by investment-grade credits in traditional fixed- and floating-rate markets.

Given that a swap is a package of futures/forward contracts, the shorter-term swap spreads respond directly to fluctuations in Eurodollar CD futures prices. As noted, there is a liquid market for Eurodollar CD futures contracts with maturities every three months for approximately five years. A market participant can create a synthetic fixed-rate security or a fixed-rate funding vehicle by taking a position in a bundle of Eurodollar CD futures contracts (i.e., a position in every 3-month Eurodollar CD futures contract up to the desired maturity date).

[5] The default risk component of a swap spread will be smaller than for a comparable bond credit spread. The reasons are straightforward. First, since only net interest payments are exchanged rather than both principal and coupon interest payments, the total cash flow at risk is lower. Second, the probability of default depends jointly on the probability of the counterparty defaulting and whether or not the swap has a positive value. See John C. Hull, *Introduction to Futures and Options Markets, Third Edition* (Upper Saddle River, NJ: Prentice Hall, 1998).

For example, consider a financial institution that has fixed-rate assets and floating-rate liabilities. Both the assets and liabilities have a maturity of three years. The interest rate on the liabilities resets every three months based on 3-month Libor. This financial institution can hedge this mismatched asset/liability position by buying a 3-year bundle of Eurodollar CD futures contracts. By doing so, the financial institution is receiving Libor over the 3-year period and paying a fixed dollar amount (i.e., the futures price). The financial institution is now hedged because the assets are fixed rate and the bundle of long Eurodollar CD futures synthetically creates a fixed-rate funding arrangement. From the fixed dollar amount over the three years, an effective fixed rate that the financial institution pays can be computed. Alternatively, the financial institution can synthetically create a fixed-rate funding arrangement by entering into a 3-year swap in which it pays fixed and receives 3-month Libor. Other things equal, the financial institution will use the vehicle that delivers the lowest cost of hedging the mismatched position. That is, the financial institution will compare the synthetic fixed rate (expressed as a percentage over U.S. Treasuries) to the 3-year swap spread. The difference between the synthetic spread and the swap spread should be within a few basis points under normal circumstances.

For swaps with tenors greater than five years, we cannot rely on the Eurodollar CD futures due to diminishing liquidity of such contracts. Instead, longer-dated swaps are priced using rates available for investment-grade corporate borrowers in fixed-rate and floating-rate debt markets. Since a swap can be interpreted as a package of long and short positions in a fixed-rate bond and a floating-rate bond, it is the credit spreads in those two market sectors that will be the primary determinant of the swap spread. Empirically, the swap curve lies above the U.S. Treasury yield curve and below the on-the-run yield curve for AA-rated banks.[6] Swap fixed rates are lower than AA-rated bond yields because their lower credit risk due to netting and offsetting of swap positions.

In addition, there are a number of other technical factors that influence the level of swap spreads.[7] While the impact of some these factors is ephemeral, their influence can be considerable in the short run. Included among these factors are: (1) the level and shape of the Treasury yield curve; (2) the relative supply of fixed- and floating-rate payers in the interest rate swap market; (3) the technical factors that affect swap dealers; and (4) the level of asset-based swap activity.

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[6] For a discussion of this point, see Andrew R. Young, *A Morgan Stanley Guide to Fixed Income Analysis* (New York: Morgan Stanley, 1997).

[7] See Ellen L. Evans and Gioia Parente Bales, "What Drives Interest Rate Swap Spreads," Chapter 13 in Carl R. Beidleman (ed.), *Interest Rate Swaps* (Burr Ridge, IL: Irwin Professional Publishing, 1991).

The level, slope, and curvature of the U.S. Treasury yield curve is an important influence on swap spreads at various maturities. The reason is that embedded in the yield curve are the market's expectations of the direction of future interest rates. While these expectations are sometimes challenging to extract, the decision to borrow at a fixed-rate or a floating-rate will be based, in part, on these expectations. The relative supply of fixed- and floating-rate payers in the interest rate swap market should also be influenced by these expectations. For example, many corporate issuers—financial institutions and federal agencies in particular—swap their newly issued fixed-rate debt into floating using the swap market. Consequently, swap spreads will be affected by the corporate debt issuance calendar. In addition, swap spreads, like credit spreads, also tend to increase with the swap's tenor or maturity.

Swap spreads are also affected by the hedging costs faced by swap dealers. Dealers hedge the interest rate risk of long (short) swap positions by taking a long (short) position in a Treasury security with the same maturity as the swap's tenor and borrowing funds (lending funds) in the repo market. As a result, the spread between Libor and the appropriate repo rate will be a critical determinant of the hedging costs. For example, with the burgeoning U.S. government budget surpluses starting in the late 1990s, the supply of Treasury securities has diminished. One impact of the decreased supply is an increase in the spread between the yields of on-the-run and off-the-run Treasuries. As this spread widens, investors must pay up for the relatively more liquid on-the-run issues. This chain reaction continues and results in on-the-run Treasuries going "on special" in repo markets. When on-the-run Treasuries go "on special," it is correspondingly more expensive to use these Treasuries as a hedge. This increase in hedging costs results in wider swap spreads.[8]

Another influence on the level of swap spreads is the volume of asset-based swap transactions. An asset-based swap transaction involves the creation of a synthetic security via the purchase of an existing security and the simultaneous execution of a swap. For example, after the Russian debt default and ruble devaluation in August 1998, risk-averse investors sold corporate bonds and fled to the relative safety of U.S. Treasuries. Credit spreads widened considerably and liquidity diminished. A contrary-minded floating-rate investor (like a financial institu-

[8] Traders often use the repo market to obtain specific securities to cover short positions. If a security is in short supply relative to demand, the repo rate on a specific security used as collateral in repo transaction will be below the general (i.e., generic) collateral repo rate. When a particular security's repo rate falls markedly, that security is said to be "on special." Investors who own these securities are able to lend them out as collateral and borrow at bargain basement rates.

tion) could have taken advantage of these circumstances by buying newly issued investment grade corporate bonds with relatively attractive coupon rates and simultaneously taking a long position in an interest rate swap (pay fixed/receive floating). Because of the higher credit spreads, the coupon rate that the financial institution receives is higher than the fixed-rate paid in the swap. Accordingly, the financial institution ends up with a synthetic floating-rate asset with a sizeable spread above Libor.

By similar reasoning, investors can use swaps to create a synthetic fixed-rate security. For example, during the mid-1980s, many banks issued perpetual floating-rate notes in the Eurobond market. A perpetual floating-rate note is a security that delivers floating-rate cash flows forever. The coupon is reset and paid usually every three months with a coupon formula equal to the reference rate (e.g., 3-month Libor) plus a spread. When the perpetual floating-rate note market collapsed in late 1986, the contagion spread into other sectors of the floaters market.[9] Many floaters cheapened considerably. As before, contrary-minded fixed-rate investors could exploit this situation through the purchase of a relatively cheap (from the investor's perspective) floater while simultaneously taking a short position in an interest rate swap (pay floating/receive fixed) thereby creating a synthetic fixed-rate investment. The investor makes floating-rate payments (say based on Libor) to their counterparty and receives fixed-rate payments equal to the Treasury yield plus the swap spread. Accordingly, the fixed rate on this synthetic security is equal to the sum of the following: (1) the Treasury bond yield that matches the swap's tenor; (2) the swap spread; and (3) the floater's index spread.

NONGENERIC INTEREST-RATE SWAPS

The swap market is very flexible and instruments can be tailor-made to fit the requirements of individual customers. A wide variety of swap contracts are traded in the market. Although the most common reference rate for the floating-leg of a swap is 6-month Libor for a semiannual paying floating leg, other reference rates that have been used include 3-month Libor, the prime rate (for dollar swaps), the 1-month commercial paper rate, and the Treasury bill rate, and the municipal bond rate.

The term of a swap need not be fixed; swaps may be *extendible* or *putable*. In an extendible swap, one of the parties has the right but not the obligation to extend the life of the swap beyond the fixed maturity

[9] Suresh E. Krishman, "Asset-Based Interest Rate Swaps," Chapter 8 in *Interest Rate Swaps*.

date, while in a putable swap one party has the right to terminate the swap prior to the specified maturity date.

It is also possible to transact options on swaps, known as *swaptions*. A swaption is the right to enter into a swap agreement at some point in the future, during the life of the option. Essentially a swaption is an option to exchange a fixed-rate bond cash flow for a floating-rate bond cash flow structure. Swaptions will be described in more detail later.

Other swaps are described below.

Constant Maturity Swap

In a constant maturity swap, the parties exchange a Libor rate for a fixed swap rate. For example, the terms of the swap might state that 6-month Libor is exchanged for the 5-year swap rate on a semiannual basis for the next five years, or for the 5-year government bond rate. In the U.S. market, the second type of constant maturity swap is known as a *constant maturity Treasury swap*.

Accreting and Amortizing Swaps

In a plain vanilla swap, the notional principal remains unchanged during the life of the swap. However it is possible to trade a swap where the notional principal varies during its life. An accreting (or step-up) swap is one in which the principal starts off at one level and then increases in amount over time. The opposite, an amortizing swap, is one in which the notional reduces in size over time. An accreting swap would be useful where for instance, a funding liability that is being hedged increases over time. The amortizing swap might be employed by a borrower hedging a bond issue that featured sinking fund payments, where a part of the notional amount outstanding is paid off at set points during the life of the bond. If the principal fluctuates in amount, for example increasing in one year and then reducing in another, the swap is known as a *roller-coaster swap*. Another application of an amortizing swap is as a hedge for a loan that is itself an amortizing one. Frequently this is combined with a forward-starting swap, to tie in with the cash flows payable on the loan. The pricing and valuation of an amortizing swap is no different in principle to a vanilla interest-rate swap; a single swap rate is calculated using the relevant discount factors, and at this rate the net present value of the swap cash flows will equal zero at the start of the swap.

Zero-Coupon Swap

A zero-coupon swap replaces the stream of fixed-rate payments with a single payment at the end of the swap's life, or less common, at the beginning. The floating-rate payments are made in the normal way. Such a swap exposes the floating-rate payer to some credit risk because it makes regular payments but does not receive any payment until the termination date of the swap.

Libor-in-Arrears Swap

In a Libor-in-arrears swap (also known as a *back-set swap*), the reset date is just before the end of the accrual period for the floating-rate rather than just before the start. Such a swap would be attractive to a counterparty who had a different view on interest rates compared to the market consensus. For instance in a rising yield curve environment, forward rates will be higher than current market rates, and this will be reflected in the pricing of a swap. A Libor-in-arrears swap would be priced higher than a conventional swap. If the floating-rate payer believed that interest rates would in fact rise more slowly than forward rates (and the market) were suggesting, he or she may wish to enter into an arrears swap as opposed to a conventional swap.

Basis Swap

In a conventional swap one leg comprises fixed-rate payments and the other floating-rate payments. In a *basis swap* both legs are floating-rate, but linked to different money market indices. One leg is normally linked to Libor, while the other might be linked to the CD rate or the commercial paper rate. This type of swap would be used by a bank in the United States that had made loans that paid at the prime rate and funded its loans at Libor. A basis swap would eliminate the basis risk between the bank's income and interest expense. Other basis swaps are traded in which both legs are linked to Libor, but at different maturities; for instance one leg might be at three-month Libor and the other at 6-month Libor. In such a swap, the basis is different as is the payment frequency: One leg pays out semiannually, while the other would be paying on a quarterly basis.

Margin Swap

It is common to encounter swaps where there is a margin above or below Libor on the floating leg, as opposed to a floating leg of Libor flat. Such swaps are called *margin swaps*. If a bank's borrowing is financed at Libor + 25 bps, it may wish to receive Libor + 25 bps in the swap so that its cash flows match exactly. The fixed-rate quote for a swap must be adjusted correspondingly to allow for the margin on the floating side. So in our example if the fixed-rate quote is say, 6.00%, it would be adjusted to around 6.25%; differences in the margin quoted on the fixed leg might arise if the day-count convention or payment frequency were to differ between fixed and floating legs. Another reason why there may be a margin is if the credit quality of the counterparty demanded it, so that highly rated counterparties may pay slightly below Libor, for instance.

Off-Market Swap

When a swap is transacted, its fixed rate is quoted at the current market rate for that maturity. When the fixed rate is different from the market rate, this type of swap is an *off-market swap*, and a compensating payment is made by one party to the other. An off-market rate may be used for particular hedging requirements for example, or when a bond issuer wishes to use the swap to hedge the bond as well as to cover the bond's issue costs.

Differential Swap

A differential swap is a basis swap but with one of the legs calculated in a different currency. Typically one leg is floating rate, while the other is floating rate but with the reference rate stated in another currency but denominated in the domestic currency. For example, a differential swap may have one party paying 6-month pound sterling Libor, in pound sterling, on a notional principal of £10 million, and receiving euro-Libor minus a margin, payable in sterling and on the same notional principal. Differential swaps are not very common and are the most difficult for a bank to hedge. The hedging is usually carried out using what is known as a *quanto option*.

Forward-Start Swap

A *forward-start swap* is an obligation, where two counterparties agree to enter into a swap contract at some future date under terms negotiated today.[10] Accordingly, the swap's effective (i.e., start) date is not the usual one or two days after the trade date but some time afterwards, say, six months after the trade date. For example, an interest rate swap with a tenor of three years that has an effective date one year from today. Once the effective date is reached, a forward-start swap is identical to a normal interest rate swap. Earlier in the chapter, we noted that it is useful to think of the generic interest rate swap market as one where two counterparties trade the floating reference rate in a series of exchanges for a fixed price. Extending this intuition, the forward-start interest rate swap market is a forward market for trading the floating reference rate as opposed to the spot market.

A forward start swap contract will specify the swap's fixed rate at which the two counterparties agree to exchange cash flows during the

[10] Forward-start swaps are also referred to as *forward swaps*, *delayed swaps*, and *deferred swaps*.

swap's life which begins on some future effective date. This rate is referred to as the *forward swap fixed rate*. In order to determine the forward swap fixed rate, we need a valuation tool that allows for changing interest rates in the future due to interest rate volatility. One obvious choice is the lattice approach presented in Chapter 2. Buetow and Fabozzi present a procedure for valuing forward start swaps using a cumulative swap valuation lattice. They demonstrate, among other things, that a change in the assumed interest rate volatility does not affect a generic interest rate swap's value or its fixed rate.[11] Conversely, a change in volatility does impact the value of a forward start swap. Specifically, the higher the interest rate volatility assumed, the higher the value of a forward start swap.

CANCELLING A SWAP

When financial institutions enter into a swap contract in order to hedge interest-rate liabilities, the swap will be kept in place until its expiration. However, circumstances may change or a financial institution may alter its view on interest rates, and so circumstances may arise such that it may be necessary to terminate the swap. The most straightforward option is for the corporation to take out a second contract that negates the first. This allows the first swap to remain in place, but there may be residual cash flows unless the two swaps cancel each other out precisely. The terms for the second swap, being nonstandard (and unlikely to be a exactly whole years to maturity, unless traded on the anniversary of the first), may also result in it being more expensive than a vanilla swap. As it is unlikely that the second swap will have the same rate, the two fixed legs will not net to zero. And if the second swap is not traded on an anniversary, payment dates will not match.

For these reasons, an entity may wish to cancel the swap entirely. To do this it will ask a swap market maker for a quotation on a *cancellation fee*. The bank will determine the cancellation fee by calculating the net present value of the remaining cash flows in the swap, using the relevant discount factor for each future cash flow. In practice just the fixed leg will be present valued, and then netted with Libor. The net present value of all the cash flows is the fair price for canceling the swap. The valuation principles we established earlier will apply; that is, if the fixed rate payer is asking to cancel the swap when interest rates have fallen, he will pay the cancellation fee, and vice-versa if rates have risen.

[11] See Chapter 7 in Gerald W. Buetow and Frank J. Fabozzi, *Valuation of Interest Rate Swaps and Swaptions* (Hoboken, NJ: John Wiley & Sons, Inc., 2000).

CREDIT RISK

The rate quoted for swaps in the interbank market assumes that the counterparty to the transaction has a lending line with the swap bank, so the swap rate therefore reflects the credit risk associated with an interbank quality counterparty. This credit risk is reflected in the spread between the swap rate and the equivalent-maturity government bond, although, as noted, the spread also reflects other considerations such as liquidity and supply and demand. The credit risk of a swap is separate from its interest-rate risk or market risk, and arises from the possibility of the counterparty to the swap defaulting on its obligations. If the present value of the swap at the time of default is net positive, then a bank is at risk of loss of this amount. While market risk can be hedged, it is more problematic to hedge credit risk. The common measures taken include limits on lending lines, collateral, and diversification across counterparty sectors, as well as a form of credit value-at-risk to monitor credit exposures.

A bank therefore is at risk of loss due to counterparty default for all its swap transactions. If at the time of default, the net present value of the swap is positive, this amount is potentially at risk and will probably be written off. If the value of the swap is negative at the time of default, in theory this amount is a potential gain to the bank, although in practice the counterparty's administrators will try to recover the value for their client. In this case then, there is no net gain or loss to the swap bank. The credit risk management department of a bank will therefore often assess the ongoing credit quality of counterparties with whom the swap transactions are currently positive in value.

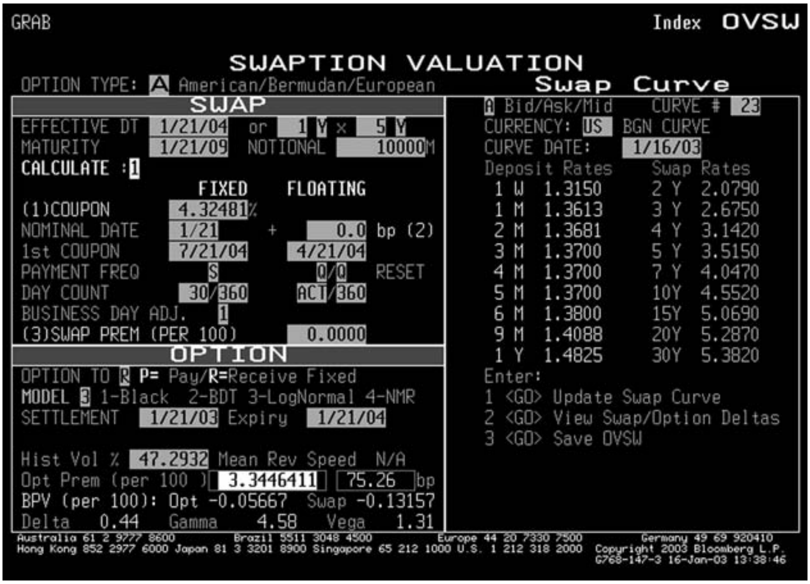
SWAPTIONS

Swaptions are options to establish a position in an interest rate swap at some future date. The swaption contract specifies the swaption's expiration date as well as the fixed rate and tenor of the underlying swap. The swap's fixed rate is called the swaption's *strike rate*. There are two types of swaptions—pay fixed or receive fixed. A pay (receive) fixed swaption gives the buyer the right to establish a position in an interest rate swap where he/she will pay (receive) the fixed rate cash flows and receive (pay) the floating rate cash flows.[12]

Let's illustrate a hypothetical swaption with Bloomberg's OVSW (swaption valuation) screen in Exhibit 10.13. This swaption presented

[12] Pay fixed swaptions are also known as *call swaptions* and receive fixed swaptions are also known as *put swaptions*.

EXHIBIT 10.13 Bloomberg's Swaption Valuation Screen

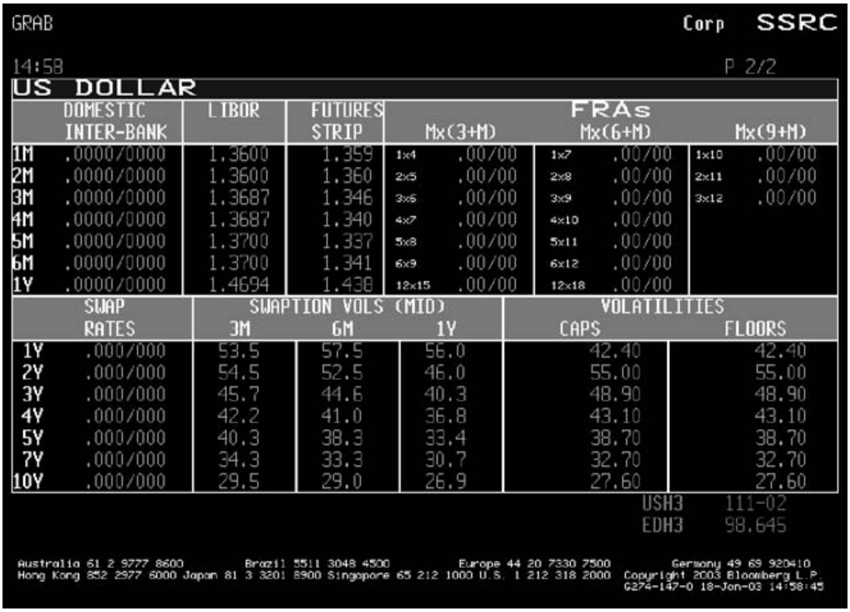


Source: Bloomberg Financial Markets

in the screen is a 1-year swaption on a 5-year generic interest rate swap in which the buyer will receive fixed cash flows and pay floating-rate cash flows. Note at the top of the screen, this is an American-style option and is therefore exercisable on any day for the next year. If the swaption is exercised on its expiration date (January 21, 2004), the 5-year swap begins on January 21, 2004 (the "effective date") and ends on January 21, 2009 ("maturity"). The swap's fixed rate is 4.32841% while the floating rate is 3-month Libor flat. As can be seen in the row labeled "Payment Freq," the fixed-rate cash flows are delivered semiannually while the floating-rate cash flows paid and reset quarterly. Note that the day counts for the fixed and floating cash flows differ.

In the "Option" box in the right-hand corner of the screen, we see that one of four valuation models can be employed to value the option. Using a lognormal interest rate tree (model 3, discussed in Chapter 2), this swaption's value is \$3.3446411 per \$100 of notional principal. However, market prices are usually quoted in terms of implied volatility. As an illustration, Exhibit 10.14 presents the second page of the Bloomberg Swapsource page (function SSRC). The implied volatilities for swaptions are located at the bottom-center of the screen. There are 3-month, 6-month, and 1-year swaptions where the underlying swaps

EXHIBIT 10.14 Bloomberg's Swapsource Screen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US DOLLAR

	DOMESTIC INTER-BANK	LIBOR	FUTURES STRIP	FRAs Mx(3+M)		Mx(6+M)		Mx(9+M)	
1M	.0000/0000	1.3600	1.359	1x4	.00/00	1x7	.00/00	1x10	.00/00
2M	.0000/0000	1.3600	1.360	2x5	.00/00	2x8	.00/00	2x11	.00/00
3M	.0000/0000	1.3687	1.346	3x6	.00/00	3x9	.00/00	3x12	.00/00
4M	.0000/0000	1.3687	1.340	4x7	.00/00	4x10	.00/00		
5M	.0000/0000	1.3700	1.337	5x8	.00/00	5x11	.00/00		
6M	.0000/0000	1.3700	1.341	6x9	.00/00	6x12	.00/00		
1Y	.0000/0000	1.4694	1.438	12x15	.00/00	12x18	.00/00		

	SWAP RATES	SWAPTION VOLS (MID) 3M	6M	1Y	VOLATILITIES CAPS	FLOORS
1Y	.000/000	53.5	57.5	56.0	42.40	42.40
2Y	.000/000	54.5	52.5	46.0	55.00	55.00
3Y	.000/000	45.7	44.6	40.3	48.90	48.90
4Y	.000/000	42.2	41.0	36.8	43.10	43.10
5Y	.000/000	40.3	38.3	33.4	38.70	38.70
7Y	.000/000	34.3	33.3	30.7	32.70	32.70
10Y	.000/000	29.5	29.0	26.9	27.60	27.60

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Hong Kong 852 2977 6000 Japan 81 3 3201 8900 Singapore 65 212 1000 U.S. 1 212 318 2000 Copyright 2003 Bloomberg L.P.
G274-147-0 18-Jun-03 14:58:45

Source: Bloomberg Financial Markets

have tenors ranging from one to ten years. The "(MID)" indicates that the implied volatilities are the mid point between the best bid price's implied volatility and the best offer price's implied volatility.

SWAPTION VALUATION

When valuing interest rate derivatives or bonds with embedded options, it is essential to model expected future interest rate volatility. Accordingly, the lattice approach discussed in Chapter 2 is a commonly used method to value swaptions.[13] A swaption's value will depend on a few critical parameters which include market inputs (e.g., the current yield curve) as well as terms of the swaption contract (e.g., time to expiration). To solidify our intuition about how swaptions work, we examine how changes in key factors impact swaption values. In particular, we will consider changes in the following: yield curve (level and slope), volatility, strike rate, and time to expiration.

[13] See Chapter 8 in Buetow and Fabozzi, *Valuation of Interest Rate Swaps and Swaptions*.

Changes in the Yield Curve

As with conventional call and put options, pay-fixed or receive-fixed swaptions tend to react in an opposite manner to changes in the underlying parameters. For example, a pay-fixed swaption increases in value with an upward parallel shift in the yield curve and a receive swaption becomes more valuable with a downward parallel shift in the yield curve. To see this, consider a 1-year European pay-fixed swaption on a 5-year generic interest rate swap. The notional principal is $10 million and the strike rate is 6%. On the expiration date, the buyer will either exercise it (i.e., enter into the 5-swap to pay 6% fixed-rate cash flows and receive floating-rate cash flows) or let the swaption expire. If the 5-year swap rate is above 6% on the expiration date, the buyer of this pay fixed swaption will exercise it. Conversely, if the 5-year swap rate is below 6%, the pay fixed swaption will expire worthless. The principle is the same for a receive swaption, only in reverse.

Next, we consider the impact of a change in the yield curve's shape on swaption values. In particular, we will discuss the impact of a steepening and an inverting yield curve. If the yield curve steepens, the value of pay fixed swaption increases and the value of receive fixed swaption decreases. The intuition is straightforward. A steepening yield curve indicates that the implied forward rates are increasing at a faster rate than suggested by the initial yield curve. The higher rates indicate that the floating-rate cash flows of the underlying swap contract are going to be higher than previously expected. This effect works to the advantage of the pay-fixed swaption buyer since she will receive higher floating-rate cash flows if the swaption is exercised. The opposite is true for a receive-fixed swaption buyer. By analogous reasoning, an inverted yield curve indicates that the implied forward rates are decreasing. If this occurs, the value of a pay-fixed swaption decreases and the value of a receive-fixed swaption increases.

Volatility

There is a positive relationship between swaption values and the assumed interest rate volatility. If interest volatility increases, all else held constant, the greater the chance the underlying swap's value will move in a favorable direction (i.e., higher floating-rate cash flows for the pay fixed swaption and higher fixed-rate cash flows for the receive-fixed swaption). As explained in Chapter 12, vega measures the impact of a change in interest rate volatility on an option's value. For the swaption illustrated in Exhibit 10.13, vega is located at the bottom center of the screen in the "Option" box. For a swaption, vega tells us the sensitivity of the swaption's value (in basis points) to a 1% change in the assumed interest rate volatility. In this illustration, vega is 1.31 basis points. To find the dollar price change in the swaption value due to the volatility change, one needs only to multiply 1.31 basis points (in decimal form) by the swaption's notional principal.

Strike Rate

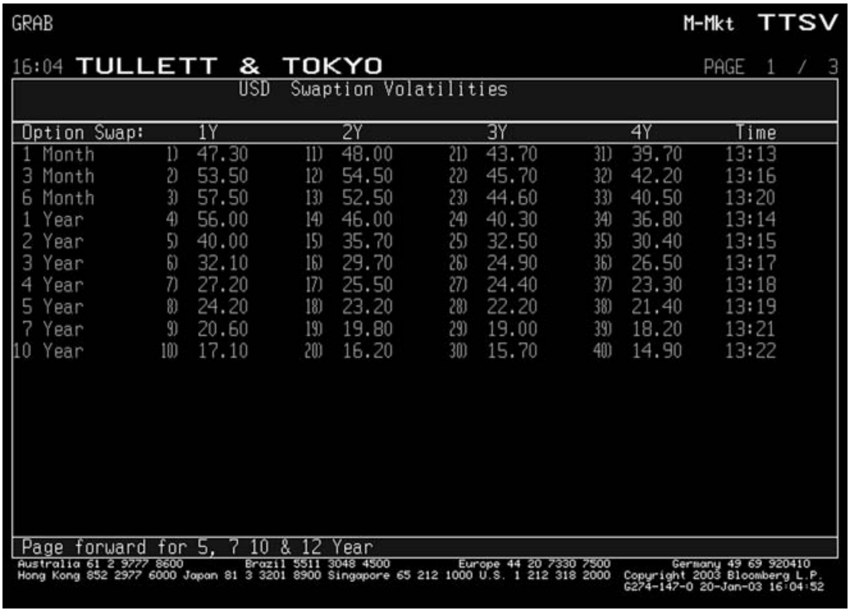
The value of a swaption is essentially the difference between the strike rate and prevailing swap rate at the time it is being valued. At expiration, a pay-fixed swaption is only exercised when the swap rate is higher than the strike rate. Conversely, at expiration, a receive-fixed swaption is only exercised when the swap rate is lower than the strike rate. Given this backdrop, it is apparent that as the strike rate changes, a pay-fixed swaption and a receive-fixed swaption will behave in a opposite manner. An increase in the strike rate, all else equal, will decrease the value of a pay-fixed swaption and increase the value of a receive-fixed swaption. The reasoning is as follows. As the strike rate increases, the pay-fixed swaption buyer will pay higher fixed-rate cash flows over the swap's life if the swaption is exercised. This is obviously less valuable than paying a lower fixed-rate for the same floating-rate cash flows in return. For the receive-fixed swaption buyer, an increase in the strike rate means that the receive-fixed swaption buyer will receive higher fixed-rate cash flows over the swap's life if the swaption is exercised. For decreases in the strike rate, the effects are reversed.

Time to Expiration

For most options (calls and puts) traded in the financial markets, increasing the option's time to expiration makes it more valuable. This is not the case for swaptions. Increasing a swaption's time to expiration can either increase or decrease its value. This ambiguity is due to the interaction of increasing the time to expiration and the other variables that drive a swaption's value—the current yield curve, volatility, and the strike rate.

As an illustration, Exhibit 10.15 presents a Bloomberg screen of the prices (in the form of implied volatilities) of swaptions of various times to expiration for generic interest rate swaps of various tenors. The times to expiration of the swaptions are listed in the first column and range from one month to ten years. The tenors of the swaps underlying these options are listed across the top and range from one year to four years. Holding the tenor of the swap constant, note there is no consistent pattern in the prices as time to expiration increases.

EXHIBIT 10.15 Prices of Swaptions of Various Times to Expiration



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16:04 TULLETT & TOKYO PAGE 1 / 3

USD Swaption Volatilities

Option Swap:		1Y		2Y		3Y		4Y	Time
1 Month	1)	47.30	11)	48.00	21)	43.70	31)	39.70	13:13
3 Month	2)	53.50	12)	54.50	22)	45.70	32)	42.20	13:16
6 Month	3)	57.50	13)	52.50	23)	44.60	33)	40.50	13:20
1 Year	4)	56.00	14)	46.00	24)	40.30	34)	36.80	13:14
2 Year	5)	40.00	15)	35.70	25)	32.50	35)	30.40	13:15
3 Year	6)	32.10	16)	29.70	26)	24.90	36)	26.50	13:17
4 Year	7)	27.20	17)	25.50	27)	24.40	37)	23.30	13:18
5 Year	8)	24.20	18)	23.20	28)	22.20	38)	21.40	13:19
7 Year	9)	20.60	19)	19.80	29)	19.00	39)	18.20	13:21
10 Year	10)	17.10	20)	16.20	30)	15.70	40)	14.90	13:22

Page forward for 5, 7 10 & 12 Year

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Hong Kong 852 2977 6000 Japan 81 3 3201 8900 Singapore 65 212 1000 U.S. 1 212 318 2000 Copyright 2003 Bloomberg L.P.
G274-147-0 20-Jan-03 16:04:52

Source: Bloomberg Financial Markets

KEY POINTS

1. An interest rate swap is an agreement between two parties to exchange interest payments at designated times in the future based on a notional principal amount.

2. In a generic interest rate swap, one party agrees to make fixed-rate payments and receive floating-rate payments while the counter-party agrees to make floating-rate payments and receive fixed-rate payments.

3. The most common reference rate for the floating-rate payments is Libor.

4. Interest rate swaps are over-the-counter instruments.

5. The default risk in a swap agreement is called *counterparty risk*.

6. A swap position can be interpreted as either a package of forward contracts or a package of cash flows from buying and selling cash market instruments.

7. The convention that has evolved for quoting swap levels is that a swap dealer sets the floating rate equal to the reference rate and then quotes the fixed rate that will apply.

8. The swap rate is determined by finding the rate that will make the present value of the cash flow of both sides of the swap equal.

9. In a Libor-based swap, the cash flow of the floating-rate side is determined from the Eurodollar CD futures contract.

10. The discount rates used to calculate the present value of the cash flows in a swap are forward rates.

11. The value of an existing swap is equal to the difference in the present value of the two payments.

12. The swap spread is the spread over the Treasury par curve specified at the initiation of the swap that the fixed-rate payer must pay.

13. Nongeneric swaps include constant maturity, accreting/amortizing, zero-coupon, Libor-in-arrears, basis, margin, off-market, differential and forward-start.

14. The net present value of all cash flows is the fair price for canceling a swap.

15. Swaptions are options to establish a position in an interest rate swap at some future date.

16. The lattice approach is a commonly used method to value swaptions.

17. The key factors that impact a swaption's value are the yield curve, volatility, strike rate, and time to expiration.

CHAPTER 11

Exchange-Traded Options

An option contract is a derivative instrument that differs from those previously discussed (forwards, futures, and swaps) in terms of its risk and return characteristics. As such, an option can be employed to control interest rate risk in ways that are either not possible or too costly to achieve using forwards, futures, or swaps. Options, like most other financial instruments, can be traded either on an organized exchange or in an over-the-counter market. The focus of this chapter is on exchange-traded options. The most popular form of an exchange-traded option is an option on a futures contract, which we discuss in detail. In the next chapter, we examine over-the-counter option contracts and other derivative products with option-like features.

The objectives of this chapter are to:

1. Describe the basic features of options contracts.
2. Explain the differences between options and futures.
3. Describe what futures options are, their trading mechanics, and the reasons for their popularity.
4. Review the various futures options currently traded.
5. Explain the risk and return characteristics for basic option positions.
6. Explain the two components of the option price and the factors that affect the value of an option.
7. Discuss the limitations of applying the Black-Scholes pricing model to value futures options and options on fixed-income instruments.
8. Explain how to measure the sensitivity of an option to changes in the factors that affect its value.
9. Explain how to estimate the duration of an option.

THE BASIC OPTION CONTRACT

An *option* is a contract in which the writer of the option grants the buyer of the option the right, but not the obligation, to purchase from or sell to the writer something at a specified price within a specified period of time (or at a specified date). The *writer*, also referred to as the *seller,* grants this right to the buyer in exchange for a certain sum of money, which is called the *option price* or *option premium*. In effect, the writer is selling a promise in exchange for the option price. Conversely, the buyer pays the option price to obtain the writer's promise. The price at which the underlying may be bought or sold is called the *exercise* or *strike price*. The date after which an option is void is called the *expiration date*. Our focus in this chapter is on options where the "something" underlying the option is a interest rate instrument.

When an option grants the buyer the right to purchase the designated instrument from the writer (seller), it is referred to as a *call option*, or *call*. When the option buyer has the right to sell the designated instrument to the writer, the option is called a *put option*, or *put*.

An option is also categorized according to when the option buyer may exercise the option. There are options that may be exercised at any time up to and including the expiration date. Such an option is referred to as an *American option*. There are options that may be exercised only at the expiration date. An option with this feature is called a *European option*. There are also *Bermudan* option contracts that are hybrids between American and European option contracts. The distinguishing feature of a Bermudan option contract is that early exercise is possible but is restricted to certain dates in the option's life.

The maximum amount that an option buyer can lose is the option price. The maximum profit that the option writer can realize is the option price. The option buyer has substantial upside return potential, while the option writer faces substantial downside risk. We'll investigate the risk and reward profile for option positions later.

There are no margin requirements for the buyer of an option once the option price has been paid in full. Because the option price is the maximum amount that the investor can lose, no matter how adverse the price movement of the underlying instrument, there is no need for margin. Because the writer of an option has agreed to accept all of the risk (and none of the reward) of the position in the underlying instrument, the writer is generally required to put up the option price received as margin. In addition, as price changes occur that adversely affect the writer's position, the writer is required to deposit additional margin (with some exceptions) as the position is marked to market.

EXHIBIT 11.1 Description of the Four Basic Option Positions

	Long	Written
Call	Option to buy	Obligation to sell
Put	Option to sell	Obligation to buy

DIFFERENCES BETWEEN OPTIONS AND FUTURES CONTRACTS

Notice that unlike in a futures contract, one party to an option contract is not obligated to transact. Specifically, the option buyer has the right but not the obligation to transact. The option writer does have the obligation to perform. In the case of a futures contract, both buyer and seller are obligated to perform. Of course, the buyer of a futures contract does not pay the seller to accept the obligation, while an option buyer pays the seller the option price.

To illustrate the rights and obligations of these option positions, consider the 2 × 2 box in Exhibit 11.1. The two rows are labeled call and put, respectively. Likewise, the two columns are labeled "Long" and "Written." The words in each of the four boxes describe the four basic option positions—long call, written call, long put, and written put.

Consequently, the risk and reward characteristics of the two contracts are also different. In the case of a futures contract, the buyer of the contract realizes a dollar-for-dollar gain when the price of the futures contract increases and suffers a dollar-for-dollar loss when the price of the futures contract drops. The opposite occurs for the seller of a futures contract. Options do not provide this symmetric risk and reward characteristic. The most that the buyer of an option can lose is the option price. While the buyer of an option retains all the potential benefits, the gain is always reduced by the amount of the option price. The maximum profit that the writer may realize is the option price; this is offset against substantial downside risk. This difference is extremely important because managers can use futures to protect against symmetric risk and options to protect against asymmetric risk.

EXCHANGE-TRADED VERSUS OTC OPTIONS

There are exchange-traded options and over-the-counter options. Exchange-traded options have two advantages. First, the exercise price and expiration date of the contract are standardized. Second, as in the case of futures contracts, the direct link between buyer and seller is severed after the order is executed because of the interchangeability of exchange-traded options. The clearinghouse associated with the exchange where the option trades performs the same function in the options market that it does in the futures market.

OTC options are used in the many situations where an institutional investor needs to have a tailor-made option because the standardized exchange-traded option does not satisfy its investment objectives. Investment banking firms and commercial banks act as principals as well as brokers in the OTC options market.

OTC options can be customized in any manner sought by an institutional investor. There are plain vanilla options such as options on a specific Treasury issue. The more complex OTC options created are called *exotic options*. Examples of OTC options are given in the next chapter. While an OTC option is less liquid than an exchange-traded option, this is typically not of concern since institutional investors who use OTC options as part of a hedging or asset/liability strategy intend to hold them to expiration.

In the absence of a clearinghouse the parties to any over-the-counter contract are exposed to counterparty risk. In the case of a forward contract (an OTC contract) both parties face counterparty risk since both parties are obligated to perform. Thus, there is bilateral counterparty risk. In contrast, for an OTC option, once the option buyer pays the option price, it has satisfied its obligation. It is only the seller that must perform if the option is exercised. Thus, the option buyer is exposed to unilateral counterparty risk—the risk that the option seller will fail to perform.

FUTURES OPTIONS

The underlying for an interest rate option can be a fixed-income security or an interest rate futures contract. The former options are called *options on physicals*. In the United States, there are no actively exchange-traded options on physicals. Options on interest rate futures are called *futures options*. The actively traded interest rate options on exchanges are futures options.

The Basics of Futures Options

A futures option gives the buyer the right to buy from or sell to the writer a designated futures contract at the strike price at any time during the life of the option. If the futures option is a call option, the buyer has the right to purchase one designated futures contract at the strike price. That is, the buyer has the right to acquire a long futures position in the designated futures contract. If the buyer exercises the call option, the writer acquires a corresponding short position in the futures contract.

EXHIBIT 11.2 Description of the Four Basic Futures Options Positions

	Long	Written
Call on Futures	Option to establish a long futures position	Obligation to establish a short futures position
Put on Futures	Option to establish a short futures position	Obligation to establish a long futures position

A put option on a futures contract grants the buyer the right to sell a designated futures contract to the writer at the strike price. That is, the option buyer has the right to acquire a short position in the designated futures contract. If the put option is exercised, the writer acquires a corresponding long position in the designated futures contract.

As we did before with option contracts, let's summarize the rights and obligations of positions in futures options with the 2×2 box in Exhibit 11.2. These four boxes describe the four basic positions in futures options—long call on a futures contract, written call on a futures contract, long put on a futures contract, and a written put on a futures contract.

As the parties to the futures option will realize, a position in a futures contract when the option is exercised, the question is: what will the futures price be? That is, at what price will the long be required to pay for the instrument underlying the futures contract, and at what price will the short be required to sell the instrument underlying the futures contract?

Upon exercise, the futures price for the futures contract will be set equal to the strike price. The position of the two parties is then immediately marked-to-market in terms of the then-current futures price. Thus, the futures position of the two parties will be at the prevailing futures price. At the same time, the option buyer will receive from the option seller the economic benefit from exercising. In the case of a call futures option, the option writer must pay the difference between the current futures price and the strike price to the buyer of the option. In the case of a put futures option, the option writer must pay the option buyer the difference between the strike price and the current futures price.

For example, suppose an investor buys a call option on some futures contract in which the strike price is 85. Assume also that the futures price is 95 and that the buyer exercises the call option. Upon exercise, the call buyer is given a long position in the futures contract at 85 and the call writer is assigned the corresponding short position in the futures contract at 85. The futures positions of the buyer and the writer are immediately marked-to-market by the exchange. Because the prevailing futures price is 95 and the strike price is 85, the long futures position (the position of the call buyer) realizes a gain of 10, while the short futures position (the position of the call writer) realizes a loss of 10. The call writer pays the exchange 10 and the call buyer receives from the exchange 10. The call buyer, who now has a long futures position at 95, can either liquidate the futures position at 95 or maintain a long futures position. If the former course of action is taken, the call buyer sells a futures contract at the prevailing futures price of 95. There is no gain or loss from liquidating the position. Overall, the call buyer realizes a gain of 10. The call buyer who elects to hold the long futures position will face the same risk and reward of holding such a position, but still realizes a gain of 10 from the exercise of the call option.

Suppose instead that the futures option is a put rather than a call, and the current futures price is 60 rather than 95. Then if the buyer of this put option exercises it, the buyer would have a short position in the futures contract at 85; the option writer would have a long position in the futures contract at 85. The exchange then marks the position to market at the then-current futures price of 60, resulting in a gain to the put buyer of 25 and a loss to the put writer of the same amount. The put buyer who now has a short futures position at 60 can either liquidate the short futures position by buying a futures contract at the prevailing futures price of 60 or maintain the short futures position. In either case the put buyer realizes a gain of 25 from exercising the put option.

There are no margin requirements for the buyer of a futures option once the option price has been paid in full. Because the option price is the maximum amount that the buyer can lose, regardless of how adverse the price movement of the underlying instrument, there is no need for margin.

Because the writer (seller) of an option has agreed to accept all of the risk (and none of the reward) of the position in the underlying instrument, the writer (seller) is required to deposit not only the margin required on the interest rate futures contract position, but also (with certain exceptions) the option price that is received for writing the option. In addition, as prices adversely affect the writer's position, the writer would be required to deposit variation margin as it is marked to market.

Exchange-Traded Futures Options

In Chapter 9, we described several interest rate and bond futures contracts traded on various exchanges throughout the world. Options on these interest rate and bond futures contracts are also traded on these same exchanges. Exhibit 11.3 shows Bloomberg's Option Table Menu for options on bond futures contracts ranked (high to low) by open interest. Options on the U.S. Treasury note and bond futures contracts are almost always the most actively traded contracts in the world. Exhibit 11.4 shows Bloomberg's Option Table Menu for options on interest rate

EXHIBIT 11.3 **Bloomberg's Option Table Menu For Options on Bond Futures Contracts**

GRAB Govt OTM
Enter # <Go> to view a list of contracts.
* SYNTHETIC, ✓ FUT OPTS OPTION TABLE MENU → SPOT OPTS Page 1/2

BOND FUTURES			OPEN INTEREST	BOND FUTURES			OPEN INTEREST
1) ✓TY	US 10YR NOTE	CBT	998695	18) ✓YM	AU 6% 3YR BND	SFE	1988
2) ✓FV	US 5YR NOTE	CBT	654894	19) ✓NT	SPAN 10YR BND	MFB	1469
3) ✓US	US LONG BOND	CBT	474081	20) ✓JJ	JPN 5YR BOND	TSE	615
4) ✓OK	3 YR NGHT OPT	SFE	351276	21) ✓ZY	NZ 3YR BOND	NZF	130
5) ✓OA	10YR NGHT OPT	SFE	159848	22) ✓KE	3YR TSY FUT	KFE	100
6) ✓RM	R153 BOND	SAF	146595	23) ✓EDZ	5Y AGY A/C/E	CBT	55
7) ✓TU	US 2YR NOTE	CBT	106030	24) ✓XD	10YR DAY OPT	SFE	27
8) ✓G	LONG GILT	LIF	89796	25) ✓XB	US LONG BOND	MCE	n.a.
9) ✓BI	10YR JGB FUT	SGX	82152	26) ✓MN	EURO 10Y BOND	MAT	n.a.
10) ✓CN	CAN 10YR BOND	MSE	64355	27) ✓A	GER 10YR BOND	LIF	n.a.
11) ✓YV	DGB 6% 09 FUT	COP	58980	28) ✓SK	SWED 5YR BOND	PMI	n.a.
12) ✓JB	JPN 10YR BOND	TSE	48502	29) ✓AR	R150 LONG BND	SAF	n.a.
13) ✓DU	EURO-SCHATZ	EUX	36507	30) ✓NZT	NZ 10YR BD NT	NZF	n.a.
14) ✓CV	R150 BOND-V	SAF	16273	31) ✓NZY	NZ 3YR BD NGT	NZF	n.a.
15) ✓BJ	10YR MINI JGB	SGX	13779	32) ✓FM	EURO 5Y BOND	MAT	n.a.
16) ✓DD	R153 BOND-V	SAF	9178	33) ✓FU	R157 BOND FUT	SAF	n.a.
17) ✓MB	MUNI BOND	CBT	5020	34) ✓RX	EURO-BUND FUT	EUX	n.a.

SORT BY 0   SORT IN 0   DISPLAY 0
A ALPHABETICAL   A ASCENDING   99 <GO> FOR DISPLAY OPTIONS
P PUT IMPL VOLAT   N # OF CONTRACTS   D DESCENDING
C CALL IMPL VOLAT   O OPEN INTEREST
S HIST. VOLAT   R CURRENCY   V VOLUME   YELLOW KEY: <CMDTY> <INDEX> <CRNCY>
Australia 61 2 9777 8600 Brazil 5511 3048 4500 Europe 44 20 7330 7500 Germany 49 69 920410
Hong Kong 852 2977 6000 Japan 81 3 3201 8900 Singapore 65 212 1000 U.S. 1 212 318 2000 Copyright 2002 Bloomberg L.P.
G768-147-0 10-Oct-02 10:42:57

Source: Bloomberg Financial Markets

EXHIBIT 11.4 **Bloomberg's Option Table Menu For Options on Interest Rate Futures Contracts**

GRAB Govt OTM
Enter # <Go> to view a list of contracts.
* SYNTHETIC, ✓ FUT OPTS OPTION TABLE MENU → SPOT OPTS Page 1/2

INTEREST RATE			OPEN INTEREST	INTEREST RATE			OPEN INTEREST
1) ✓ED	EURO$ 3-MONTH	CME	4221851	18) ✓NYE	3MO EURO¥ NGT	TIF	90
2) ✓IR	90DAY BK BILL	SFE	1954707	19) ✓2E	EURO$ 2Y MCRV	CME	n.a.
3) ✓L	STERLING 3-MO	LIF	885660	20) ✓Y5	5Y EURO$ BNDL	CME	n.a.
4) ✓YE	EURO¥ 3-MONTH	TIF	582933	21) ✓DN	EURO$ 3M AON	CME	n.a.
5) ✓EY	EURO¥ TIBOR	SGX	240051	22) ✓W0	E$ 1Y MCV AON	CME	n.a.
6) ✓DE	EURO$ 3-MONTH	SGX	188566	23) ✓W2	E$ 2Y MCV AON	CME	n.a.
7) ✓IS	91D BANK BILL	SAF	146595	24) ✓FP	3M EURIBOR	EUX	n.a.
8) ✓VO	EURIBR 3M EST	MFB	42868	25) ✓EM	LIBOR 1 MONTH	CME	n.a.
9) ✓FJ	EURIBOR 3MO	MAT	24656	26) ✓ER	EURO EURIBOR	LIF	n.a.
10) ✓IZ	90DAY BNK OVN	SFE	4706	27) ✓EF	EURO¥ LIBOR	SGX	n.a.
11) ✓OD	BANK DEP 1DAY	BMF	3656	28) ✓ZB	NZ 3MO BILL	NZF	n.a.
12) ✓RK	3-MONTH JIBAR	SAF	3556	29) ✓OR	EURIBOR MD-CV	LIF	n.a.
13) ✓ES	EUROSWISS 3MO	LIF	2331	30) ✓MY	EURO¥ 3-MONTH	CME	n.a.
14) ✓NN	91D BNK BIL-V	SAF	330	31) ✓DY	3MO Euroyen	TIF	n.a.
15) ✓NZB	NZ 3M BILL NT	NZF	202	32) ✓RW	2YR SWAPNOTE	LIF	n.a.
16) ✓TB	US T-BILL 3MO	CME	169	33) ✓T	5YR SWAPNOTE	LIF	n.a.
17) ✓OE	EURO$ 1Y MCRV	CME	115	34) ✓P	10Y SWAPNOTE	LIF	n.a.

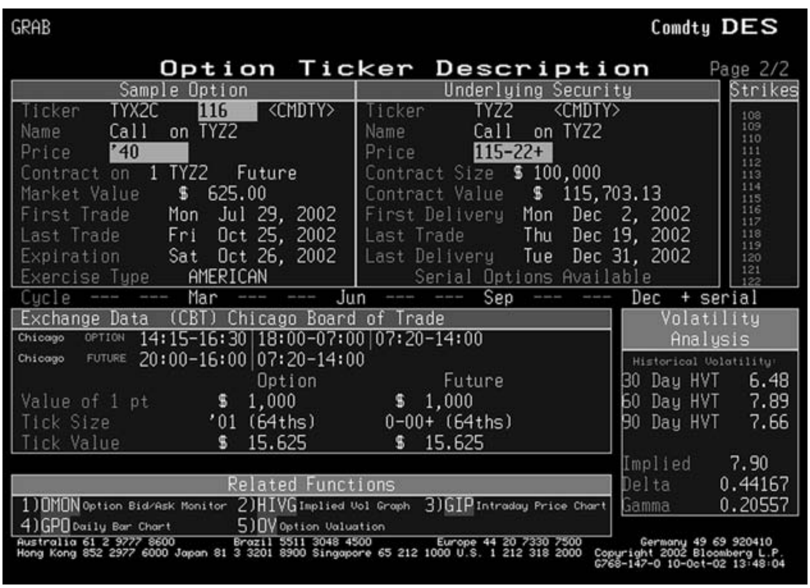
SORT BY 0   SORT IN 0   DISPLAY 0
A ALPHABETICAL   A ASCENDING   99 <GO> FOR DISPLAY OPTIONS
P PUT IMPL VOLAT   N # OF CONTRACTS   D DESCENDING
C CALL IMPL VOLAT   O OPEN INTEREST
S HIST. VOLAT   R CURRENCY   V VOLUME   YELLOW KEY: <CMDTY> <INDEX> <CRNCY>
Australia 61 2 9777 8600 Brazil 5511 3048 4500 Europe 44 20 7330 7500 Germany 49 69 920410
Hong Kong 852 2977 6000 Japan 81 3 3201 8900 Singapore 65 212 1000 U.S. 1 212 318 2000 Copyright 2002 Bloomberg L.P.
G768-147-0 10-Oct-02 10:44:18

Source: Bloomberg Financial Markets

futures contracts ranked by open interest. Not surprisingly, options on the Eurodollar CD futures contracts traded on the Chicago Mercantile Exchange are the most actively traded. Almost all futures options are of the American type. If the option buyer elects to exercise early, he or she must notify the clearing corporation which then randomly selects a clearing member that must select a short from among its customers.

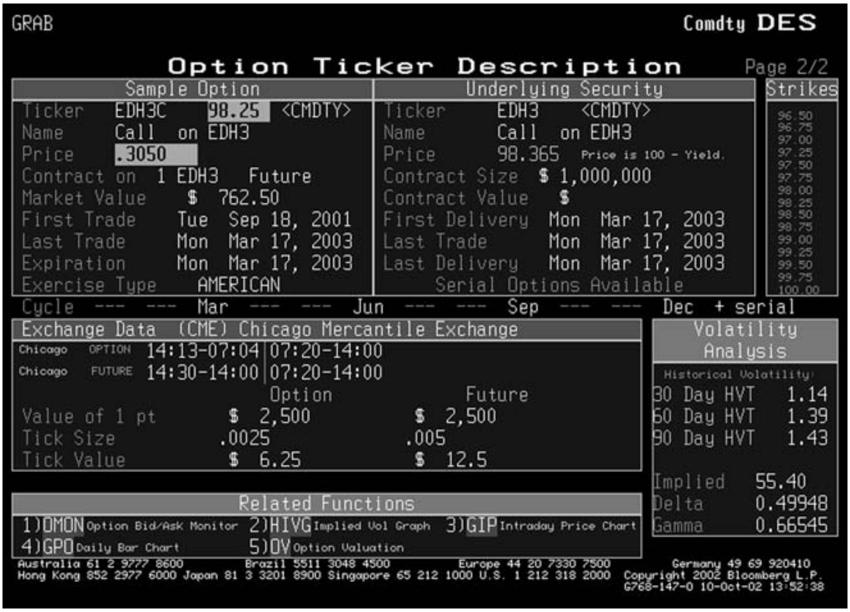
Exhibit 11.5 shows the Bloomberg Option Ticker Description screen for an American call option on the December 10-year Treasury note futures contract. The exercise or strike price on this contract is 116. Over on the right-hand side of the screen is a box labeled "Strikes," this box indicates the strike prices of available options. This contract expires on October 26, 2002. There are always five contract (expiration) months available for trading: the next three consecutive months plus the next two months in the quarterly cycle (March, June, September, and December). The price of futures options on a Treasury note is quoted in a 64*th* of 1% of par value. Since the face value of the 10-year Treasury note futures contract is $100,000, the value of one point (1% of par value) is $1,000 and the value of a 64*th* (i.e., one tick) is $15.625. Options on the 2 and 5 year Treasury notes and 30-year Treasury bond are structured similarly.

EXHIBIT 11.5 **Bloomberg's Option Ticker Description Screen for a Call Option on a 10-Year Treasury Note Futures Contract**



Source: Bloomberg Financial Markets

EXHIBIT 11.6 **Bloomberg's Option Ticker Description Screen for a Call Option on a Eurodollar CD Futures Contract**



Source: Bloomberg Financial Markets

In an attempt to compete with the OTC option market, the CBOT introduced in 1994 the *flexible Treasury futures options*. These futures options allow counterparties to customize options within certain limits. Specifically, the exercise price, expiration date, and type of exercise (American or European) can be customized subject to CBOT constraints. For example, the exercise price can be set to any 1⁄32 of a point and the expiration date can be set to any trading day but cannot exceed that of the longest standard option traded on the CBOT. Unlike an OTC option where the option buyer is exposed to counterparty risk, a flexible Treasury futures option is guaranteed by the CBOT Clearing Corporation. The minimum size requirements for the launching of a flexible futures option is 50 contracts.

Exhibit 11.6 shows the Bloomberg Option Ticker Description screen for an American call option on the March 3-month Eurodollar CD futures contract that trades on the CME. This contract has an exercise price of 98.25. The "Strikes" box indicates the exercise prices of available options, which are at intervals of 0.25 of an index point. This call option expires on March 17, 2003. Options are listed for eight months in the March quarterly cycle and two serial months not in the

March cycle. Note that futures options and futures expire on the same date. The tick size is 0.0025 of an index point with a dollar value of \$6.25. As indicated in the bottom center of the screen, the futures option and futures contract have different tick sizes.

RISK AND RETURN CHARACTERISTICS OF OPTIONS

Here we illustrate the risk and return characteristics of the four basic option positions—buying a call option, writing a call option, buying a put option, and writing a put option. The illustrations assume that each option position is held to the expiration date and not exercised early. In our illustrations we will use an option on a physical since the principles apply equally to futures options. To keep the illustration simple, we ignore transactions costs.

Buying Call Options

The purchase of a call option creates a financial position referred to as a *long call position*. To illustrate this position, assume that there is a call option on Asset XYZ that expires in one month and has a strike price of \$100. The option price is \$3. Suppose that the current price of Asset XYZ is \$100. For an investor who purchases this call option, the profit or loss at the expiration date is shown in the second column of Exhibit 11.7. The maximum loss is the option price and there is substantial upside potential.

It is worthwhile to compare the profit and loss profile of the call option buyer to taking a long position in one unit of Asset XYZ. The payoff from the position depends on Asset XYZ's price at the expiration date. Exhibit 11.7 compares the long call position and the long position in Asset XYZ. This comparison clearly demonstrates the way in which an option can change the risk/return profile. An investor who takes a long position in Asset XYZ realizes a profit of \$1 for every \$1 increase in Asset XYZ's price. As Asset XYZ's price falls, however, the investor loses dollar-for-dollar. If the price drops by more than \$3, the long position in Asset XYZ results in a loss of more than \$3. The long call position, in contrast, limits the loss to only the option price of \$3 but retains the upside potential, which will be \$3 less than for the long position in Asset XYZ.

Writing (Selling) Call Options

The writer of a call option is said to be in a *short call position*. To illustrate the option seller's (writer's) position, we use the same call option we used to illustrate buying a call option. The profit and loss profile of the short call position (that is, the position of the call option writer) is the

EXHIBIT 11.7 Comparison of Long Call Position and Long Asset Position

Assumptions:	Price of Asset XYZ = $100
	Option price = $3
	Strike price = $100
	Time to expiration = 1 month

Price of Asset XYZ	Net Profit/Loss for	
at Expiration Date	Long Call[a]	Long Asset XYZ[b]
$150	$47	$50
140	37	40
130	27	30
120	17	20
115	12	15
114	11	14
113	10	13
112	9	12
111	8	11
110	7	10
109	6	9
108	5	8
107	4	7
106	3	6
105	2	5
104	1	4
103	0	3
102	–1	2
101	–2	1
100	–3	0
99	–3	–1
98	–3	–2
97	–3	–3
96	–3	–4
95	–3	–5
94	–3	–6
93	–3	–7
92	–3	–8
91	–3	–9
90	–3	–10
89	–3	–11
88	–3	–12
87	–3	–13
86	–3	–14
85	–3	–15
80	–3	–20
70	–3	–30
60	–3	–40

[a] Price at expiration – $100 – $3, Maximum loss = $3
[b] Price at expiration – $100

mirror image of the profit and loss profile of the long call position (the position of the call option buyer). That is, the profit of the short call position for any given price for Asset XYZ at the expiration date is the same as the loss of the long call position. Consequently, the maximum profit that the short call position can produce is the option price. The maximum potential loss is the highest price realized by Asset XYZ on or before the expiration date, less the option price; this price can be indefinitely high.

Buying Put Options

The buying of a put option creates a financial position referred to as a *long put position*. To illustrate this position, we assume a hypothetical put option on one unit of Asset XYZ with one month to maturity and a strike price of $100. Assume the put option is selling for $2. The current price of Asset XYZ is $100. The profit or loss for this position at the expiration date depends on the market price of Asset XYZ. The profit and loss profile for the long put position is shown in the second column of Exhibit 11.8.

As with all long option positions, the loss is limited to the option price. The profit potential, however, is substantial: the theoretical maximum profit is generated if Asset XYZ's price falls to zero. Contrast this profit potential with that of the buyer of a call option. The theoretical maximum profit for a call buyer cannot be determined beforehand because it depends on the highest price that can be reached by Asset XYZ before or at the option expiration date.

To see how an option alters the risk and return profile we again compare it to a position in Asset XYZ. The long put position is compared to taking a short position in Asset XYZ because this is the position that would realize a profit if the price of the asset falls. Suppose an investor sells Asset XYZ short for $100. Exhibit 11.8 compares the profit and loss profile for the long put position and short position in Asset XYZ.

While the investor who takes a short position in Asset XYZ faces all the downside risk as well as the upside potential, the long put position limits the downside risk to the option price while still maintaining upside potential (reduced only by an amount equal to the option price).

Writing (Selling) Put Options

Writing a put option creates a financial position referred to as a *short put position*. The profit and loss profile for a short put option is the mirror image of the long put option. The maximum profit from this position is the option price. The theoretical maximum loss can be substantial should the price of the underlying asset fall; at the outside, if the price were to fall all the way to zero, the loss would be as large as the strike price less the option price.

EXHIBIT 11.8 **Profit/Loss Profile for a Long Put Position and Comparison with a Short Asset Position**

Assumptions:	Price of Asset XYZ = $100 Option price = $2 Strike price = $100 Time to expiration = 1 month

Price of Asset XYZ	Net Profit/Loss for	
at Expiration Date	Long Put[a]	Short Asset XYZ[b]
$150	–$2	–$50
140	–2	–40
130	–2	–30
120	–2	–20
115	–2	–15
110	–2	–10
105	–2	–5
100	–2	0
99	–1	1
98	0	2
97	1	3
96	2	4
95	3	5
94	4	6
93	5	7
92	6	8
91	7	9
90	8	10
89	9	11
88	10	12
87	11	13
86	12	14
85	13	15
84	14	16
83	15	17
82	16	18
81	17	19
80	18	20
75	23	25
70	28	30
65	33	35
60	38	40

[a] $100 – Price at expiration – $2, Maximum loss = $2
[b] $100 – Price at expiration

To summarize, buying calls or selling puts allows the investor to gain if the price of the underlying asset rises. Selling calls and buying puts allows the investor to gain if the price of the underlying asset falls.

VALUATION OF OPTIONS

In this section we will look at how to value an option and discuss models for valuing futures options. In the next chapter, we will look at models for valuing options on physicals.

Basic Components of the Option Price

The option value is a reflection of the option's *intrinsic value* and any additional amount over its intrinsic value. The premium over intrinsic value is often referred to as the *time value*. The intrinsic value of an option is its economic value if it is exercised immediately. If no positive economic value would result from exercising the option immediately, then the intrinsic value is zero.

For a call option, the intrinsic value is positive if the current price of the underlying security is greater than the strike price. The intrinsic value is then the difference between the two prices. If the strike price of a call option is greater than or equal to the current price of the security, the intrinsic value is zero. For example, if the strike price for a call option is 100 and the current price for the security is 105, the intrinsic value is 5. That is, an option buyer exercising the option and simultaneously selling the underlying security would realize 105 from the sale of the security, which would be covered by acquiring the security from the option writer for 100, thereby netting a 5 gain.

When an option has intrinsic value, it is said to be *in the money*. When the strike price of a call option exceeds the current price of the security, the call option is said to be *out of the money*; it has no intrinsic value. An option for which the strike price is equal to the current price of the security is said to be *at the money*. Both at-the-money and out-of-the-money options have an intrinsic value of zero because they will not generate a positive payoff if exercised.

For a put option, the intrinsic value is equal to the amount by which the current price of the security is below the strike price. For example, if the strike price of a put option is 100 and the current price of the security is 92, the intrinsic value is 8. The buyer of the put option who exercises it and simultaneously buys the underlying security will net 8 by exercising this option since the security will be sold to the writer for 100 and purchased in the market for 92. The intrinsic value is zero if the strike price is less than or equal to the current market price.

EXHIBIT 11.9 Relationship Between Security Price, Strike Price, and Intrinsic Value

If Security Price > Strike Price	Call Option	Put Option
Intrinsic value	Security price – Strike price	Zero
Jargon	In-the-money	Out-of-the money
If Security Price < Strike Price	**Call Option**	**Put Option**
Intrinsic value	Zero	Security price – Stock price
Jargon	Out-of-the-money	In-the-money
If Security Price = Strike Price	**Call Option**	**Put Option**
Intrinsic value	Zero	Zero
Jargon	At-the-money	At-the-money

For our put option with a strike price of 100, the option would be: (1) in the money when the security's price is less than 100, (2) out of the money when the security's price exceeds 100, and (3) at the money when the security's price is equal to 100. These relations are summarized in Exhibit 11.9.

The time value of an option is the amount by which the option price exceeds its intrinsic value. The option buyer hopes that, at some time prior to expiration, changes in the market price of the underlying security will increase the value of the rights conveyed by the option. For this prospect, the option buyer is willing to pay a premium above the intrinsic value.

For example, if the price of a call option with a strike price of 100 is 9 when the current price of the security is 105, the time value of this option is 4 (9 minus its intrinsic value of 5). Had the current price of the security been 90 instead of 105, then the time value of this option would be the entire 9 because the intrinsic value is zero.

Factors that Influence the Value of an Option on a Fixed-Income Instrument

There are six factors that influence the value of an option in which the underlying is a fixed-income instrument:

1. Current price of the underlying security.
2. Strike price.
3. Time to expiration of the option.
4. Expected yield volatility over the life of the option.
5. Short-term risk-free interest rate over the life of the option.
6. Coupon interest payment over the life of the option.

EXHIBIT 11.10 Summary of Factors that Affect the Price of an Option on a Fixed-Income Instrument

Increase in Factor with all Other Factors Held Constant	Effect on Call Option	Effect on Put Option
Current price of underlying security	Increase	Decrease
Strike price	Decrease	Increase
Time to expiration (American options)	Increase	Increase
Expected yield volatility	Increase	Increase
Short-term risk-free rate	Increase	Decrease
Coupon interest payments	Decrease	Increase

The impact of each of these factors may depend on whether (1) the option is a call or a put, and (2) the option is an American option or a European option. A summary of the effect of each factor on put and call option prices is presented in Exhibit 11.10.

Current Price of the Underlying Security

The option price will change as the price of the underlying security changes. For a call option, as the price of the underlying security increases (holding all other factors constant), the option price increases. In other words, the option to buy the underlying instrument at a fixed price becomes more valuable as the underlying instrument's price increases. The opposite holds for a put option: As the price of the underlying security increases, the price of a put option decreases.

Strike Price

All other factors equal, the lower the strike price, the higher the price of a call option. Although for a particular option contract the strike price is fixed for the option's life, this relationship is apparent when comparing two call options on the same underlying asset that are alike in every aspect except their strike prices.

Time to Expiration

Holding all other factors equal, the longer the time to expiration, the more valuable the option. For a call option, the longer the time to expiration, there is more time for the underlying asset's price to rise above the exercise price. The higher the underlying asset price, the higher the call option's expected payoff. While this is certainly true, the converse is also true. Namely, the longer the time to expiration, there is also more time for the underlying asset's price to fall below the exercise price. Why then does more time to expiration make a call more rather than less valuable? This puzzle is easily resolved by recalling that the most an option buyer can lose if the underlying asset's price moves in an unfavorable direction is the price paid for the option. While there is certainly more time for the underlying asset's price to fall, this prospect does not affect the call buyer's loss. More time to expiration can only help the option buyer and not hurt. The reasoning is analogous for American puts—more time to expiration, holding other factors equal, makes puts more valuable.[1]

Expected Yield Volatility

Options feed off volatility. Other factors held equal, the greater the expected yield volatility (as measured by standard deviation), the more an investor would be willing to pay for the option and the more an option writer would demand for it. The intuition for this result is similar to the time to expiration explanation. If volatility increases, all else held constant, the greater to chance the underlying asset's price will move in favorable direction (i.e., up for the call buyer and down for the put buyer). However, does not volatility work in both directions meaning that the underlying asset's price can take on higher highs and lower lows? This is true of course but does not matter to an option buyer whose loss is fixed to the price paid for the option. Increases in volatility can only help and never hurt the option buyer.

Short-Term Risk-Free Interest Rate

Buying the underlying security ties up one's money. Buying an option on the same quantity of the underlying security makes the difference between the security price and the option price available for investment at the risk-free rate. All other factors constant, the higher the short-term risk-free interest rate, the greater the cost of buying the underlying security and carrying it to the expiration date of the call option. Hence, the higher the short-term risk-free interest rate, the more attractive the call option will be relative to the direct purchase of the underlying security. As a result, the higher the short-term risk-free interest rate, the greater the price of a call option. The reverse is true for a put option.

Coupon Payments

Coupon interest payments on the underlying security tend to decrease the price of a call option because they make it more attractive to hold

[1] The relationship between the value of a European put and time to expiration is not straightforward. In some cases, a European put with a longer time to expiration may be less valuable than an otherwise identical European put with a shorter time to expiration.

the underlying security than to hold the option. For put options, coupon interest payments on the underlying security tend to increase their price.

Factors that Influence the Value of a Futures Option

There are five factors that influence the value of an option in which the underlying is a futures contract:

1. Current futures price.
2. Strike price.
3. Time to expiration of the option.
4. Expected yield volatility over the life of the option.
5. Short-term risk-free interest rate over the life of the option.

These are the same factors that affect the value of an option on a fixed-income instrument. Notice that the coupon payment is not a factor since the underlying is a futures contract.

Exhibit 11.11 summarizes how each factor affects the value of a futures option. The primary difference between factors that influence the price of a futures option and an option on a fixed-income instrument is the short-term risk-free rate. For both a call and a put, the option price decreases when the short-term risk-free rate increases.

Option Pricing Model

At any time, the intrinsic value of an option can be determined. The question is, what is the time value of an option worth. To answer this question, option pricing models have been developed.

The most popular model for the pricing of equity options is the Black-Scholes option pricing model.[2] By imposing certain assumptions

EXHIBIT 11.11 **Summary of Factors that Affect the Price of a Futures Option**

Increase in Factor with All Other Factors Held Constant	Effect on Call Option	Effect on Put Option
Current futures price	Increase	Decrease
Strike price	Decrease	Increase
Time to expiration	Increase	Increase
Expected yield volatility	Increase	Increase
Short-term risk-free rate	Decrease	Decrease

[2] Fischer Black and Myron Scholes, "The Pricing of Corporate Liabilities," *Journal of Political Economy*, May–June 1973, pp. 637–659.

and using arbitrage arguments, the Black-Scholes option pricing model computes the fair (or theoretical) price of a European call option on a nondividend-paying stock.

There are problems with using the model to value an option on a fixed-income instrument and a futures option due to its underlying assumptions. The Black-Scholes model would price a call option on a zero-coupon bond with a strike price of $103 at some positive value. Such an option will always be worthless since the price of a zero-coupon bond will never exceed $100.

There are three assumptions underlying the Black-Scholes model that limit its use in pricing options on fixed-income instruments. First, the probability distribution for the prices assumed by the Black-Scholes option pricing model permits some probability—no matter how small—that the price can take on any positive value. But in the case of a zero-coupon bond, the price cannot take on a value above $100. In the case of a coupon bond, we know that the price cannot exceed the sum of the coupon payments plus the maturity value. For example, for a 5-year 10% coupon bond with a maturity value of $100, the price cannot be greater than $150 (five coupon payments of $10 plus the maturity value of $100). Thus, unlike stock prices, bond prices have a maximum value. The only way that a bond's price can exceed the maximum value is if negative interest rates are permitted. This is not likely to occur, so any probability distribution for prices assumed by an option pricing model that permits bond prices to be higher than the maximum bond value could generate nonsensical option prices. The Black-Scholes model does allow bond prices to exceed the maximum bond value (or, equivalently, allows negative interest rates).

The second assumption of the Black-Scholes option pricing model is that the short-term interest rate is constant over the life of the option. Yet the price of an interest rate option will change as interest rates change. A change in the short-term interest rate changes the rates along the yield curve. Therefore, to assume that the short-term rate will be constant is inappropriate for interest rate options. The third assumption is that the variance of prices is constant over the life of the option. As a bond moves closer to maturity its price volatility declines. Therefore, the assumption that price variance is constant over the life of the option is inappropriate.[3]

The more commonly used model for valuing futures options is the Black model.[4] The model was developed to value European options on

[3] While we have discussed the problems of using the Black-Scholes model to price interest rate options, it can also be shown that the binomial option pricing model based on the price distribution of the underlying bond suffers from the same problems.
[4] Fischer Black, "The Pricing of Commodity Contracts," *Journal of Financial Economics*, March 1976, pp. 161–179.

futures contracts. There are two problems with this model. First, the Black model does not overcome the problems cited earlier for the Black-Scholes model. Failing to recognize the yield curve means that there will not be a consistency between pricing bond futures and options on bond futures. Second, the Black model was developed for pricing European options on futures contracts. Futures options, however, are American options.

The second problem can be overcome. The Black model was extended by Barone-Adesi and Whaley to American options on futures contracts.[5] This is the model used by the CBT to settle the flexible Treasury futures options. However, this model was also developed for equities and is subject to the first problem noted above.

Sensitivity of Option Price to Change in Factors

The use options to control risk, a manager would like to know how sensitive the price of an option is to a change in every factor that affects its price. Here we look at the sensitivity of a call option's price to changes in the price of the underlying bond, the time to expiration, and expected yield volatility.

The Call Option Price and the Price of the Underlying Bond

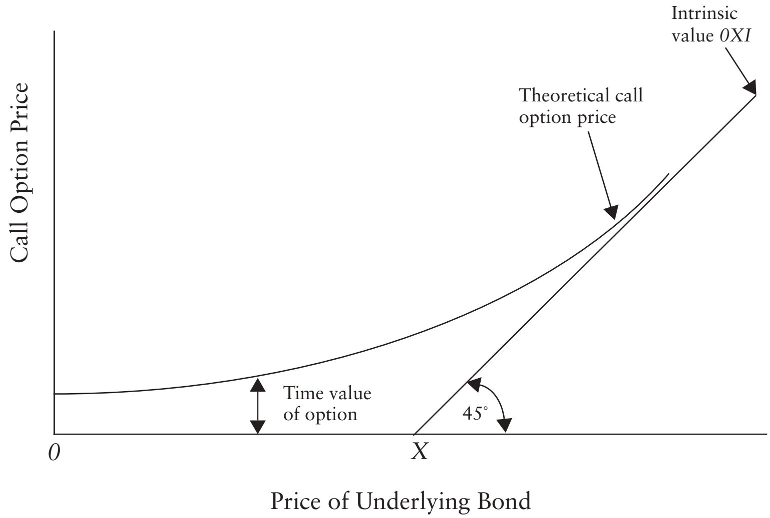
Exhibit 11.12 shows the theoretical price of a call option based on the price of the underlying bond. The horizontal axis is the price of the underlying bond at any point in time. The vertical axis is the call option price. The shape of the curve representing the theoretical price of a call option, given the price of the underlying bond, would be the same regardless of the actual option pricing model used. In particular, the relationship between the price of the underlying bond and the theoretical call option price is convex. Thus, option prices also exhibit convexity.

The line from the origin to the strike price on the horizontal axis in Exhibit 11.12 is the intrinsic value of the call option when the price of the underlying bond is less than the strike price, since the intrinsic value is zero. The 45-degree line extending from the horizontal axis is the intrinsic value of the call option once the price of the underlying bond exceeds the strike price. The reason is that the intrinsic value of the call option will increase by the same dollar amount as the increase in the price of the underlying bond.

For example, if the strike price is \$100 and the price of the underlying bond increases from \$100 to \$101, the intrinsic value will increase by \$1. If the price of the bond increases from \$101 to \$110, the intrinsic

[5] Giovanni Barone-Adesi and Robert E. Whaley, "Efficient Analytic Approximation of American Option Values," *Journal of Finance*, June 1987, pp. 301–320.

EXHIBIT 11.12 Theoretical Call Price and Price of Underlying Bond



X = Strike price

value of the option will increase from \$1 to \$10. Thus, the slope of the line representing the intrinsic value after the strike price is reached is 1.

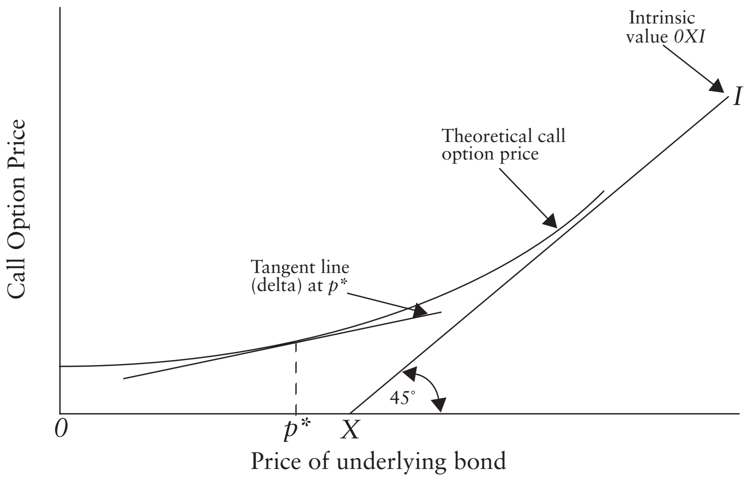
Since the theoretical call option price is shown by the convex curve, the difference between the theoretical call option price and the intrinsic value at any given price for the underlying bond is the time value of the option.

Exhibit 11.13 shows the theoretical call option price, but with a tangent line drawn at the price of p^*. The tangent line in the figure can be used to estimate what the new option price will be (and therefore what the change in the option price will be) if the price of the underlying bond changes. Because of the convexity of the relationship between the option price and the price of the underlying bond, the tangent line closely approximates the new option price for a small change in the price of the underlying bond. For large changes, however, the tangent line does not provide as good an approximation of the new option price because the curve line bends away from the straight line.

The slope of the tangent line shows how the theoretical call option price will change for small changes in the price of the underlying bond. The slope is popularly referred to as the *delta* of the option. Specifically,

$$\text{Delta} = \frac{\text{Change in price of call option}}{\text{Change in price of underlying bond}}$$

EXHIBIT 11.13 **Estimating the Theoretical Option Price with a Tangent Line**



X = Strike price

For example, a delta of 0.4 means that a $1 change in the price of the underlying bond will change the price of the call option by approximately $0.40.[6]

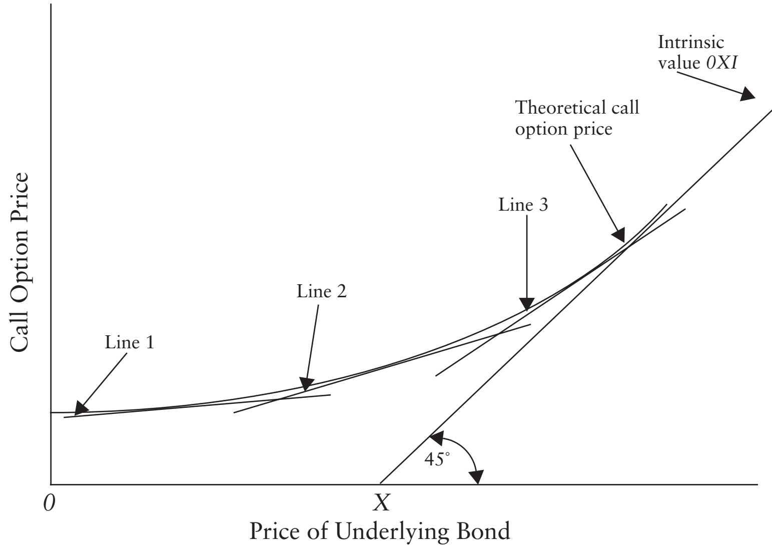
Exhibit 11.14 shows the curve of the theoretical call option price with three tangent lines drawn. The steeper the slope of the tangent line, the greater the delta. When an option is deep out of the money (that is, the price of the underlying bond is substantially below the strike price), the tangent line is nearly flat (see Line 1 in Exhibit 11.14). This means that delta is close to zero. To understand why, consider a call option with a strike price of $100 and two months to expiration. If the price of the underlying bond is $20, its price would not increase by much, if anything, should the price of the underlying bond increase by $1, from $20 to $21.

For a call option that is deep in the money, the delta will be close to one. That is, the call option price will increase almost dollar for dollar with an increase in the price of the underlying bond. In terms of Exhibit 11.14, the slope of the tangent line approaches the slope of the intrinsic value line after the strike price. As we stated earlier, the slope of that line is 1.

Thus, the delta for a call option varies from zero (for call options deep out of the money) to one (for call options deep in the money). The delta for a call option at the money is approximately 0.5.

[6] For readers who know calculus, delta is simply the partial derivative of the theoretical call price with respect to the underlying asset price.

EXHIBIT 11.14 Theoretical Option Price with Three Tangents



X = Strike price

Puts have negative deltas which mean simply that the theoretical value of a put falls as the value of the underlying asset increases. Accordingly, if we graphed the theoretical put value against the price of the underlying asset, the curve falls as we move from left to right on the graph. For deep-in-the-money puts, delta will approach –1. As the value of the underlying asset rises, the tangent line will become flatter and flatter and delta will approach zero.

The curvature of the convex relationship can also be approximated. This is the rate of change of delta as the price of the underlying bond changes. The measure is commonly referred to as *gamma* and is defined as follows:[7]

$$\text{Gamma} = \frac{\text{Change in delta}}{\text{Change in price of underlying bond}}$$

Gamma is highest for both calls and puts when options are at-the-money.

The Call Option Price and Time to Expiration

All other factors constant, the longer the time to expiration, the greater the option price. Since each day the option moves closer to the expira-

[7] Once again, for readers that know calculus, gamma is the second partial derivative of the option price with respect to the value of the underlying asset.

tion date, the time to expiration decreases. The *theta* of an option measures the change in the option price as the time to expiration decreases, or equivalently, it is a measure of *time decay*. Theta is measured as follows:

$$\text{Theta} = \frac{\text{Change in price of option}}{\text{Decrease in time to expiration}}$$

Assuming that the price of the underlying bond does not change (which means that the intrinsic value of the option does not change), theta measures how quickly the time value of the option changes as the option moves towards expiration.

An option's time value will also depend on the proximity of the underlying bond value with respect to the strike price. In particular, the time value will be low when the option is either deep in-the-money or deep out-of-the-money. This is true because in these cases there is relatively little uncertainty about whether the options will be exercised or not.

Buyers of options prefer a low theta so that the option price does not decline quickly as it moves toward the expiration date. An option writer benefits from an option that has a high theta.

The Call Option Price and Expected Yield Volatility

The vega of an option measures the impact of changes in volatility on the option's price.[8] Specifically, vega measures the dollar price change in the option's price and can be written as

$$\text{Vega} = \frac{\text{Change in option price}}{\text{1\% change in expected yield volatility}}$$

Vega is positive for both calls and puts which means increases the volatility of the underlying bond makes calls and puts worth more all else equal. Vega is highest when options are at-the-money.

Duration of an Option

The duration of an option measures the price sensitivity of the option to changes in interest rates and can be shown to be equal to:

[8] The volatility of the underlying asset is never known with certainty and so must be estimated. Moreover, it is impossible to predict with certainty how volatility of the underlying asset will change in the future.

$$\text{Duration for an option} = \text{Duration of underlying instrument} \times \text{Delta} \times \frac{\text{Price of underlying instrument}}{\text{Price of option}}$$

As expected, the duration of an option depends on the duration of the underlying bond. It also depends on the price responsiveness of the option to a change in the underlying instrument, as measured by the option's delta. The leverage created by a position in an option comes from the last ratio in the formula. The higher the price of the underlying instrument relative to the price of the option, the greater the leverage (i.e., the more exposure to interest rates for a given dollar investment).

It is the interaction of all three factors that affects the duration of an option. For example, a deep out-of-the-money option offers higher leverage than a deep-in-the-money option, but the delta of the former is less than that of the former.

Since the delta of call option is positive, the duration of an interest rate call option will be positive. Thus, when interest rates decline, the value of an interest rate call option will rise. A put option, however, has a delta that is negative. Thus, duration is negative. Consequently, when interest rates rise, the value of a put option rises.

Measuring the Sensitivity

Exhibit 11.15 presents Bloomberg Call Sensitivity Table screen (function COST) for November call options on the December 10-year U.S. Treasury note futures contract. At the time of the analysis, these options had four days until expiration on October 26, 2002. Let's examine the information presented on this screen. The first three columns indicate the type of option (call or put), the strike price, and the time of the last trade respectively. The next two columns labeled "Option" contains the most recent market price quoted in points and ticks (tick = $\frac{1}{64}$) and the implied volatility using the Black-Scholes option pricing model. The columns labeled "Hedge" contain the delta and the gamma for each option. As conjectured, when the strike price decreases and the call option gets deeper in-the-money, delta increases. Further, gamma is highest for the option that closest to being at-the-money (i.e., strike price = underlying asset price). The column labeled "Time Value" indicates the option's time value which is the difference between the option's price and its intrinsic value. The next column (4-Day Decay) tells us how much the option price will decline with the passage of time until the expiration date holding both the volatility and the price of the underlying asset remains constant. The option price change for a 1% change in the volatility of the underlying asset price is contained in the column labeled

EXHIBIT 11.15 **Bloomberg's Call Option Table for Call Options on a 10-Year Treasury Note Futures Contract**

GRAB Comdty COST
Option type: American Calc type: European Hit 1<GO> for calc defaults

10:21 CALL SENSITIVITY TABLE (TICKER)
Tue 10/22 Nov Options On US 10YR NOTE FUT Dec02
MARKET IS OPEN

TYZ2 112-05+ + 01 10:20 Hi 112-11+ Lo 111-30+

	STRIKE	Trade TIME	Option PRICE	Option I.VOL	Hedge DEL	Hedge GAM	Time VALUE	4 DAY DECAY	VEGA	I.VOL CHANGE OPTN + '01	I.VOL CHANGE FUTR + 00+
1) C	111	Mon Impd	1'21e	11.75%	.81	.200	.150	.1475	.0333	.48%	-.40%
2) C	112	10:00	'38	10.75%	.56	.313	.422	.3059	.0464	.34%	-.19%
3) C	113	09:50	'12	10.53%	.25	.259	.188	.1803	.0384	.41%	-.11%
4) C	114	09:11	'03	10.97%	.08	.116	.047	.0469	.0191	.83%	-.07%
5) C	115	09:27	'01	12.41%	.03	.044	.016	.0156	.0087	1.65%	-.06%
6) C	116		n/a	n/a							
7) C	117		n/a	n/a							
8) C	118		n/a	n/a							

4 days to Sat 10/26/02 option expiration(1.69 $ finance rate)

FOR ---> Option Horizon Analysis Historical Volatility Table
ENTER ---> OHT <GO> HVT <GO>

Source: Bloomberg Financial Markets

"Vega." Note that vega is also highest for the option that is closest to being at-the-money. The last two columns are labeled "I. Vol Change" which stands for "implied volatility change" for the option and the underlying futures contract. Namely, the change in the implied volatility for a one tick increase in the option price and the change in implied volatility for a one tick increase in the underlying futures price. Note a tick for both the option and the underlying futures contract is 1⁄64 but they are presented differently. Specifically, for the option price, 1⁄64 is presented as '01 and for the futures price it is presented as 00+.

KEY POINTS

1. An option is a contract in which the writer of the option grants the buyer the right, but not the obligation, to purchase from or sell to the writer something at a specified price within a specified period of time (or on a specified date).

2. The option buyer pays the option writer (seller) a fee, called the *option price*.

3. A call option allows the option buyer to purchase the underlying from the option writer at the strike price; a put option allows the option buyer to sell the underlying to the option writer at the strike price.

4. Interest rate options include options on fixed-income securities and options on interest rate futures contracts, called *futures options*.

5. There are exchange-traded options and over-the-counter options.

6. The only actively-traded exchange-traded options are futures options.

7. Futures options are usually American-type options.

8. The Chicago Board of Trade has introduced customized futures options called *flexible Treasury futures options*.

9. The value of an option is composed of its intrinsic value and its time value.

10. The six factors that affect the value of an option on a fixed-income instrument are the current price of the underlying security, the strike price, the time to expiration of the option, the expected yield volatility over the life of the option, the short-term risk-free interest rate over the life of the option, and the coupon interest payment over the life of the option.

11. With the exception of the coupon interest payment, the value of a futures option is affected by the same factors that affect an option on a fixed-income instrument.

12. With the exception of the short-term risk-free interest rate, how an option changes when one of the factors changes is the same for futures options and options on fixed-income instruments.

13. Several assumptions of the Black-Scholes model limit its use in pricing options on interest rate instruments and futures options.

14. The Black model is used for valuing futures options but is limited because it deals with European-type options.

15. The Black model was extended by Adesi-Barone and Whaley to futures options that are of the American type.

16. The Black model and the Adesi-Barone and Whaley model were originally developed for equities and as a result did not take into account the Treasury yield curve.

17. Failure to take into account the yield curve can result in an inconsistent valuation of bonds, bond futures, and futures options.

18. Managers need to know how sensitive an option's value is to changes in the factors that affect the value of an option.

19. The delta of an option measures how sensitive the option price is to changes in the price of the underlying bond and varies from zero (for call options deep out of the money) to one (for call options deep in the money).

20. The gamma of an option measures the rate of change of delta as the price of the underlying bond changes.

21. The theta of an option measures the change in the option price as the time to expiration decreases.

22. The vega of an option measures the dollar price change in the price of the option for a 1% change in expected yield volatility.

23. The duration of an interest rate option is a measure of its price sensitivity to small changes in interest rates and depends on the option's delta, the option's leverage, and the duration of the underlying bond.

CHAPTER 12

OTC Options and Related Products

As explained in the previous chapter, there are exchange-traded and over-the-counter (OTC) interest rate options. The product traded on exchanges is a futures option that we described in the previous chapter. In this chapter we focus on OTC options. We also look at option-related products. These include compound options, caps, and floors. With all of these products there is counterparty risk faced by the buyer of the option or option-related product.

> The objectives of this chapter are to:
> 1. Describe the different types of OTC options and how they can be structured.
> 2. Demonstrate how to value an option on a fixed-income security using the binomial model.
> 3. Explain how the binomial model can be extended to value futures options.
> 4. Describe what a compound option is.
> 5. Describe what a cap and a floor are and how they can be used to create a collar.
> 6. Demonstrate how caps and floors can be valued using the binomial model.

OVER-THE-COUNTER INTEREST RATE OPTIONS

OTC interest rate options are created by commercial banks and investment banks for their clients. Dealers can customize the expiration date, the underlying, and the type of exercise. For example, the underlying could be a specific fixed-income security or a spread between yields in two sectors of the fixed-income market.

In addition to American- and European-type options, an OTC option can be created in which the buyer may exercise prior to the expiration date but only on designated dates. Such options are referred to as *Bermuda* options. With an OTC option, the buyer need not pay the option price at the time of purchase. Instead, the option price can be paid at the expiration or exercise date. For such options, the option writer is exposed to counterparty risk in addition to the option buyer.

In the OTC option market there are plain vanilla and exotic options. Plain vanilla options are options on specific securities or on the spread between two sectors of the bond market. Exotic options have more complicated payoffs and we do not review these in this chapter.

Options on a Specific Security

Institutional investors who want to purchase an option on a specific Treasury security or a Ginnie Mae passthrough can do so on an over-the-counter basis. There are government and mortgage-backed securities dealers who make a market in options on specific securities. Over-the-counter (or dealer) options typically are purchased by institutional investors or mortgage bankers who want to hedge the risk associated with a specific security. Typically, the maturity of the option coincides with the time period over which the buyer of the option wants to hedge, so the buyer is usually not concerned with the option's liquidity.

A popular option used by mortgage originators for hedging forward delivery is an option on a specific mortgage-backed security (MBS). Typically, the underlying security is a TBA (pools to be arranged) agency passthrough security (Ginnie Mae, Fannie Mae, or Freddie Mac). The settlement process in the MBS market is forward delivery. The exercise of a mortgage option means the delivery of that security in the month specified in the option. Options are of the European type.

Spread Options

Some institutional investors may have exposure not only to the level of rates but the spread between two yields. It is difficult to hedge against spread risk with current exchange-traded options. As a result, several dealer firms have developed proprietary products for this purpose. These options can be structured with a payoff in one of the following ways should the option expire in the money. First, there could be a cash settlement based on the amount that the option expires in the money. Second, there could be an exchange of ownership of the two securities underlying the option. It is difficult to structure options with a settlement based on an exchange of securities, but there are institutional investors who desire this type of structure.[1]

Below we discuss two types of spread options—an option on the yield curve and an option on the spread between mortgage-backed securities (MBS) and Treasury securities.[2]

Yield Curve Spread Option

The reason for the popularity of yield curve spread options is that there are many institutional investors whose performance is affected by a change in the shape of the yield curve. We discussed yield curve risk in Chapter 4. As an example of a yield curve spread option, consider the Goldman Sachs' product called SYCURVE. This option represents the right to buy (in the case of a call option) or sell (in the case of a put option) specific segments of the yield curve. "Buying the curve" means buying the shorter maturity and selling the longer maturity; "selling the curve" means selling the shorter maturity and buying the longer maturity. The curve is defined by the spread between two specific maturities. They could be the 2-year/10-year spread, the 2-year/30-year spread, or the 10-year/30-year spread. The strike is quoted in basis points.

The yield spread is measured by the long maturity yield minus the short maturity yield. For a call option to be in the money at the expiration date, the yield spread must be positive; for a put option to be in the money at the expiration date, the yield spread must be negative. For example, a 25 basis point call option on the 2-year/10-year spread will be in the money at the expiration date if

$$\text{10-year yield} - \text{2-year yield} > \text{25 basis points}$$

A 35-basis-point put option on the 10-year/30-year spread will be in the money at the expiration date if

$$\text{30-year yield} - \text{10-year yield} < \text{35 basis points}$$

Yield curve options such as the SYCURVE are cash settlement contracts. In the case of the SYCURVE, if the option expires in the money, the option buyer receives \$0.01 per \$1 of notional amount, per in-the-money basis point at exercise. That is

[1] Goldman Sachs refers to such structures as dual exercise options (DUOPs).

[2] For a discussion of the various Goldman Sachs spread options, see Scott McDermott, "A Survey of Spread Options for Fixed-Income Investors," Chapter 4 in Frank J. Fabozzi (ed.), *The Handbook of Fixed-Income Options* (Burr Ridge, IL: Irwin Professional Publishing, 1996).

$$\text{Amount option expires in money (in bp)} \times \$0.01 \times \text{Notional amount}$$

For example, suppose that \$10 million notional amount of a 2-year/10-year call is purchased with a strike of 25 basis points. Suppose at the expiration date the yield spread is 33 basis points. Then the option expires 8 basis points in the money. The cash payment to the buyer of this option is

$$8 \times \$0.01 \times \$10{,}000{,}000 = \$800{,}000$$

From this amount, the option premium must be deducted.

MBS/Treasury Spread Option

Some institutional investors seek to control the spread risk between the yield on MBS and Treasuries. One example of an option on this spread is Goldman Sachs' MOTTO (mortgages over Treasury) option. The buyer of a MOTTO call option benefits if MBS outperform Treasuries; the buyer of a MOTTO put option benefits if Treasuries outperform MBS.

As noted earlier in discussing MBS options, the structuring of MOTTO options is complicated by the nuances of the MBS market. For the particular Treasury, the calculation of its yield at the expiration date is straightforward given its price at the expiration date. On the other hand, at the expiration date, while the market price of a generic agency MBS with a given coupon rate is known, its yield is not uniquely determined. The yield depends on the prepayment assumption that determines the particular security's cash flow. This yield is called the *cash flow yield* and the prepayment assumption is commonly called the *prepayment speed*. Each MBS dealer has a proprietary prepayment model to project the speed. One important factor in a prepayment model is the yield level relative to the coupon rate paid on the mortgages in the underlying mortgage pool. Thus, the yield on an MBS depends on the prepayment speed that, in turn, depends on the yield level.

One possible way to handle this problem is to specify at the outset of the option the prepayment speed that should be used to determine the yield on an MBS given the Treasury yield at the expiration date. Specifically, the higher the Treasury yield, the lower the prepayment speed. However, it is not only the yield level but also the shape of the yield curve that affects the prepayment speed. Structuring a MOTTO such that the prepayment speed for all possible combinations of yield curves and yield levels would be difficult. Consequently, a MOTTO is structured so that an in-the-money option at the expiration date is settled by the exchange of the two underlying securities.

Valuation of Options on Fixed-Income Securities

The proper way to value options on a fixed-income security is to use an arbitrage-free model that takes into account the yield curve. In Chapter 2, an arbitrage-free binomial model was introduced and used to value a fixed-income security. The same model can be used to value an option on a fixed-income security. Thus, there will be consistency in the pricing of cash market instruments and options on those instruments. A popular model employed by dealer firms is the Black-Derman-Toy model.[3]

We have already developed the basic principles for employing this model. In Chapter 2, we explained how to construct a binomial interest rate tree such that the tree would be arbitrage free. We used the interest rate tree to value bonds (both option-free and bonds with embedded options). But the same tree can be used to value a stand-alone option on a bond.

Valuing a Treasury Call Option

To illustrate how this is done, let's consider a 2-year call option on a 6.5% 4-year Treasury bond with a strike price of 100.25. We will assume that the yield for the on-the-run Treasuries is the one in Chapter 2 and that the volatility assumption is 10% per year. Exhibit 2.14 in Chapter 2 repeated here as Exhibit 12.1 shows the binomial interest rate tree along with the value of the Treasury bond at each node.

It is a portion of Exhibit 12.1 that we use to value the call option. Specifically, Exhibit 12.2 shows the value of our Treasury bond (excluding coupon interest) at each node at the end of year 2. There are three values shown: 97.925, 100.418, and 102.534. Given these three values, the value of a call option struck at 100.25 can be determined at each node. For example, if at the end of year 2 the price of this Treasury bond is 97.925, then since the strike price is 100.25, the value of the call option would be zero. In the other two cases, since the price at the end of year 2 is greater than the strike price, the value of the call option is the difference between the price of the bond and 100.25.

Exhibit 12.2 shows the value of the call option at the end of year 2 (the option expiration date) for each of the three nodes. (The values are shown to four decimal places.) Given these values, the binomial interest rate tree is used to find the present value of the call option. The backward induction procedure is used. The discount rates are those from the binomial interest rate tree. For years 0 and 1, the discount rate is the second number shown at each node. The first number at each node for year 1 is the average present value found by discounting the call option

[3] Fischer Black, Emanuel Derman, and William Toy, "A One-Factor Model of Interest Rates and Its Application to Treasury Bond Options," *Financial Analysts Journal,* January–February 1990, pp. 24–32.

EXHIBIT 12.1 Valuing a Treasury Bond with Four Years to Maturity and a Coupon Rate of 6.5% (10% Volatility Assumed)

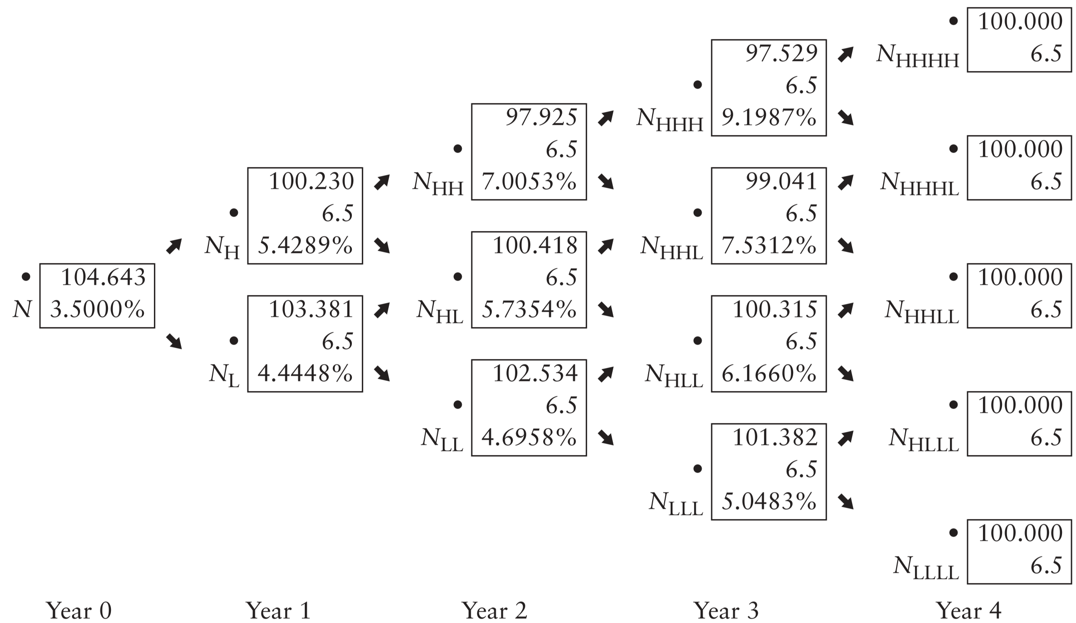
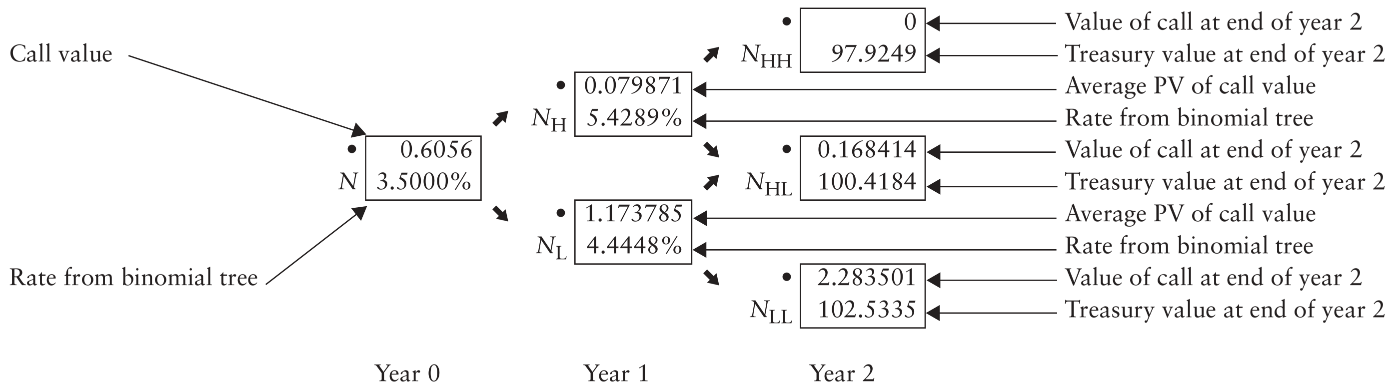


EXHIBIT 12.2 **Valuing a European Call Option Using the Arbitrage-Free Binomial Method**
Expiration: 2 years; Strike price: 100.25; Current price: 104.643; Volatility assumption: 10%



value of the two nodes to the right using the discount rate at the node. The value of the option is the first number shown at the root, 0.6056.

Valuing a Treasury Put Option

The same procedure is used to value a put option. This is illustrated in Exhibit 12.3 assuming that the put option has two years to expiration and that the strike price is 100.25. The value of the put option at the end of year 2 is shown at each of the three nodes.

Put-Call Parity Relationship

There is a relationship between the price of a call option and the price of a put option on the same underlying instrument, with the same strike price and the same expiration date. This relationship is commonly referred to as the *put-call parity relationship*. For European options on coupon bearing bonds, the relationship is

$$\begin{aligned}\text{Put price} = {} & \text{Call price} + \text{Present value of strike price} \\ & + \text{Present value of coupon payments} \\ & - \text{Price of underlying bond}\end{aligned}$$

To demonstrate that the arbitrage-free binomial model satisfies the put-call parity relationship for European options, let's use the values from our illustration. We just found that the put price is 0.6056 and the call price is 0.5327. In Chapter 2, we showed that the theoretical price for the 6.5% 4-year option-free bond is 104.643. In the same chapter we showed the spot rates for each year. The spot rate for year 2 is 4.2147%. Therefore,

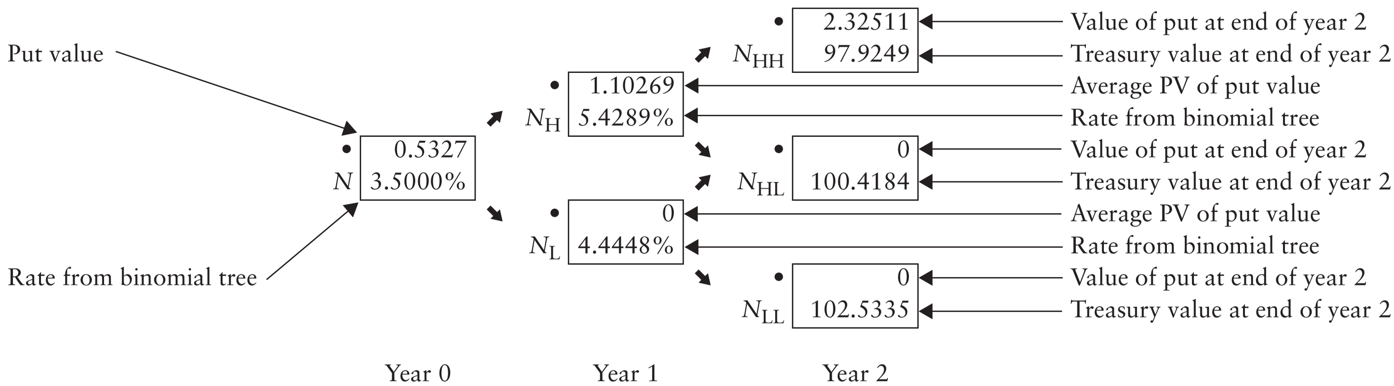
$$\text{Present value of strike price} = \frac{100.25}{(1.042147)^2} = 92.3053$$

The present value of the coupon payments are found by discounting the two coupon payments of 6.5 by the spot rates. As just noted, the spot rate for year 2 is 4.2147%; the spot rate for year 1 is 3.5%. Therefore,

$$\text{Present value of coupon payments} = \frac{6.5}{(1.035)^1} + \frac{6.5}{(1.042147)^2} = 12.2650$$

Substituting the values into the right-hand side of the put-parity relationship we find

EXHIBIT 12.3 Valuing a European Put Option Using the Arbitrage-Free Binomial Method
Expiration: 2 years; Strike price: 100.25; Current price: 104.643; Volatility assumption: 10%



$$0.6056 + 92.3053 + 12.2650 - 104.643 = 0.5319$$

The put value that we found is 0.5327. The discrepancy is due simply to rounding error. Therefore, put-call parity holds.

Extension to Futures Options

The binomial model can be extended to value futures options. For each node at the expiration date of the futures option, a yield is given. Given the acceptable issues that can be delivered, the conversion factors, and the yield at the expiration date of the futures option, the cheapest-to-deliver Treasury issue can be determined at each node. Therefore at each node at the expiration date of the futures option, there is a cheapest-to-deliver Treasury issue and a value for that issue. From the value of the cheapest-to-deliver Treasury issue and its conversion factor, the value of the underlying Treasury bond futures can be determined.

Based on the strike price, the value of the option at each node at the expiration date of the futures option can be determined. The backward induction method is then used to determine the value of the futures option.

The binomial model allows the consistent valuation of Treasury bonds, Treasury bond futures, and options on Treasury bond futures.

COMPOUND OPTIONS

A *compound* or *split-fee option* is an option to purchase an option. We can explain the elements of a compound option by using a long call option on a long put option. This compound option gives the buyer of the option the right but not the obligation to require the writer of the compound option to sell the buyer a put option. The compound option would specify the following terms:

1. The day on which the buyer of the compound option has the choice of either requiring the writer of the option to sell the buyer a put option or allowing the option to expire. This date is called the *extension date*.
2. The strike price and the expiration date of the put option that the buyer acquires from the writer. The expiration date of the put option is called the *notification date*.

The payment that the buyer makes to acquire the compound option is called the *front fee*. If the buyer exercises the call option in order to acquire the put option, a second payment is made to the writer of the option. That payment is called the *back fee*.

An option that allows the option buyer to purchase a put option is called a *caput*. A *cacall* grants the option buyer the right to purchase a call option.

Compound options are most commonly used by mortgage originators to hedge pipeline risk. They can also be used in any situation when a manager needs additional time to gather information about the need to purchase an option.

CAPS AND FLOORS

An important option combination in debt markets is the *cap* and *floor*, which are used to control interest-rate risk exposure. Caps and floors are combinations of the same types of options (calls or puts) with identical strike prices but arranged to run over a range of time periods. In an earlier chapter, we reviewed the main instruments used to control interest-rate risk, including short-dated interest-rate futures and FRAs. For example, a corporation that desires to protect against a rise in future borrowing costs could buy FRAs or sell futures. These instruments allow the user to lock in the forward interest rate available today. However, such positions do not allow the hedger to gain if market rates actually move as feared/anticipated. Hedging with FRAs or futures can prevent loss but at the expense of any extra gain. To overcome this, the hedger might choose to construct the hedge using options. For interest rate hedges, primary instruments are the cap and floor.[4]

Caps and floors are agreements between two parties, whereby one party for an upfront fee agrees to compensate the other if a designated interest rate (called the *reference rate*) is different from a predetermined level. The party that benefits, if the reference rate differs from a perdetermined level, is called the *buyer,* and the party that must potentially make payments is called the *seller.* The predetermined interest rate level is called the *strike rate*. An interest rate cap specifies that the seller agrees to pay the buyer if the reference rate exceeds the strike rate. An interest rate floor specifies that the seller agrees to pay the buyer if the reference rate is below the strike rate.

The terms of an interest rate agreement include: (1) the reference rate; (2) the strike rate that sets the cap or floor; (3) the length of the agreement; (4) the frequency of reset; and (5) the notional amount (which determines the size of the payments). If a cap or a floor are in-the-money on the reset date, the payment by the seller is typically made in arrears.

[4] The term *cap and floor* is not to be confused with floating-rate note products that have caps and/or floors which restrict how much a floater's coupon rate can float.

Some commercial banks and investment banks now write options on interest rate caps and floors for customers. Options on caps are called *captions*. Options on floors are called *flotions*.

Caps

A cap is essentially a strip of options. A borrower with an existing interest-rate liability can protect against a rise in interest rates by purchasing a cap. If rates rise above the cap, the borrower will be compensated by the cap payout. Conversely, if rates fall the borrower gains from lower funding costs and the only expense is the upfront premium paid to purchase the cap. The payoff for the cap buyer at a reset date if the value of the reference rate exceeds the cap rate on that date is as follows:

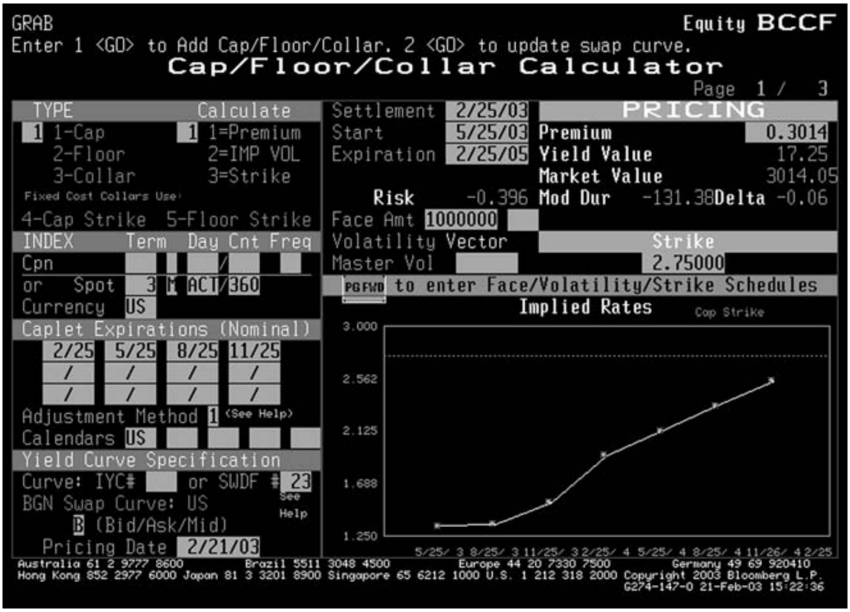
Notional amount × (Value of the reference rate – Cap rate)
× (Number of days in settlement period/Number of days in year)

Naturally, if the reference rate is below the cap rate, the payoff is zero.

A cap is composed of a series of individual options or caplets. The price of a cap is obtained by pricing each of the caplets individually. Each caplet has a strike interest rate that is the rate of the cap. For example, a borrower might purchase a 3% cap (Libor reference rate), which means that if rates rise above 3% the cap will pay out the difference between the cap rate and the actual Libor rate. A 1-year cap might be composed of a strip of three individual caplets, each providing protection for successive 3-month periods. The first 3-month period in the 1-year term is usually not covered, because the interest rate for that period, as it begins immediately, will be known already. A caplet runs over two periods, the exposure period and the protection period. The exposure period runs from the date the cap is purchased to the interest reset date for the next borrowing period. At this point, the protection period begins and runs to the expiration of the caplet. The protection period is usually three months, six months or one year, and will be set to the interest rate reset liability that the borrower wishes to hedge. Therefore, the protection period is usually identical for all the caplets in a cap.

As an illustration, let's utilize Bloomberg's Cap, Floor, Collar Calculator presented in Exhibit 12.4. Consider a hypothetical 2-year cap on 3-month Libor with a strike rate of 2.75%. The settlement date for the agreement is February 25, 2003 and the expiration date is February 25, 2005. The first reset date is May 25, 2003, which is labeled "Start" in the top center of the screen. If 3-month Libor is above the strike rate on this date, say, 3.25%, the payoff of the cap assuming the notional principal is $1,000,000 is computed as follows:

EXHIBIT 12.4 Bloomberg's Cap/Floor/Collar Calculator



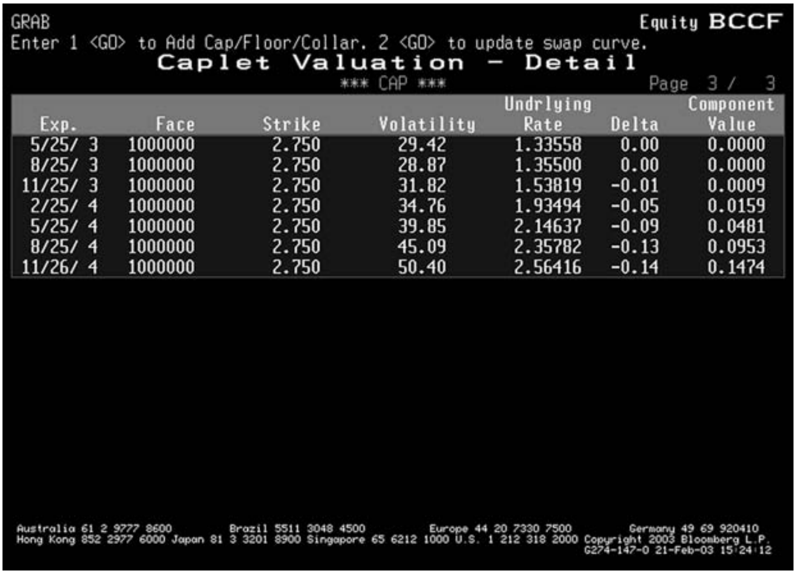
Source: Bloomberg Financial Markets

$$\$1{,}000{,}000 \times (3.25\% - 2.75\%) \times 92/369 = \$1{,}277.78$$

This payment is made on August 25, 2003. Note that the day count convention is Actual/360 in the U.S. markets and Actual/365 in the U.K. The second reset date is August 25, 2003 for which payment is made, if necessary, on November 25, 2003.

As noted above, each cap can be thought of a series of call options or caplets on the underlying reference rate in this case, 3-month Libor. The first caplet expires on the next reset date, May 25, 2003, the second caplet expires on August 28, 2003, and so forth. Accordingly, the value of the cap is the sum of the values of all the caplets. In the "PRICING" box, the "Premium" represents the value of our hypothetical cap as a percentage of the notional amount. For our hypothetical cap, the premium is 0.3014% or approximately \$3,014 when the notional principal is \$1,000,000. Exhibit 12.5 presents Bloomberg's Caplet Valuation screen that shows the value of each caplet in the column labeled "Component Value." Bloomberg uses a modified Black-Scholes model to value each caplet and users can choose whether to use the same volatility estimate

EXHIBIT 12.5 Bloomberg Screen with the Valuation of a Hypothetical Cap


GRAB Equity BCCF
Enter 1 <GO> to Add Cap/Floor/Collar. 2 <GO> to update swap curve.
Caplet Valuation – Detail
*** CAP *** Page 3 / 3

Exp.	Face	Strike	Volatility	Undrlying Rate	Delta	Component Value
5/25/ 3	1000000	2.750	29.42	1.33558	0.00	0.0000
8/25/ 3	1000000	2.750	28.87	1.35500	0.00	0.0000
11/25/ 3	1000000	2.750	31.82	1.53819	-0.01	0.0009
2/25/ 4	1000000	2.750	34.76	1.93494	-0.05	0.0159
5/25/ 4	1000000	2.750	39.85	2.14637	-0.09	0.0481
8/25/ 4	1000000	2.750	45.09	2.35782	-0.13	0.0953
11/26/ 4	1000000	2.750	50.40	2.56416	-0.14	0.1474

Australia 61 2 9777 8600 Brazil 5511 3048 4500 Europe 44 20 7330 7500 Germany 49 69 920410
Hong Kong 852 2977 6000 Japan 81 3 3201 8900 Singapore 65 6212 1000 U.S. 1 212 318 2000 Copyright 2003 Bloomberg L.P.
G274-147-0 21-Feb-03 15:24:12

Source: Bloomberg Financial Markets

for each caplet or allow the volatility for each caplet to differ. Binomial lattice models are also extensively in practice to value caps.

Floors

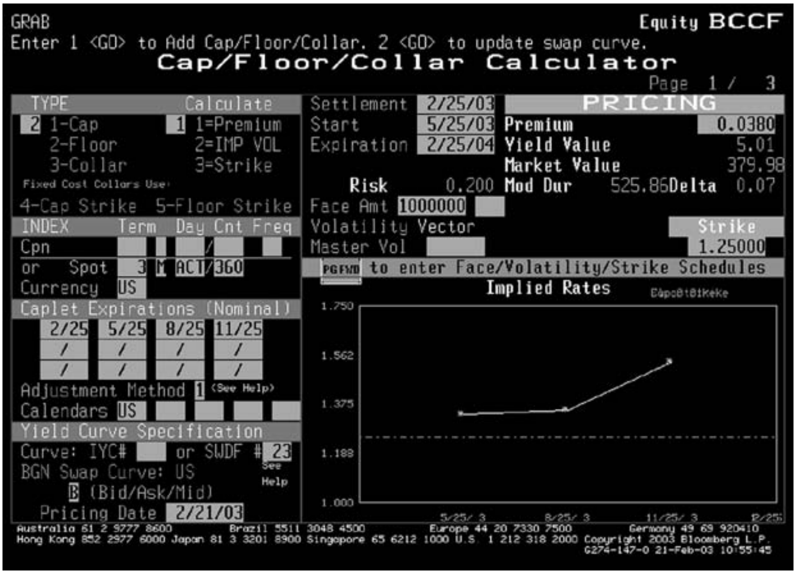
It is possible to protect against a drop in interest rates by purchasing a floor. This is exactly opposite of a cap in that a floor pay outs when the reference rate falls below the strike rate. This would be used by an institution that wished to protect against a fall in income caused by a fall in interest rate—for example, a commercial bank with a large proportion of floating-rate assets. For the floor buyer, the payoff at a reset date is as follows if the value of the reference rate at the reset date is less than the floor rate:

Notional amount × (Floor rate – Value of the reference rate)
× (Number of days in settlement period/Number of days in a year)

The floor's payoff is zero if the reference rate is higher than the floor rate.

To illustrate, let's once again utilize Bloomberg's Cap, Floor, Collar Calculator presented in Exhibit 12.6. Consider a hypothetical one-year floor on 3-month Libor with a strike rate of 1.25%. The settlement date for the agreement is February 25, 2003 and the expiration date is Febru-

EXHIBIT 12.6 Bloomberg's Cap/Floor/Collar Calculator



Source: Bloomberg Financial Markets

ary 25, 2004. If 3-month Libor is below the strike rate on this date, say, 1%, the payoff of the floor assuming the notional amount is $1,000,000 is computed as follows:

$$\$1{,}000{,}000 \times (1.25\% - 1.0\%) \times 92/360 = \$638.89$$

This payment is made on May 25, 2003.

A floor can be thought of as a series of put options on the underlying reference rate in this case, 3-month Libor. The value of the floor is the sum of the values of all the individual put options. In the "PRICING" box, the "Premium" for our hypothetical floor, the premium is 0.0380% or approximately $379.98.

Collars

The combination of a cap and a floor creates a collar, which is a corridor that fixes interest payment or receipt levels. A collar is sometimes advantageous for borrowers because it is a lower cost than a straight cap. A collar protects against a rise in rates, and provides some gain if there is a fall down to the floor rate. The cheapest structure is a collar with a narrow spread between cap and floor rates.

Risk and Return Characteristics

In an interest rate cap and floor, the buyer pays an upfront fee, which represents the maximum amount that the buyer can lose and the maximum amount that the seller of the agreement can gain. The only party that is required to perform is the seller of the interest rate agreement. The buyer of an interest rate cap benefits if the reference rate rises above the strike rate because the seller must compensate the buyer. The buyer of an interest rate floor benefits if the reference rate falls below the strike rate because the seller must compensate the buyer.

How can we better understand interest rate caps and interest rate floors? In essence these contracts are equivalent to a package of interest rate options. As with a swap, a complex contract can be seen to be a package of basic contracts—options in the case of caps and floors.

The question is what type of package of options is a cap and a floor. Recall from Chapter 10 when we discussed the relationship between forwards and swaps, that the relationship depends whether the underlying is a rate or a fixed-income instrument. The same applies to call options, put options, caps, and floors.

If the underlying is considered a fixed-income instrument, its value changes inversely with interest rates. Therefore,

- For a call option on a fixed-income instrument:
  1. Interest rates increase → Fixed-income instrument's price decreases → Call option value decreases
  2. Interest rates decrease → Fixed-income instrument's price increases → Call option value increases

- For a put option on a fixed-income instrument
  1. Interest rates increase → Fixed-income instrument's price decreases → Put option value increases
  2. Interest rates decrease → Fixed-income instrument's price increases → Put option value decreases

To summarize:

	When interest rates	
Value of:	**Increase**	**Decrease**
Long call	Decrease	Increase
Short call	Increase	Decrease
Long put	Increase	Decrease
Short put	Decrease	Increase

For a cap and floor, the situation is as follows:

	When interest rates	
Value of:	**Increase**	**Decrease**
Short cap	Decrease	Increase
Long cap	Increase	Decrease
Short floor	Increase	Decrease
Long floor	Decrease	Increase

Therefore, buying a cap (long cap) is equivalent to buying a package of puts on a fixed-income instrument and buying a floor (long floor) is equivalent to buying a package of calls on a fixed-income instrument.

On the other hand, if the underlying is viewed as an option on an interest rate, then buying a cap (long cap) is equivalent to buying a package of calls on interest rates. Buying a floor (long floor) is equivalent to buying a package of puts on interest rates.

Valuing a Cap and Floor

The binomial method can also be used to value a cap and a floor. Remember that a cap and a floor are nothing more than a package of options. More specifically, they are a package of European options on interest rates. Thus, to value a cap, the value of each period's cap is found and all the period caps are then summed. The same can be done for a floor.

To illustrate how this is done, we will once again use the binomial tree given in Exhibit 2.13 of Chapter 2. Consider first a 5.2% 3-year cap with a notional principal amount of \$10 million. The reference rate is the 1-year rate in the binomial tree. The payoff for the cap is annual.

Exhibits 12.7a, 12.7b, and 12.7c show how this cap is valued by valuing the cap for each year individually. The value for the cap for any year, say year *X*, is found as follows. First, calculate the payoff in year *X* at each node as either:

1. Zero if the 1-year rate at the node is less than or equal to 5.2%; or
2. The notional principal amount of \$10 million times the difference between the 1-year rate at the node and 5.2% if the 1-year rate at the node is greater than 5.2%.

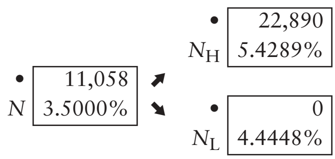
Mathematically, this is expressed as follows:

$$\$10{,}000{,}000 \times \text{Maximum}\ [(\text{Rate at node} - 5.2\%), 0]$$

EXHIBIT 12.7 **Valuation of a 3-Year 5.2% Cap (10% Volatility Assumed) by Valuing Each Year's Cap**

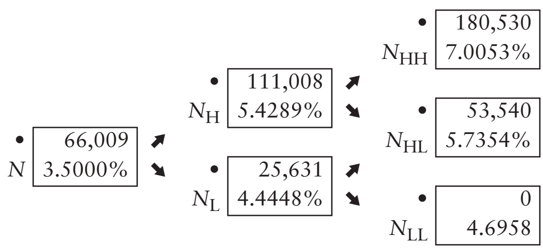
Assumptions:
Cap rate: 5.2%
Notional principal amount: $10,000,000
Payment frequency: Annual

Panel A: The Value of the Year 1 Cap



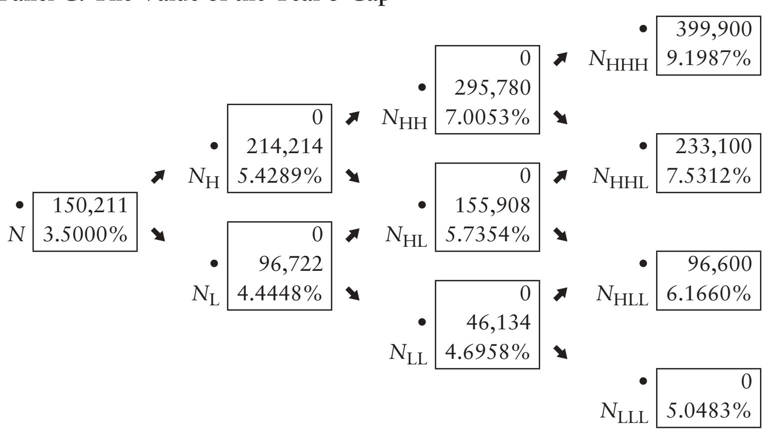
Value of Year 1 cap = $11,058

Panel B: The Value of the Year 2 Cap



Value of Year 2 cap = $66,009

Panel C: The Value of the Year 3 Cap



Value of Year 3 cap = $150,211

Summary: Value of 3-Year Cap = $150,211 + $66,009 + $11,058 = $227,278
Note on calculations: Payoff in last box of each exhibit is

$$\$10{,}000{,}000 \times \text{Maximum}[(\text{Rate at node} - 5.2\%),0]$$

Then the backward induction method is used to determine the value of the year *X* cap.

For example, consider the year 3 cap. At the top node in year 3 of Exhibit 12.7c, the 1-year rate is 9.1990%. Since the 1-year rate at this node exceeds 5.2%, the payoff in year 3 is

$$\$10{,}000{,}000 \times (9.1990\% - 5.2\%) = \$399{,}212$$

Using the backward induction method, the value of the year 3 cap is \$150,211. Following the same procedure, the value of the year 2 cap is \$66,009, and the value of the year 1 cap is \$11,058. The value of the cap is then the sum of the cap for each of the three years. Thus, the value of the cap is \$227,278, found by adding \$150,211, \$66,009, and \$11,058.

An alternative procedure is to calculate the value of the cap as follows:

Step 1: For each year, determine the payoff of the cap at each node based on the reference rate at the node. Mathematically, the payoff is:

$$\text{Notional principal amount} \times \text{Maximum } [(\text{Rate at node} - \text{cap rate}), 0]$$

Step 2: At each node one period prior to the maturity of the cap, the value of the cap at a node is found as follows:

$$\frac{\text{Average of the value at two nodes in next period}}{1 + \text{Rate at node}} + \text{Value found in Step 1}$$

Step 3: Use the backward induction method to determine the value of the cap in year 0.

This is illustrated in Exhibit 12.8. Notice that the value of the 3-year cap is \$227,278, the same value as found earlier.

The value of a floor can be found using the same three-step procedure. However, in Step 1, the payoff is

$$\text{Notional principal amount} \times \text{Maximum } [(\text{Floor rate} - \text{Rate at node}), 0]$$

Exhibit 12.9 illustrates the calculation of a 3-year floor with a strike rate of 4.8% and a \$10 million notional principal amount. The value of the floor is \$19,569.

EXHIBIT 12.8 Valuation of a 3-Year 5.2% Cap (10% Volatility Assumed)

Assumptions:
Cap rate: 5.2%
Notional principal amount: $10,000,000
Payment frequency: Annual

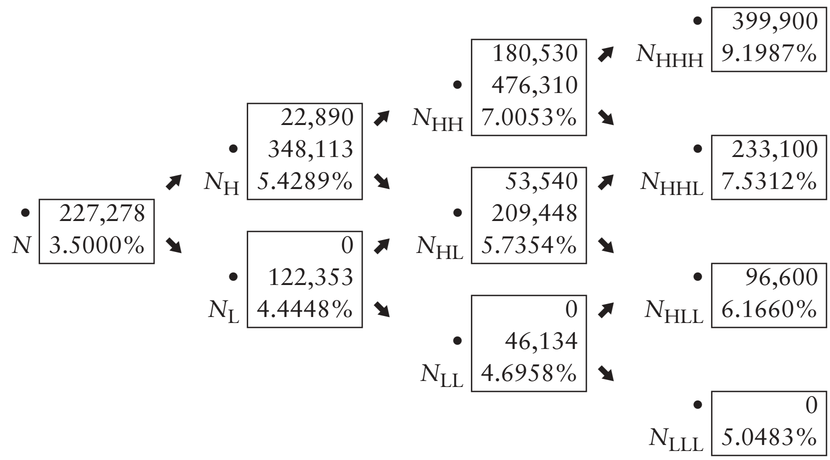
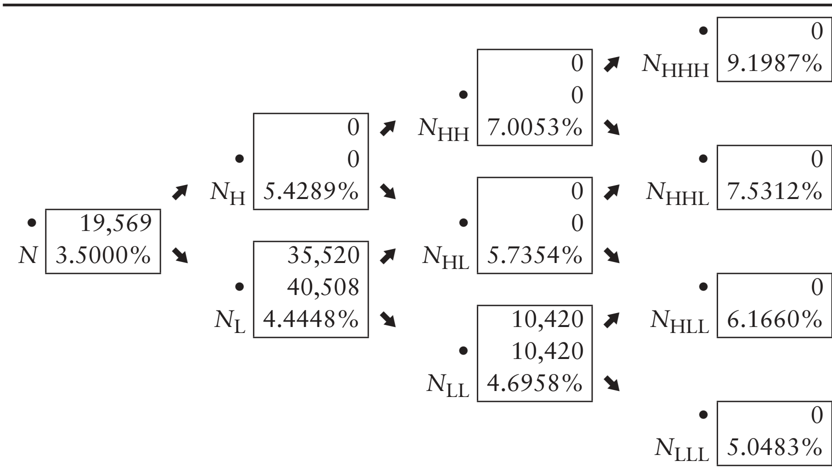


EXHIBIT 12.9 Valuation of a 3-Year 4.8% Floor (10% Volatility Assumed)

Assumptions:
Cap rate: 4.8%
Notional principal amount: $10,000,000
Payment frequency: Annual



KEY POINTS

1. OTC interest rate options are customized by dealers for their clients in terms of the expiration date, the underlying, and the type of exercise.
2. An OTC option can be created in which the buyer may exercise prior to the expiration date but only on designated dates (so called modified American or Atlantic or Bermuda options).
3. An OTC option can be created whereby the buyer pays the premium at the expiration date.
4. There are OTC options on specific securities.
5. There are OTC options on the spread between two yields.
6. Spread options can be structured with a payoff that is either cash settled or requires an exchange of ownership of the two securities underlying the option.
7. Two common spread options are options on the yield curve and options on the spread between mortgages and Treasuries.
8. The arbitrage-free binomial model is the proper model to value options on fixed-income securities since it takes into account the yield curve.
9. The arbitrage-free binomial model allows for the consistent pricing of Treasury bonds, Treasury bond futures, and options on Treasury bonds.
10. The put-call parity relationship is the pricing relationship between the price of a call option and the price of a put option on the same underlying instrument, with the same strike price and the same expiration date.
11. The put-call parity relationship is satisfied by the binomial model.
12. A compound option (also called a split-fee option) is an option to purchase an option.

13. The front fee for a compound option is the initial payment that the buyer makes.

14. The back fee for a compound option is the fee paid by the buyer if the option is exercised.

15. An interest rate cap is an agreement whereby the seller agrees to pay the buyer if the reference rate exceeds the strike rate.

16. An interest rate floor is an agreement whereby the seller agrees to pay the buyer if the reference rate is below the strike rate.

17. The terms of a cap and floor set forth the reference rate, the strike rate, the length of the agreement, the frequency of reset, and the notional principal amount.

18. An interest rate collar can be created by buying an interest rate cap and selling an interest rate floor.

19. In an interest rate cap and floor, the buyer pays an upfront fee, which represents the maximum amount that the buyer can lose and the maximum amount that the seller of the agreement can gain.

20. Buying a cap is equivalent to buying a package of puts on a fixed-income security and buying a floor is equivalent to buying a package of calls on a fixed-income security.

21. If an option is viewed as one in which the underlying is an interest rate, then buying a cap is equivalent to buying a package of calls on interest rates and buying a floor is equivalent to buying a package of puts on interest rates.

22. The binomial method can be used to value a cap or a floor by valuing the cap or floor for each period and then summing these values.

CHAPTER 13

Controlling Interest Rate Risk with Derivatives

In this chapter we look at how to control interest rate risk with derivative instruments. As explained in Chapters 3 and 4, interest rate risk includes level risk and yield curve risk. A risk control strategy can be employed to control the interest rate risk of a portfolio without regard to the price movement of any individual bond comprising the portfolio. This type of risk control strategy is called a *macro strategy*. Alternatively, a risk control strategy can be implemented to control the risk of an individual bond or a group of bonds with similar characteristics. This type of risk control strategy is called a *micro strategy*. With a micro strategy, there may be considerably less exposure to yield curve risk.

The objectives of this chapter are to:

1. Describe the preliminary steps in any risk control strategy.
2. Explain the basic principles of controlling risk with interest rate futures.
3. Explain what hedging and cross hedging are.
4. Explain the basis risk associated with hedging.
5. Demonstrate how interest rate futures can be used to hedge.
6. Explain the basic principles of three common hedge strategies employing options—the protective put buying strategy, covered call writing strategy, and collar strategy.
7. Illustrate the complexities associated with hedging with futures options.
8. Illustrate how an interest rate swap and a swaption can be used to alter the risk exposure of a position.
9. Explain what an asset swap is.
10. Explain how caps and floors can be used.

In this chapter, our illustration will involve micro strategies using derivative instruments. In the next chapter, we will look at how several derivative instruments can be used in combination to control the level risk and yield curve risk of a complex mortgage-backed securities portfolio.

PRELIMINARY STEPS IN ANY RISK CONTROL STRATEGY

There are four preliminary steps that a risk manager or portfolio manager should take before implementing any strategy to control interest rate risk:

1. Determine which instruments are the most appropriate to employ to control risk.
2. Determine the objectives of the strategy.
3. Determine the position that should be taken in a risk control instrument.
4. Assess the potential outcome of the risk control strategy.

These steps are essential for two reasons. First, by taking these steps, the manager can assess what a risk control strategy can and cannot accomplish. Second, the steps ensure that if the risk control strategy is employed, it is set up in the proper way.

Determining Which Instruments Are the Most Appropriate to Employ

To control the interest rate risk of a position or portfolio, a position must be taken in another instrument or instruments. We shall focus on the use of derivative instruments as the risk control instruments. A primary factor in determining which instrument or instruments to use for controlling risk is the degree of correlation between the rate on the derivative instrument and the interest rate that creates the underlying risk that the manager seeks to control. For example, the rate risk associated with a long-term corporate bond portfolio can be better controlled with an instrument that is affected by long-term Treasury rates rather than short-term Treasury bill rates because long-term corporate bond rates are more highly correlated with the former than with the latter.

Correlation is not the only consideration if liquidity is of concern. For a position that requires liquidity, it may not be desirable to control its risk with an illiquid instrument or an instrument in which the value is determined solely by a counterparty. For example, managers who sought to control the risk of mortgage-backed securities found that when their positions were declining in value, the mortgage derivative products they used did not perform as expected because liquidity dried up for these instruments. A manager who uses some of the more complex over-the-counter derivative instruments that are priced by a dealer faces a similar risk. When size is an important consideration, even derivative instruments that are generally viewed as highly liquid may have a liquidity problem. In such cases, it may be necessary for the manager to use several vehicles rather than one.

Determining the Objectives of the Strategy

The measures described earlier in this book provide information about the potential loss from a position. Given the potential loss and the appropriate risk control instruments to employ, the manager should then determine what is expected from the risk control strategy. For example, hedging is a special case of risk control. Suppose that manager wants to hedge the risk associated with a current or anticipated future position of an individual bond (i.e., a micro hedging strategy). The manager should then determine what is expected from the hedge—that is, what rate will, on average, be locked in by the hedge. This is the *target rate* or *target price*. If this target rate is too high (if hedging a sale) or too low (if hedging a purchase), hedging may not be the right strategy for dealing with the unwanted risk.

Determining the Position that Should Be Taken in a Risk Control Instrument

Given the risk control instruments and the objectives of the strategy, the position that should be taken in the risk control instruments must be determined. A position has two dimensions. The first dimension is whether the position should be a long position or a short position. For example, if a manager seeks to reduce the interest rate risk exposure of a long position in a Treasury bond using Treasury bond futures, the appropriate position is a short position in the futures contract. The second dimension is the size of the position in the risk control instrument selected. For example, when using futures and options, it is the number of contracts. In the case of a swap, cap, and floor it is the notional principal amount. The amount of the position will depend on the dollar price volatility of the position whose risk the manager seeks to control relative to the derivative instrument used to control that risk. Later we will explain how this is done.

Assessing the Potential Outcome of the Risk Control Strategy

Given the position in the risk control instrument or instruments, the next step is to determine the potential outcome of the strategy. In many instances, this involves determining the outcome of the strategy under various scenarios that might be expected. That is, scenario analysis is performed. The scenarios analyzed will obviously involve different future interest rate levels.

In addition, because all risk control strategies make certain assumptions, it will be necessary to stress test the outcomes. For example, in the case of Treasury bond futures, a common assumption is that the cheapest-to-deliver issue will not change. In fact, the cheapest-to-deliver will change as interest rates change. The outcome of a risk control strategy can assess the potential impact of a change in the cheapest-to-deliver issue at different interest rate levels. As another example, it is common to make an assumption about the spread between two rates. So, a manager might make an assumption about the spread between single-A corporates and Treasuries when using Treasury bond futures to control the interest rate risk of a single-A corporate bond.

The scenarios analyzed can then be compared to the objectives established for the risk control strategy. It might be found, for example, that under a wide range of scenarios the objectives may be realized. On the other hand, it may turn out that for some scenarios that are reasonably likely to occur, the risk control strategy results in outcomes that are inferior to doing nothing at all.

In complex portfolios, the interaction among the random variables might require that simulation be employed. The product of a simulation is a probability distribution. Given this distribution, the manager can assess the strategy in light of the objectives.

CONTROLLING INTEREST RATE RISK WITH FUTURES

We begin with the application of interest rate futures to control risk. The price of an interest futures contract moves in the opposite direction from the change in interest rates: when rates rise, the futures price will fall; when rates fall, the futures price will rise. By buying a futures contract, a portfolio's exposure to rate changes is increased. That is, the portfolio's duration increases. By selling a futures contract, a portfolio's exposure to rate changes is decreased. Equivalently, this means that the portfolio's duration is reduced.

The same exposure can be obtained by using cash market instruments. Treasury securities can be used to alter the duration of a position. Specifically, a long bond position's duration can be reduced by shorting an appropriate amount of Treasury securities and a short bond position's duration can be reduced by buying an appropriate amount of Treasury securities.

Using interest rate futures instead of Treasuries has three advantages. First, transactions costs for trading futures are lower than trading in the cash market. Second, margin requirements are lower for futures than for Treasury securities; using futures thus permits greater leverage. Finally, it is easier to sell short in the futures market than in the Treasury market. Consequently, while a manager can alter the duration of a portfolio with cash market instruments, a quick and inexpensive means for doing so (on either a temporary or permanent basis) is to use futures contracts.

General Principle

The general principle in controlling interest rate risk with futures is to combine the dollar value exposure of the current portfolio and that of a futures position so that it is equal to the target dollar exposure. This means that the manager must be able to accurately measure the dollar exposure of both the current portfolio and the futures contract employed to alter the exposure.

Dollar duration can be used to approximate the change in the dollar value of a bond or bond portfolio to changes in interest rates. In the foregoing discussion, when we refer to duration and dollar duration, we mean effective duration and effective dollar duration, respectively.

Suppose that a manager has a $200 million portfolio with a duration of 5 and wants to reduce the duration to 4. Thus, the target duration for the portfolio is 4. Given the target duration, a target dollar duration for a small number of basis point change in interest rates, say 50 basis points, can be obtained. A target duration of 4 means that for a 100-basis-point change in rates (assuming a parallel shift in rates of all maturities), the target percentage price change is 4%. For a 50-basis-point change, the target percentage price change is 2%. Multiplying the 2% by $250 million gives a target dollar duration of $5 million for a 50-basis-point change in rates.

The manager must then determine the dollar duration of the current portfolio for a 50-basis-point change in rates. Since the current duration for the portfolio is 5, the current dollar duration for a 50-basis-point change in interest rates is $6.25 million. The target dollar duration is then compared to the current dollar duration. The difference between the two dollar durations is the dollar exposure that must be provided by a position in the futures contract. If the target dollar duration exceeds the current dollar duration, a futures position must increase the dollar exposure by the difference. To increase the dollar exposure, an appropriate number of futures contracts must be purchased. If the target dollar duration is less than the current dollar duration, an appropriate number of futures contracts must be sold.

Once a futures position is taken, the portfolio's dollar duration is equal to the current dollar duration without futures and the dollar duration of the futures position. That is,

$$\text{Portfolio's dollar duration} = \text{Current dollar duration without futures} + \text{Dollar duration of futures position}$$

The objective is to control the portfolio's interest rate risk by establishing a futures position such that the portfolio's dollar duration is equal to the target dollar duration. That is,

$$\text{Portfolio's dollar duration} = \text{Target dollar duration}$$

Or, equivalently,

$$\text{Target dollar duration} = \text{Current dollar duration without futures} + \text{Dollar duration of futures position} \tag{13.1}$$

Over time, the portfolio's dollar duration will move away from the target dollar duration. The manager can alter the futures position to adjust the portfolio's dollar duration to the target dollar duration.

Determining the Number of Contracts

Each futures contract calls for a specified amount of the underlying interest rate instrument. When interest rates change, the value of the underlying interest rate instrument changes, and therefore the value of the futures contract changes. How much the futures dollar value will change when interest rates change must be estimated. This amount is called the *dollar duration per futures contract*. For example, suppose the futures price of an interest rate futures contract is 70 and that the underlying interest rate instrument has a par value of $100,000. Thus, the futures delivery price (i.e., converted price) is $70,000 (0.70 times $100,000). Suppose that a change in interest rates of 50 basis points results in the futures price changing by about 3 points. Then the dollar duration per futures contract is $3,000 (0.03 times $100,000). Or equivalently, it is $3,000 per $100,000 par value of the underlying.

The dollar duration of a futures position is then the number of futures contracts multiplied by the dollar duration per futures contract. That is,

$$\begin{aligned}&\text{Dollar duration of futures position}\\&= \text{Number of futures contracts}\\&\quad\times \text{Dollar duration per futures contract}\end{aligned} \tag{13.2}$$

To determine how many futures contracts are needed to obtain the target dollar duration, we can substitute equation (13.2) into equation (13.1). The result is

$$\begin{aligned}&\text{Number of futures contracts} \times \text{Dollar duration per futures contract}\\ &= \text{Target dollar duration} - \text{Current dollar duration without futures}\end{aligned} \tag{13.3}$$

Solving for the number of futures contracts we have

$$\begin{aligned}&\text{Number of futures contracts}\\ &= \frac{\text{Target dollar duration} - \text{Current dollar duration without futures}}{\text{Dollar duration per futures contract}}\end{aligned} \tag{13.4}$$

Equation (13.4) gives the approximate number of futures contracts that are necessary to adjust the portfolio's dollar duration to the target dollar duration. A positive number means that the futures contract must be purchased; a negative number means that the futures contract must be sold. Notice that if the target dollar duration is greater than the current dollar duration without futures, the numerator is positive and therefore futures contracts are purchased. If the target dollar duration is less than the current dollar duration without futures, the numerator is negative and therefore futures contracts are sold.

HEDGING WITH FUTURES

Hedging with futures calls for taking a futures position as a temporary substitute for transactions to be made in the cash market at a later date. If cash and futures prices move together, any loss realized by the hedger from one position (whether cash or futures) will be offset by a profit on the other position. Hedging is a special case of controlling interest rate risk. In a hedge, the manager seeks a target duration or target dollar duration of zero.

Typically the bond or portfolio to be hedged is not identical to the bond underlying the futures contract. This type of hedging is referred to as *cross hedging*. There may be significant risks in cross hedging.

A *short* (or *sell*) *hedge* is used to protect against a decline in the cash price of a bond. To execute a short hedge, futures contracts are sold. By establishing a short hedge, the manager has fixed the future cash price and transferred the price risk of ownership to the buyer of the futures contract. A *long* (or *buy*) *hedge* is undertaken to protect against an increase in the cash price of a bond.

Hedge Effectiveness and Residual Hedging Risk

Earlier we described the four preliminary steps that a manager should undertake prior to the employment of a risk control strategy. In the case of hedging, the manager must try to assess the hedge effectiveness and the residual hedging risk. *Hedge effectiveness* lets the manager know what percent of risk is eliminated by hedging. For example, if the hedge effectiveness is determined to be 85% effective, over the long run a hedged position will have only 15% of the risk (that is, the standard deviation) of an unhedged position.

The *residual hedging risk* is the absolute level of risk in the hedged position. This risk tells the manager how much risk remains after hedging. While it may be comforting to know, for example, that 85% of the risk is eliminated by hedging, without additional statistics the manager still does not know how much risk remains. The residual hedging risk in a hedged position is expressed as a standard deviation. For example, it might be determined that the hedged position has a standard deviation of 10 basis points. Assuming a normal distribution of hedging errors, the manager will then obtain the target rate plus or minus 10 basis points 66% of the time. The probability of obtaining the target rate plus or minus 20 basis points is 95%, and the probability of obtaining the target rate plus or minus 30 basis points is greater than 99%.

The target rate, the hedge effectiveness, and the residual hedging risk determine the basic trade-off between risk and expected return. Consequently, these statistics give the manager the information needed to decide whether to employ a hedge strategy. Using these statistics, the manager can construct confidence intervals for hedged and unhedged positions. Comparing these confidence intervals, the manager can determine whether hedging is the best alternative. Furthermore, if hedging is the right decision, the level of confidence in the hedge is defined in advance.

It is important for a manager to realize that the hedge effectiveness and the residual hedging risk are not necessarily constant from one hedge to the next. Hedges for dates near a futures delivery date will tend to be more effective and have less residual hedging risk than those lifted on other dates. The life of the hedge, that is, the amount of time between when the hedge is set and when it is lifted, also generally has a significant impact on hedge effectiveness and residual hedging risk. For example, a hedge held for six months might be 95% effective, whereas a hedge held for one month might be only 30% effective. This is because the security to be hedged and the hedging instrument might be highly correlated over the long run, but only weakly correlated over the short run. On the other hand, residual hedging risk usually increases as the life of the hedge increases. The residual hedging risk on a 6-month hedge may be 80 basis points while the residual hedging risk for a 1-month hedge may be only 30 basis points. It may seem surprising that hedges for longer periods have more risk if they are also more effective. However, hedge effectiveness is a measure of relative risk, and because longer time periods exhibit greater swings in interest rates, the greater percentage reduction in risk for longer hedges does not mean that there is less risk left over.

The target rate, the residual risk, and the effectiveness of a hedge are relatively simple concepts. However, because these statistics are usually estimated using historical data, the manager who plans to hedge should be sure that these figures are estimated correctly.

Risk and Expected Return in a Hedge

In a micro hedge strategy, when a manager enters into a hedge, the objective is to "lock in" a rate for the sale or purchase of a security. However, there is much disagreement about what rate a manager should expect to lock in when futures are used to hedge. One view is that the manager can, on average, lock in the current spot rate for the security. The opposing view is that the manager will, on average, lock in the rate at which the futures contracts are bought or sold. The truth usually lies somewhere in between these two positions. However, as the following cases illustrate, each view is entirely correct in certain situations.

The Target for Hedges Held to Delivery

Minimum variance hedges that are held until the futures delivery date provide an example of a hedge that locks in the futures rate. The complication in the case of using Treasury bond futures and Treasury note futures to hedge the value of intermediate- and long-term bonds, is that because of the delivery options the manager does not know for sure when delivery will take place or which bond will be delivered.

To illustrate how a Treasury bond futures held to the delivery date locks in the futures rate, assume for the sake of simplicity, that the manager knows which Treasury bond will be delivered and that delivery will take place on the last day of the delivery month. Consider the 7⅝s Treasury bonds maturing on February 15, 2007.[1] For delivery on the June 1985 contract, the conversion factor for these bonds was 0.9660, implying that the investor who delivers the 7⅝s would receive from the buyer 0.9660 times the futures settlement price, plus accrued interest. Consequently, at delivery, the (flat) spot price and the futures price times the conversion factor must

[1] This example is taken from Chapter 9 in Mark Pitts and Frank J. Fabozzi, *Interest Rate Futures and Options* (Chicago, IL: Probus Publishing, 1989).

converge. *Convergence* refers to the fact that at delivery there can be no discrepancy between the spot and futures price for a given security. If convergence does not take place, arbitrageurs would buy at the lower price and sell at the higher price and earn risk-free profits. Accordingly, a manager could lock in a June sale price for the 7⅝s by selling Treasury bond futures contracts equal to 0.9660 times the face value of the bonds. For example, $100 million face value of 7⅝s would be hedged by selling $96.6 million face value of bond futures (rounded to 967 contracts).

The sale price that the manager locks in would be 0.9660 times the futures price. Thus, if the futures price is 70 when the hedge is set, the manager locks in a sale price of 67.62 (70 times 0.9660) for June delivery, regardless of where rates are in June. Exhibit 13.1 shows the cash

EXHIBIT 13.1 Treasury Bond Hedge Held to Delivery

Instrument to be hedged: 7⅝ Treasury bonds of 2/15/07
Conversion factor for June 1985 delivery = 0.9660
Price of futures contract when sold = 70
Target price = 0.9660 × 70 = 67.62

Actual Sale Price for 7⅝ Treasury Bond ($)	Final Futures Price ($)[a]	Gain or Loss on 967 Contracts ($; $10/0.01/Contract)[b]	Effective Sale Price ($)[c]
62	64.182	5,620,188	67,620,118
63	65.217	4,620,378	67,620,378
64	66.253	3,619,602	67,619,602
65	67.288	2,619,792	67,619,792
66	68.323	1,619,982	67,619,982
67	69.358	620,172	67,620,172
68	70.393	–379,638	67,620,362
69	71.429	–1,380,414	67,619,568
70	72.464	–2,380,224	67,619,776
71	73.499	–3,380,034	67,619,966
72	74.534	–4,379,844	67,620,156
73	75.569	–5,379,654	67,620,346
74	76.605	–6,380,430	67,619,570
75	77.640	–7,380,240	67,619,760

[a] By convergence, must equal bond price divided by the conversion factor. In 1985, the notional coupon on the Treasury bond futures contract was 8%. This explains why the conversion factors are less than one.
[b] Bond futures trade in even increments of 1/32. Accordingly, the futures prices and margin flows are only approximate.
[c] Transaction costs and the financing of margin flows are ignored.

flows for a number of final prices for the 7⅝s and illustrates how cash flows on the futures contracts offset gains or losses relative to the target price of 67.62. In each case, the effective sale price is very close to the target price (and, in fact, would be exact if enough decimal places were carried through the calculations). However, the target price is determined by the futures price, so the target price may be higher or lower than the cash market price when the hedge is set.

When we admit the possibility that bonds other than the 7⅝s of 2007 can be delivered, and that it might be advantageous to deliver other bonds, the situation becomes somewhat more involved. In this more realistic case, the manager may decide not to deliver the 7⅝s, but if she does decide to deliver them, the manager is still assured of receiving an effective sale price of approximately 67.62. If the manager does not deliver the 7⅝s, it would be because another bond can be delivered more cheaply, and thus the manager does better than the targeted price.

In summary, if a manager sets a risk minimizing futures hedge that is held until delivery, the manager can be assured of receiving an effective price dictated by the futures rate (not the spot rate) on the day the hedge is set.

The Target for Hedges with Short Holding Periods

When a manager must lift (remove) a hedge prior to the delivery date, the effective rate that is obtained is much more likely to approximate the current spot rate than the futures rate the shorter the term of the hedge. The critical difference between this hedge and the hedge held to the delivery date is that convergence will generally not take place by the termination date of the hedge. This will be the case regardless of whether the manager is hedging with one of the short-term contracts (such as Eurodollar CD futures or Treasury bill futures) or hedging longer-term instruments with the intermediate- and long-term contracts.

To illustrate why a manager should expect the hedge to lock in the spot rate rather than the futures rate for very short-lived hedges, let's return to the simplified example used earlier to illustrate a hedge to the delivery date. It is assumed that the 7⅝s of 2007 were the only deliverable Treasury bonds for the Treasury bond futures contract. Suppose that the hedge is set three months before the delivery date and the manager plans to lift the hedge after one day. It is much more likely that the spot price of the bond will move parallel to the converted futures price (that is, the futures price times the conversion factor) than that the spot price and the converted futures price will converge by the time the hedge is lifted.

A 1-day hedge is, admittedly, an extreme example. However, it is not uncommon for traders and risk managers to have such a short horizon.

Few money managers are interested in such a short horizon. The very short-term hedge does illustrate a very important point: When hedging, a manager should not expect to lock in the futures rate (or price) just because he is hedging with futures contracts. The futures rate is locked in only if the hedge is held until delivery, at which point convergence must take place. If the hedge is held for only one day, the manager should expect to lock in the 1-day forward rate, which will very nearly equal the spot rate. Generally hedges are held for more than one day, but not necessarily to delivery.

How the Basis Affects the Target Rate for a Hedge

The proper target for a hedge that is to be lifted prior to the delivery date depends on the basis. The *basis* is simply the difference between the spot (cash) price of a security and its futures price. That is,

$$\text{Basis} = \text{Spot price} - \text{Futures price}$$

In the bond market, a problem arises when trying to make practical use of the concept of the basis. The quoted futures price does not equal the price that one receives at delivery. For the Treasury bond and note futures contracts, the actual futures price equals the quoted futures price times the appropriate conversion factor. Consequently, to be useful the basis in the bond market should be defined using actual futures delivery prices rather than quoted futures prices. Thus, the price basis for bonds should be redefined as

$$\text{Price basis} = \text{Spot price} - \text{Futures delivery price}$$

Unfortunately, problems still arise due to the fact that bonds age over time. Thus, it is not exactly clear what is meant by the "spot price." Does spot price mean the current price of the actual instrument that can be held and delivered in satisfaction of a short position, or does it mean the current price of an instrument that currently has the characteristics called for in the futures contract? For example, when the basis is defined for a 3-month Treasury bill contract maturing in three months, should spot price refer to the current price of a 6-month Treasury bill, which is the instrument that will actually be deliverable on the contract (because in three months it will be a 3-month Treasury bill), or should spot price refer to the price of the current 3-month Treasury bill? In most cases the former definition of the spot price makes the most sense.

For hedging purposes it is also frequently useful to define the basis in terms of interest rates rather than prices. The rate basis is defined as

$$\text{Rate basis} = \text{Spot rate} - \text{Futures rate}$$

where spot rate refers to the current rate on the instrument to be hedged and the futures rate is the interest rate corresponding to the futures delivery price of the deliverable instrument.

The rate basis is helpful in explaining why the two types of hedges explained earlier are expected to lock in such different rates. To see this, we first define the *target rate basis*. This is defined as the expected rate basis on the day the hedge is lifted. A hedge lifted on the delivery date is expected to have, and by convergence will have, a zero rate basis when the hedge is lifted. Thus, the target rate for the hedge should be the rate on the futures contract plus the expected rate basis of zero, or in other words, just the futures rate. When a hedge is lifted prior to the delivery date, one would not expect the basis to change very much in one day, so the target rate basis equals the futures rate plus the current difference between the spot and futures rate, i.e., the current spot rate.

The manager can set the target rate for any hedge equal to the futures rate plus the target rate basis. That is,

$$\text{Target rate for hedge} = \text{Futures rate} + \text{Target rate basis}$$

If projecting the basis in terms of price rather than rate is more manageable (as is often the case for intermediate- and long-term futures), it is easier to work with the target price basis instead of the target rate basis. The target price basis is just the projected price basis for the day the hedge is to be lifted. For a deliverable security, the target for the hedge then becomes

$$\text{Target price for hedge} = \text{Futures delivery price} + \text{Target price basis}$$

The idea of a target price or rate basis explains why a hedge held until the delivery date locks in a price with certainty, and other hedges do not. As is often said, hedging substitutes basis risk for price risk, and the examples have shown that this is true. For the hedge held to delivery, there is no uncertainty surrounding the target basis; by convergence, the basis on the day the hedge is lifted is certain to be zero. For the short-lived hedge, the basis will probably approximate the current basis when the hedge is lifted, but its actual value is not known. For hedges longer than one day but ending prior to the futures delivery date, there can be considerable risk because the basis on the day the hedge is lifted can end up being anywhere within a wide range. Thus, the uncertainty surrounding the outcome of a hedge is directly related to the uncertainty surrounding the basis on the day the hedge is lifted, that is, the uncertainty surrounding the target basis.

For a given investment horizon hedging substitutes basis risk for price risk. Thus, one trades the uncertainty of the price of the hedged security for the uncertainty of the basis. Consequently, when hedges do not produce the desired results, it is customary to place the blame on "basis risk." However, basis risk is the real culprit only if the target for the hedge is properly defined. Basis risk should refer only to the unexpected or unpredictable part of the relationship between cash and futures. The fact that this relationship changes over time does not in itself imply that there is basis risk.

Basis risk, properly defined, refers only to the uncertainty associated with the target rate basis or target price basis. Accordingly, it is imperative that the target basis be properly defined if one is to correctly assess the risk and expected return in a hedge.

Cross Hedging

Earlier, we defined a cross hedge in the futures market as a hedge in which the security to be hedged is not deliverable into the futures contract used in the hedge. For example, a manager who wants to hedge the sale price of long-term corporate bonds might hedge with the Treasury bond futures contract, but since corporate bonds cannot be delivered in satisfaction of the contract, the hedge would be considered a cross hedge. Similarly, on the short end of the yield curve, a manager might want to hedge a 3-month rate that does not perfectly track the Treasury bill rate or Libor. A manager might also want to hedge a rate that is of the same quality as the rate specified in one of the contracts, but that has a different maturity. For example, it is necessary to cross hedge to hedge a Treasury bond, note, or bill with a maturity that does not qualify for delivery on any futures contract. Thus, when the security to be hedged differs from the futures contract specification in terms of either quality or maturity, one is led to the cross hedge.

Conceptually, cross hedging is somewhat more complicated than hedging deliverable securities, because it involves two relationships. First, there is the relationship between the cheapest-to-deliver (CTD) issue and the futures contract. Second, there is the relationship between the security to be hedged and the CTD. Practical considerations may at times lead a manager to shortcut this two-step relationship and focus directly on the relationship between the security to be hedged and the futures contract, thus ignoring the CTD altogether. However, in so doing, a manager runs the risk of miscalculating the target rate and the risk in the hedge. Furthermore, if the hedge does not perform as expected, the shortcut makes it difficult to tell why the hedge went awry.

The Hedge Ratio

The key to minimize risk in a cross hedge is to choose the right *hedge ratio*. The hedge ratio depends on the relative dollar duration of the bond to be hedged and the futures position. Equation (13.4) indicates the number of futures contract to achieve a particular target dollar duration. The objective in hedging is make the target dollar duration equal to zero. Substituting zero for target dollar duration in equation (13.4), we obtain:

$$\text{Number of futures contracts} \\ = -\frac{\text{Current dollar duration without futures}}{\text{Dollar duration per futures contract}} \tag{13.5}$$

To calculate the dollar duration of a bond, the manager must know the precise point in time that the dollar duration is to be calculated (because volatility generally declines as a bond seasons) as well as the price or yield at which to calculate dollar duration (because higher yields generally reduce dollar duration for a given yield change). The relevant point in the life of the bond for calculating volatility is the point at which the hedge will be lifted. Dollar duration at any other point is essentially irrelevant because the goal is to lock in a price or rate only on that particular day. Similarly, the relevant yield at which to calculate dollar duration initially is the target yield. Consequently, the numerator of equation (13.5) is the dollar duration on the date the hedge is expected to be delivered. The yield that is to be used on this date in order to determine the dollar duration is the forward rate.

An example for a single bond rather than a portfolio shows why dollar duration weighting leads to the correct hedge ratio.[2] Suppose that on April 19, 1985, a money manager owned \$10 million face value of the Southern Bell 11¾% bonds of 2023 and sold June 1985 Treasury bond futures to hedge a future sale of the bonds. This is an example of a cross hedge. Suppose that (1) the Treasury 7⅝s of 2007 were the cheapest-to-deliver issue on the contract and that they were trading at 11.50%, (2) the Southern Bell bonds were at 12.40%, and (3) the Treasury bond futures were at a price of 70. To simplify, assume also that the yield spread between the two bonds remains at 0.90% (i.e., 90 basis points) and that the anticipated sale date was the last business day in June 1985.

Because the conversion factor for the deliverable 7⅝s for the June 1985 contract was 0.9660, the target price for hedging the 7⅝s would be 67.62 (70×0.9660), and the target yield would be 11.789% (the yield at a price of 67.62).[3] The yield on the telephone bonds is assumed

[2] This example is adapted from Pitts and Fabozzi, *Interest Rate Futures and Options*.
[3] The notional coupon of the Treasury bond futures contract was 8% in 1985.

to stay at 0.90% above the yield on the 7⅝s, so the target yield for the Southern Bell bonds would be 12.689%, with a corresponding price of 92.628. At these target levels, the dollar duration for a 50-basis-point change in rates for the 7⅝s and telephone bonds per $100 of par value are, respectively, $2.8166 and $3.6282. As indicated earlier, all these calculations are made using a settlement date equal to the anticipated sale date, in this case the end of June 1985. The dollar duration for $10 million par value of the Southern Bell bonds is $362,820 ($10 million/100 times $3.6282). Per $100,000 of par value for the futures contract, the dollar duration per futures contract is $2,817 ($100,000/100 times $2.8166). Therefore,

Current dollar duration without futures
= Dollar duration of the Southern Bell bonds = $362,820

and

Dollar duration of the CTD = $2,817

However, to calculate the hedge ratio, we need the dollar duration not of the CTD, but of the hedging instrument, that is, of the futures contract. Fortunately, knowing the dollar duration of the bond to be hedged relative to the CTD and the dollar duration of the CTD relative to the futures contract, we can easily obtain the hedge ratio:

$$\text{Hedge ratio} = -\frac{\text{Current dollar duration without futures}}{\text{Dollar duration of the CTD}} \times \frac{\text{Dollar duration of the CTD}}{\text{Dollar duration per futures contract}} \tag{13.6}$$

Assuming a fixed yield spread between the bond to be hedged and the CTD, the hedge ratio given by equation (13.6) can be rewritten as

$$\text{Hedge ratio} = -\frac{\text{Current dollar duration without futures}}{\text{Dollar duration of the CTD}} \times \text{Conversion factor for the CTD} \tag{13.7}$$

Substituting the values from our example into equation (13.7):

$$\text{Hedge ratio} = -\frac{\$362{,}820}{\$2{,}817} \times 0.9660 = -124 \text{ contracts}$$

Thus, to hedge the Southern Bell position, 124 Treasury bond futures contracts must be shorted.

Scenario analysis can be used to show the potential outcome of this hedge. Exhibit 13.2 shows that, if the simplifying assumptions hold, a futures hedge using the recommended hedge ratio very nearly locks in the target price for $10 million face value of the telephone bonds.[4]

Another refinement in the hedging strategy is usually necessary for hedging nondeliverable securities. This refinement concerns the assumption about the relative yield spread between the CTD and the bond to be hedged. In the prior discussion, we assumed that the yield spread was constant over time. Yield spreads, however, are not constant over time. They vary with the maturity of the instruments in question and the level of rates, as well as with many unpredictable and nonsystematic factors.

Regression analysis allows the manager to capture the relationship between yield levels and yield spreads and use it to advantage. For hedging purposes, the variables are the yield on the bond to be hedged and the yield on the CTD. The regression equation takes the form:[5]

$$\text{Yield on bond to be hedged} = \alpha + \beta \times \text{Yield on CTD} + \text{error} \qquad (13.8)$$

The regression procedure provides an estimate of β (the *yield beta*), which is the expected relative yield change in the two bonds. Our example that used a constant spread implicitly assumes that the yield beta, β, equals 1.0 and α equals 90 basis points (the assumed spread).

For the two issues in question, that is, the Southern Bell 11¾s and the Treasury 7⅝s, suppose that the estimated yield beta was 1.05. Thus, yields on the corporate issue are expected to move 5% more than yields on the Treasury issue. To calculate the hedge ratio correctly, this fact must be taken into account; thus, the hedge ratio derived in our earlier example is multiplied by the factor 1.05. Consequently, instead of shorting 124 Treasury bond futures contracts to hedge $10 million of telephone bonds, the investor would short 130 contracts.

The formula for the hedge ratio is revised as follows to incorporate the impact of the yield beta:

$$\text{Hedge ratio} = -\frac{\text{Current dollar duration without futures}}{\text{Dollar duration of the CTD}} \times \text{Conversion factor for CTD} \times \text{Yield beta} \qquad (13.9)$$

[4] In practice, most of the remaining error could be eliminated by frequent adjustments to the hedge ratio to account for the fact that the dollar duration changes as rates move up or down.

[5] For an explanation of regression analysis, see Chapter 6.

EXHIBIT 13.2 **Hedging a Nondeliverable Bond to a Delivery Date with Futures: Scenario Analysis**
Instrument to be hedged: Southern Bell 11¾s of 4/19/23

Par value = \$10 million — Price of futures contract when sold = 70
Hedge ratio = 124 — Target price for Southern Bell bonds = 92.628

Actual Sale Price of Telephone Bonds (\$)	Yield at Sale (%)	Yield on Treasury 7⅝s[a] (%)	Price of Treasury 7⅝s	Futures Price[b]	Gain (Loss) on 124 Contracts (\$10/0.01/Contract) (\$)	Effective Sale Price (\$)[c]
7,600,000	15.468	14.568	54.590	56.511	1,672,636	9,272,636
7,800,000	15.072	14.172	56.167	58.144	1,470,144	9,270,144
8,000,000	14.696	13.769	57.741	59.773	1,268,148	9,268,148
8,200,000	14.338	13.438	59.313	61.401	1,066,276	9,266,276
8,400,000	13.996	13.096	60.887	63.030	864,280	9,264,280
8,600,000	13.671	12.771	62.451	64.649	663,524	9,263,524
8,800,000	13.359	12.459	64.018	66.271	462,396	9,262,396
9,000,000	13.061	12.161	65.580	67.888	261,888	9,261,888
9,200,000	12.776	11.876	67.134	69.497	62,372	9,262,372
9,400,000	12.503	11.603	68.683	71.100	(136,400)	9,263,600
9,600,000	12.240	11.340	70.233	72.705	(335,420)	9,264,580
9,800,000	11.988	11.088	71.773	74.299	(533,076)	9,266,924
10,000,000	11.745	10.845	73.312	75.892	(730.608)	9,269,392
10,200,000	11.512	10.612	74.839	77.473	(926,652)	9,273,348
10,400,000	11.287	10.387	76.364	79.052	(1,122,448)	9,277,552
10,600,000	11.070	10.170	77.884	80.625	(1,317,500)	9,282,500
10,800,000	10.861	9.961	79.394	82.188	(1,511,312)	9,288,688
11,000,000	10.659	9.759	80.889	83.746	(1,704,504)	9,295,496
11,200,000	10.463	9.563	82.403	85.303	(1,897,572)	9,302,428

[a] By assumption, the yield on the 7⅝s of 2007 is 90 basis points lower than the yield on the Southern Bell bond.
[b] By convergence, the futures price equals the price of the 7⅝s of 2007 divided by 0.9660 (the conversion factor).
[c] Transaction costs and the financing of margin flows are ignored.

where the yield beta is derived from the yield of the bond to be hedged regressed on the yield of the CTD [equation (13.8)].

The effect of a change in the CTD and the yield spread can be assessed a priori. An exhibit similar to that of Exhibit 13.2 can be constructed under a wide range of assumptions. For example, at different yield levels at the date the hedge is to be lifted (the second column in Exhibit 13.2), a different yield spread may be appropriate and a different acceptable issue will be the CTD. The manager can determine what this will do to the outcome of the hedge.

Monitoring and Evaluating the Hedge

After a target is determined and a hedge is set, there are two remaining tasks. The hedge must be monitored during its life, and evaluated after it is over. A futures hedge may require very little active monitoring during its life. In fact, overactive management may pose more of a threat to most hedges than does inactive management. The reason for this is that the manager usually will not receive enough new information during the life of the hedge to justify a change in the hedging strategy. For example, it is not advisable to readjust the hedge ratio every day in response to a new data point and a possible corresponding change in the estimated value of the yield beta.

There are, however, exceptions to this general rule. As rates change, dollar duration changes. Consequently, the hedge ratio may change slightly. In other cases, there may be sound economic reasons to believe that the yield beta has changed. While there are exceptions, the best approach is usually to let a hedge run its course using the original hedge ratio with only slight adjustments.

A hedge can normally be evaluated only after it has been lifted. Evaluation involves, first, an assessment of how closely the hedge locked in the target rate, that is, how much error there was in the hedge. To provide a meaningful interpretation of the error, the manager should calculate how far from the target the sale (or purchase) would have been, had there been no hedge at all.

One good reason for evaluating a completed hedge is to ascertain the sources of error in the hedge in the hope that a manager will gain insights that can be used to advantage in subsequent hedges. A manager will find that there are three major sources of hedging errors:

1. The projected value of the basis at the lift date can be in error.
2. The parameters estimated from the regression (α and β) can be inaccurate.
3. The error term in the regression may not equal zero.

Frequently, at least in the short run, the last two sources of error are indistinguishable. The manager will generally only know that the regression equation did not give an accurate estimate of the rate to be hedged. However, such inaccuracy could have occurred either from poor parameter estimates or from very accurate parameter estimates in conjunction with a large error term.

The first major source of errors in a hedge—an inaccurate projected value of the basis—is the more difficult problem. Unfortunately, there are no satisfactory simple models like the regression that can be applied to the basis. Simple models of the basis violate certain equilibrium relationships for bonds that should not be violated. On the other hand, theoretically rigorous models are very unintuitive and usually soluble only by complex numerical methods. Modeling the basis is undoubtedly one of the most important and difficult problems that managers seeking to hedge face.

HEDGING WITH OPTIONS

There are three popular hedge strategies employing options: (1) a protective put buying strategy, (2) a covered call writing strategy, and (3) a collar strategy. We begin with basic hedging principles for each strategy. Then we illustrate the first two strategies using futures options to hedge the Southern Bell bonds in which a futures hedge was used. Using futures options in our illustration of hedging the Southern Bell bonds is a worthwhile exercise because it shows how complicated hedging with futures options is and the key parameters involved in the process. We also compare the outcome of hedging with futures and hedging with futures options.[6]

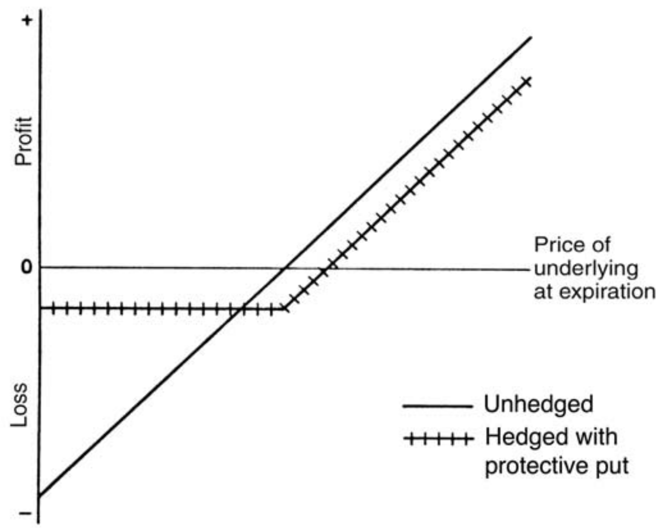
Basic Hedging Strategies

Protective Puts

Consider first a money manager who has a bond and wants to hedge against rising interest rates. The most obvious options hedging strategy is to buy puts on bonds. These *protective puts* are usually out-of-the-money puts and may be either puts on cash bonds or puts on interest rate futures. If interest rates rise, the puts will increase in value (holding other factors constant), offsetting some or all the loss on the bonds in the portfolio.

[6] The illustrations in this section are taken from Chapter 10 of Pitts and Fabozzi, *Interest Rate Futures and Options*.

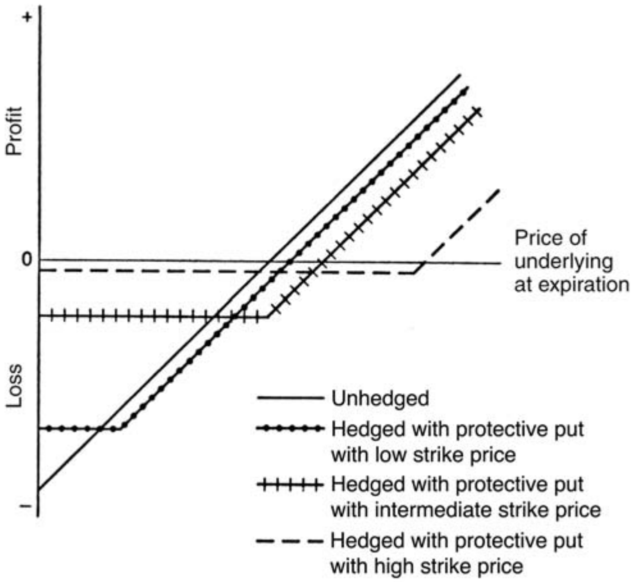
EXHIBIT 13.3 Protective Put



This strategy is a simple combination of a long put option with a long position in a cash bond. The result is a payoff pattern that resembles a long position in a call option alone. Such a position has limited downside risk, but large upside potential. However, if rates fall, the price appreciation on the securities in the portfolio will be diminished by the amount paid for the puts. Exhibit 13.3 compares the protective put strategy to an unhedged position.

The protective put strategy is very often compared to purchasing insurance. Like insurance, the premium paid for the protection is nonrefundable and is paid before the coverage begins. The degree to which a portfolio is protected depends upon the strike price of the options; thus, the strike price is often compared to the deductible on an insurance policy. The lower the deductible (that is, the higher the strike on the put), the greater the level of protection and the more the protection costs. Conversely, the higher the deductible (the lower the strike on the put), the more the portfolio can lose in value; but the cost of the insurance is lower. Exhibit 13.4 compares an unhedged position with several protective put positions, each with a different strike price, or level of protection. As the exhibit shows, no one strategy dominates any other strategy, in the sense of performing better at all possible rate levels. Consequently, it is impossible to say that one strike price is necessarily the "best" strike price, or even that buying protective puts is necessarily better than doing nothing at all.

EXHIBIT 13.4 Protective Put with Different Strike Prices



Covered Call Writing

Another options hedging strategy used by many portfolio managers is to sell calls against the bond portfolio; that is, to do *covered call writing*. The calls that are sold are usually out-of-the-money calls, and can be either calls on cash bonds or calls on interest rate futures. Covered call writing is just an outright long position combined with a short call position. The strategy thus results in a payoff pattern that resembles a short position in a put option alone. Obviously, this strategy entails much more downside risk than buying a put to protect the value of the portfolio. In fact, many portfolio managers do not consider covered call writing a hedge.

Regardless of how it is classified, it is important to recognize that while covered call writing has substantial downside risk, it has less downside risk than an unhedged long position alone. On the downside, the difference between the long position alone and the covered call writing strategy is the premium received for the calls that are sold. This premium acts as a cushion for downward movements in prices, reducing losses when rates rise. The cost of obtaining this cushion is that the manager gives up some of the potential on the upside. When rates decline, the call options become greater liabilities for the covered call writer. These incremental liabilities decrease the gains the portfolio manager would otherwise have realized on the portfolio in a declining rate environment. Thus, the covered call writer gives up some (or all) of the upside potential of the portfolio in return for a cushion on the downside. The more upside potential that is forfeited (that is, the lower the strike price on the calls), the more cushion there is on the downside. Like the protective put strategy, there is no "right" strike price for the covered call writer.

Comparing the two basic strategies for hedging with options, one cannot say that the protective put strategy or the covered call writing strategy is necessarily the better or more correct options hedge. The best strategy (and the best strike prices) depends upon the manager's view of the market. Purchasing a put and paying the required premium is appropriate if the manager is fundamentally bearish. If, on the other hand, the manager is neutral to mildly bearish, it is better to take in the premium on the covered call writing strategy. If the manager prefers to take no view on the market at all, and as little risk as possible, then a futures hedge is most appropriate. If the manager is fundamentally bullish, then no hedge at all is probably the best strategy.

Collars

There are other options hedging strategies frequently used by money managers. For example, many managers combine the protective put strategy and the covered call writing strategy. By combining a long position in an out-of-the-money put and a short position in an out-of-the-money call, the manager creates a long position in a *collar*. The manager who uses the collar eliminates part of the portfolio's downside risk by giving up part of its upside potential.

The collar in some ways resembles the protective put, in some ways resembles covered call writing, in some ways resembles an unhedged position, and in some ways resembles a futures or forward hedge. The collar is like the protective put strategy in that it limits the possible losses on the portfolio if interest rates go up. Like the covered call writing strategy, the portfolio's upside potential is limited. Like an unhedged position, within the range defined by the strike prices the value of the portfolio varies with interest rates. On the other hand, if the put strike price and the call strike price are both equal to the forward price, the collar is just like a forward hedge, and the effective sale price is not dependent upon interest rates.

Options Hedging Preliminaries

As explained earlier, there are certain preliminaries that managers should consider before undertaking a risk control strategy. The best options contract to use depends upon several factors. These include option price, liquidity, and correlation with the bond(s) to be hedged.

In price-inefficient markets, the option price is important because not all options will be priced in the same manner or with the same volatility assumption. Consequently, some options may be overpriced and some underpriced. Obviously, with other factors equal, it is better to use the underpriced options when buying and the overpriced options when selling.

Whenever there is a possibility that the option position may be closed out prior to expiration, liquidity is also an important consideration. If the particular option is illiquid, closing out a position may be prohibitively expensive, and the manager loses the flexibility of closing out positions early, or rolling into other positions that may become more attractive.

Correlation with the underlying bond(s) to be hedged is another factor in selecting the right contract. The higher the correlation, the more precisely the final profit and loss can be defined as a function of the final level of rates. Poor correlation leads to more uncertainty. While most of the uncertainty in an options hedge usually comes from the uncertainty of interest rates themselves, slippage between the bonds to be hedged and the instruments underlying the options contracts add to that risk. Thus, the degree of correlation between the two underlying instruments is one of the determinants of the risk in the hedge.

Hedging Long-Term Bonds with Puts on Futures

As explained above, managers who want to hedge their bond positions against a possible increase in interest rates will find that buying puts on futures is one of the easiest ways to purchase protection against rising rates. To illustrate this strategy, we can use the same utility bond example that we used to demonstrate how to hedge with Treasury bond futures. In that example, a manager held Southern Bell 11¾s of 2023 and used futures to lock in a sale price for those bonds on a futures delivery date. Now we want to show how the manager could have used futures options instead of futures to protect against rising rates.

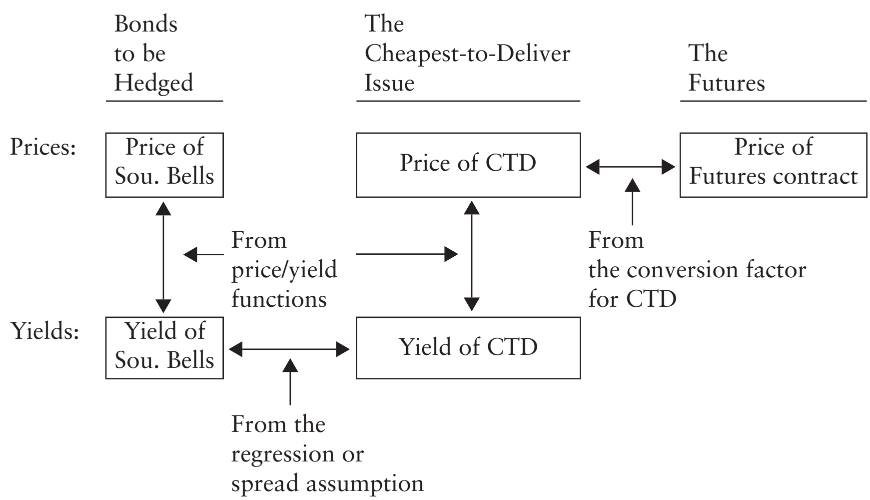
In the example, rates were already fairly high; the hedged bonds were selling at a yield of 12.40%, the Treasury 7⅝s of 2007 (the cheapest-to-deliver issue at the time) were at 11.50%. For simplicity, it was assumed that this yield spread would remain at 90 basis points.

Selecting the Strike Price

The manager must determine the minimum price that he or she wants to establish for the hedged bonds. In our illustration it is assumed that the minimum price is 87.668. This is equivalent to saying that the manager wants to establish a strike price for a put option on the hedged bonds of 87.668. But, the manager is not buying a put option on the utility bonds. She or he is buying a put option on a Treasury bond futures contract. Therefore, the manager must determine the strike price for a put option on a Treasury bond futures contract that is equivalent to a strike price of 87.668 for the utility bonds.

This can be done with the help of Exhibit 13.5. We begin at the top left hand box of the exhibit. Since the minimum price is 87.668 for the utility bonds, this means that the manager is attempting to establish a maximum yield of 13.41%. This is found from the relationship between price and yield: given a price of 87.668 for the utility bond, this equivalent to a yield of 13.41%. (This gets us to the lower left hand box in Exhibit 13.5.) From the assumption that the spread between the utility bonds and the cheapest-to-deliver issue is a constant 90 basis points, setting a maximum yield of 13.41% for the utility bond is equivalent to setting a maximum yield of 12.51% for the CTD. (Now we are at the lower box in the middle column of Exhibit 13.5.) Given the yield of 12.51% for the CTD, the minimum price can be determined (the top box in the middle column of the exhibit). A 12.51% yield for the Treasury $7\frac{5}{8}$s of 2007 (the CTD at the time) gives a price of 63.756. The corresponding futures price is found by dividing the price of the CTD by the conversion factor. This gets us to the box in the right hand column of Exhibit 13.5. Since the conversion factor is 0.9660, the futures price is about 66 (63.7567 divided by 0.9660). This means that a strike price of 66 for a put option on a Treasury bond futures contract is roughly equivalent to a put option on the utility bonds with a strike price of 87.668.

EXHIBIT 13.5 **Calculating Equivalent Prices and Yields for Hedging with Futures Options**



The foregoing steps are always necessary to obtain the appropriate strike price on a put futures option. The process is not complicated. It simply involves (1) the relationship between price and yield, (2) the assumed relationship between the yield spread between the hedged bonds and the CTD, and (3) the conversion factor for the CTD. As with hedging employing futures illustrated earlier in this chapter, the success of the hedging strategy will depend on (1) whether the CTD changes and (2) the yield spread between the hedged bonds and the CTD.

Calculating the Hedge Ratio

The hedge ratio is determined using the following equation similar to equation (13.7) since we will assume a constant yield spread between the security to be hedged and the CTD issue:

$$\text{Hedge ratio} = \frac{\text{Current dollar duration without options}}{\text{Dollar duration of the CTD}} \times \text{Conversion factor for CTD}$$

For increased accuracy, we calculate the dollar durations at the option expiration date (assumed to be June 28, 1985 in our illustration) and at the yields corresponding to the futures strike price of 66 (12.51% for the CTD and 13.41% for the hedged bonds). The dollar durations are as follows per 50-basis-point change in rates:

$$\text{Current dollar duration without options} = \$326{,}070$$

$$\text{Dollar duration of the CTD} = \$2{,}548$$

Notice that the dollar durations are different from those used in calculating the hedge ratio for the futures hedge in the previous chapter. This is because the dollar durations are calculated at prices corresponding to the strike price of the futures option (66), rather than the futures price (70). The hedge ratio is then

$$\text{Hedge ratio} = \frac{\$326{,}070}{\$2{,}548} \times 0.9660 = 124 \text{ put options}$$

Thus, to hedge the Southern Bell position with put options on Treasury bond futures, 124 put options must be purchased.

Outcome of the Hedge

To create a table for the protective put hedge, we can use some of the numbers from Exhibit 13.2. Everything will be the same except the last two columns. For the put option hedge we have to insert the value of the 124 futures put options in place of the 124 futures contracts in the next-to-last column. This is easy because the value of each option at expiration is just the strike price of the futures option (66) minus the futures price (or zero if that difference is negative), all multiplied by \$1,000. The effective sale price for the hedged bonds is then just the actual market price for the hedged bonds plus the value of the options at expiration minus the cost of the options.

Suppose that the price of the put futures option with a strike price of 66 is 24. An option price of 24 means $^{24}\!/_{64}$ of 1% of par value, or \$375. With a total of 124 options, the cost of the protection would have been \$46,500 (124 × \$375, not including financing costs and commissions). This cost, together with the final value of the options, is combined with the actual sale price of the hedged bonds to arrive at the effective sale price for the hedged bonds. These final prices are shown in the last column of Exhibit 13.6. This effective price is never less than 87.203. This equals the price of the hedged bonds equivalent to the futures strike price of 66 (i.e., 87.668), minus the cost of the puts (that is, $0.4650 = 1.24 \times {}^{24}\!/_{64}$). This minimum effective price is something that can be calculated before the hedge is ever initiated. As prices decline, the effective sale price actually exceeds the projected effective minimum sale price of 87.203 by a small amount. This is due only to rounding and the fact that the hedge ratio is left unaltered although the relative dollar durations that go into the hedge ratio calculation change as yields change. As prices increase, however, the effective sale price of the hedged bonds increases as well; unlike the futures hedge shown in Exhibit 13.2, the options hedge protects the investor if rates rise, but allows the investor to profit if rates fall.

Covered Call Writing with Futures Options

Unlike the protective put strategy, covered call writing is not entered into with the sole purpose of protecting a portfolio against rising rates. The covered call writer, believing that the market will not trade much higher or much lower than its present level, sells out-of-the-money calls against an existing bond portfolio. The sale of the calls brings in premium income that provides partial protection in case rates increase. The premium received does not, of course, provide the kind of protection that a long put position provides, but it does provide some additional income that can be used to offset declining prices. If, on the other hand, rates fall, portfolio appreciation is limited because the short call position constitutes a liability for the seller, and this liability increases as rates go down. Consequently, there is limited upside potential for the

EXHIBIT 13.6 Hedging a Nondeliverable Bond to a Delivery Date with Puts on Futures: Scenario Analysis

Instrument to be hedged: Southern Bell 11¾s of 4/19/23
Hedge ratio = 124 puts
Strike price for puts on futures = 66-0
Target minimum price for hedged bonds = 87.203
Option price per contract = $375

Actual Sale Price of Hedged Bonds ($)	Yield at Sale (%)	Futures Price[a]	Value of 124 Put Options ($)[b]	Cost of 124 Put Options ($)	Effective Sale Price ($)[c]
7,600,000	15.468	56.511	1,176,636	46,500	8,730,136
7,800,000	15.072	58.144	974,144	46,500	8,727,644
8,000,000	14.696	59.773	772,148	46,500	8,725,648
8,200,000	14.338	61.401	570,276	46,500	8,723,776
8,400,000	13.996	63.030	368,280	46,500	8,721,780
8,600,000	13.671	64.649	167,524	46,500	8,721,024
8,800,000	13.359	66.271	0	46,500	8,753,500
9,000,000	13.061	67.888	0	46,500	8,953,500
9,200,000	12.776	69.497	0	46,500	9,153,500
9,400,000	12.503	71.100	0	46,500	9,353,500
9,600,000	12.240	72.705	0	46,500	9,553,500
9,800,000	11.988	74.299	0	46,500	9,753,500
10,000,000	11.745	75.892	0	46,500	9,953,500
10,200,000	11.512	77.473	0	46,500	10,153,500
10,400,000	11.287	79.052	0	46,500	10,353,500
10,600,000	11.070	80.625	0	46,500	10,553,500
10,800,000	10.861	82.188	0	46,500	10,753,500
11,000,000	10.659	83.746	0	46,500	10,953,500
11,200,000	10.463	85.303	0	46,500	11,153,500

[a] These numbers are approximate because futures trade in even 32nds.
[b] From 124 × $1,000 × Max{(66 – Futures Price), 0}.
[c] Does not include transaction costs or the financing of the options position.

covered call writer. Of course, this is not so bad if prices are essentially going nowhere; the added income from the sale of call options is obtained without sacrificing any gains.

To see how covered call writing with futures options works for the bond used in the protective put example, we construct a table much as we did before. With futures selling around 71-24 on the hedge initiation date, a sale of a 78 call option on futures might be appropriate. As before,

EXHIBIT 13.7 Hedging a Nondeliverable Bond to a Delivery Date with Calls on Futures: Scenario Analysis

Instrument to be hedged: Southern Bell 11¾s of 4/19/23
Hedge ratio = 124 calls
Strike price for calls on futures = 78-0
Expected maximum price for hedged bonds = 103.131
Option price per contract = $375

Actual Sale Price of Hedged Bonds ($)	Yield at Sale (%)	Futures Price[a]	Liability of 124 Call Options ($)[b]	Premium from 124 Call Options ($)	Effective Sale Price ($)[c]
7,600,000	15.468	56.511	0	46,500	7,646,500
7,800,000	15.072	58.144	0	46,500	7,846,500
8,000,000	14.696	59.773	0	46,500	8,046,500
8,200,000	14.338	61.401	0	46,500	8,246,500
8,400,000	13.996	63.030	0	46,500	8,446,500
8,600,000	13.671	64.649	0	46,500	8,646,500
8,800,000	13.359	66.271	0	46,500	8,846,500
9,000,000	13.061	67.888	0	46,500	9,046,500
9,200,000	12.776	69.497	0	46,500	9,246,500
9,400,000	12.503	71.100	0	46,500	9,446,500
9,600,000	12.240	72.705	0	46,500	9,646,500
9,800,000	11.988	74.299	0	46,500	9,846,500
10,000,000	11.745	75.892	0	46,500	10,046,500
10,200,000	11.512	77.473	0	46,500	10,246,500
10,400,000	11.287	79.052	130,448	46,500	10,316,052
10,600,000	11.070	80.625	325,500	46,500	10,321,000
10,800,000	10.861	82.188	519,312	46,500	10,327,188
11,000,000	10.659	83.746	712,504	46,500	10,333,996
11,200,000	10.463	85.303	905,572	46,500	10,340,928

[a] These numbers are approximate because futures trade in even 32nds.
[b] From 124 × $1,000 × Max{(Futures Price – 76),0}.
[c] Does not include transaction costs.

it is assumed that the hedged bond will remain at a 90-basis-point spread off the CTD (the 7⅝s of 2007). We also assume for simplicity that the price of the 78 calls is 24⁄64. The number of options contracts sold will be the same, namely 124 contracts for $10 million face value of underlying bonds. Exhibit 13.7 shows the results of the covered call writing strategy given these assumptions.

To calculate the effective sale price of the bonds in the covered call writing strategy, the premium received from the sale of calls is added to the actual sale price of the bonds, while the liability associated with the short call position is subtracted from the actual sale price. The liability associated with each call is the futures price minus the strike price of 78 (or zero if this difference is negative), all multiplied by $1,000. The middle column in Exhibit 13.7 is just this value multiplied by 124, the number of options sold.

Just as the minimum effective sale price could be calculated beforehand for the protective put strategy, the maximum effective sale price can be calculated beforehand for the covered call writing strategy. The maximum effective sale price will be the price of the hedged security corresponding to the strike price of the option sold, plus the premium received. In this case, the strike price on the futures call option was 76. A futures price of 76 corresponds to a price of 75.348 (from 76 times the conversion factor), and a corresponding yield of 10.536% for the CTD (the 7⅝s of 2007). The equivalent yield for the hedged bond is 90 basis points higher, or 11.436%, for a corresponding price of 102.666. Adding on the premium received, 0.465 points, the final maximum effective sale price will be about 103.131. As Exhibit 13.7 shows, if the hedged bond does trade at 90 basis points over the CTD as assumed, the maximum effective sale price for the hedged bond is, in fact, slightly over 103. The discrepancies shown in the exhibit are due to rounding and the fact that the position is not adjusted even though the relative dollar durations change as yields change.

Comparing Alternative Strategies

We reviewed three basic strategies for hedging a bond position: (1) hedging with futures, (2) hedging with out-of-the-money puts, and (3) covered call writing with out-of-the-money calls. Similar, but opposite, strategies exist for those whose risks are that rates will decrease. As might be expected, there is no "best" strategy. Each strategy has its advantages and its disadvantages, and we never get something for nothing. To get anything of value, something else of value must be forfeited.

To make a choice among strategies, it helps to lay the alternatives side by side. Using the futures and futures options examples from this chapter, Exhibit 13.8 shows the final values of the portfolio for the various hedging alternatives. It is easy to see from Exhibit 13.8 that if one alternative is superior to another alternative at one level of rates, it will be inferior at some other level of rates.

Consequently, we cannot conclude that one strategy is the best strategy. The manager responsible for selecting the strategy makes a choice

EXHIBIT 13.8 Alternative Hedging Strategies Compared

Actual Sale Price of Bonds ($)	Yield at Sale (%)	Effective Sale Price with Futures Hedge ($)	Effective Sale Price with Protective Puts ($)	Effective Sale Price with Covered Calls ($)
7,600,000	15.468	9,272,636	8,730,136	7,646,500
7,800,000	15.072	9,270,144	8,727,644	7,846,500
8,000,000	14.696	9,268,148	8,725,648	8,046,500
8,200,000	14.338	9,266,276	8,723,776	8,246,500
8,400,000	13.996	9,264,280	8,721,780	8,446,500
8,600,000	13.671	9,263,524	8,721,024	8,646,500
8,800,000	13.359	9,262,396	8,753,500	8,846,500
9,000,000	13.061	9,261,888	8,953,500	9,046,500
9,200,000	12.776	9,262,372	9,153,500	9,246,500
9,400,000	12.503	9,263,600	9,353,500	9,446,500
9,600,000	12.240	9,264,580	9,553,500	9,646,500
9,800,000	11.988	9,266,924	9,753,500	9,846,500
10,000,000	11.745	9,269,392	9,953,500	10,046,500
10,200,000	11.512	9,273,348	10,153,500	10,246,500
10,400,000	11.287	9,277,552	10,353,500	10,316,052
10,600,000	11.070	9,282,500	10,553,500	10,321,000
10,800,000	10.861	9,288,688	10,753,500	10,327,188
11,000,000	10.659	9,295,496	10,953,500	10,333,996
11,200,000	10.463	9,302,428	11,153,500	10,340,928

among probability distributions, not usually among specific outcomes. Except for the perfect hedge, there is always some range of possible final values of the portfolio. Of course, exactly what that range is, and the probabilities associated with each possible outcome, is a matter of opinion.

Hedging with Options on Cash Instruments

Hedging a position with options on cash bonds is relatively straightforward. Most strategies, including the purchase of protective puts, covered call writing, and creating collars, are essentially the same whether futures options or options on physicals are used. As explained in Chapters 10 and 11, there are some mechanical differences in the way the two types of contracts are traded, and there may be substantial differences in the liquidity of the two types of contracts. Nonetheless, the basic economics of the strategies are virtually identical.

Using options on physicals frequently relieves the manager of much of the basis risk associated with an options hedge. For example, a manager of Treasury bonds or notes can usually buy or sell options on the exact security held in the portfolio. Using options on futures, rather than options on Treasury bonds, is sure to introduce additional elements of uncertainty.

Given the illustration presented above, and given that the economics of options on physicals and options on futures are essentially identical, additional illustrations for options on physicals are unnecessary. The only important difference is the hedge ratio calculation and the calculation of the equivalent strike. To derive the hedge ratio, we always resort to an expression of relative dollar durations. Thus, for options on physicals, assuming a constant spread the hedge ratio is

$$\text{Hedge ratio} = \frac{\text{Current dollar duration without options}}{\text{Dollar duration of underlying for option}}$$

If a relationship is estimated between the yield on the bonds to be hedged and the instrument underlying the option, the appropriate hedge ratio is

$$\text{Hedge ratio} = \frac{\text{Current dollar duration without options}}{\text{Dollar duration of underlying for option}} \times \text{Yield beta}$$

Unlike futures options, there is only one deliverable, so there is no conversion factor. When cross hedging with options on physicals, the procedure for finding the equivalent strike price on the bonds to be hedged is very similar. Given the strike price of the option, the strike yield is easily determined using the price/yield relationship for the instrument underlying the option. Then given the projected relationship between the yield on the instrument underlying the option and the yield on the bonds to be hedged, an equivalent strike yield is derived for the bonds to be hedged. Finally, using the yield-to-price formula for the bonds to be hedged, the equivalent strike price for the bonds to be hedged can be found.

CONTROLLING INTEREST RATE RISK WITH SWAPS

As we explained in Chapter 10, an interest rate swap is equivalent to a package of forward/futures contracts. Consequently, swaps can be used in the same way as futures and forwards for controlling interest rate risk. The dollar duration of an interest rate swap was explained in Chapter 10.

The following illustration demonstrates how an interest rate swap can be used to hedge interest rate risk by altering the cash flow characteristics of a portfolio so as to match assets and liabilities. In our illustration we will use two hypothetical financial institutions—a commercial bank and a life insurance company.

Suppose a bank has a portfolio consisting of 4-year term commercial loans with a fixed interest rate. The principal value of the portfolio is $100 million, and the interest rate on all the loans in the portfolio is 11%. The loans are interest-only loans; interest is paid semiannually, and the principal is paid at the end of four years. That is, assuming no default on the loans, the cash flow from the loan portfolio is $5.5 million every six months for the next four years and $100 million at the end of four years. To fund its loan portfolio, assume that the bank can borrow at 6-month Libor for the next four years.

The risk that the bank faces is that 6-month Libor will be 11% or greater. To understand why, remember that the bank is earning 11% annually on its commercial loan portfolio. If 6-month Libor is 11%, there will be no spread income. Worse, if 6-month Libor rises above 11%, there will be a loss; that is, the cost of funds will exceed the interest rate earned on the loan portfolio. The bank's objective is to lock in a spread over the cost of its funds.

The other party in the interest rate swap illustration is a life insurance company that has committed itself to pay an 8% rate for the next four years on a guaranteed investment contract (GIC) it has issued. The amount of the GIC is $100 million. Suppose that the life insurance company has the opportunity to invest $100 million in what it considers an attractive 4-year floating-rate instrument in a private placement transaction. The interest rate on this instrument is 6-month Libor plus 120 basis points. The coupon rate is set every six months. The risk that the life insurance company faces in this instance is that 6-month Libor will fall so that the company will not earn enough to realize a spread over the 8% rate that it has guaranteed to the GIC policyholders. If 6-month Libor falls to 6.8% or less, no spread income will be generated. To understand why, suppose that 6-month Libor at the date the floating-rate instrument resets its coupon is 6.8%. Then the coupon rate for the next six months will be 8% (6.8% plus 120 basis points). Because the life insurance company has agreed to pay 8% on the GIC policy, there will be no spread income. Should 6-month Libor fall below 6.8%, there will be a loss.

We can summarize the asset/liability problems of the bank and the life insurance company as follows:

Bank:
1. Has lent long term and borrowed short term.
2. If 6-month Libor rises, spread income declines.

Life insurance company:
1. Has lent short term and borrowed long term.
2. If 6-month Libor falls, spread income declines.

Now let's suppose the market has available a 4-year interest rate swap with a notional principal amount of $100 million. The swap terms available to the bank are as follows:

1. Every six months the bank will pay 9.50% (annual rate).
2. Every six months the bank will receive Libor.

The swap terms available to the insurance company are as follows:

1. Every six months the life insurance company will pay Libor.
2. Every six months the life insurance company will receive 9.40%.

Now let's look at the position of the bank and the life insurance company after the swap. Exhibit 13.9 summarizes the position of each institution before and after the swap. Consider first the bank. For every 6-month period for the life of the swap agreement, the interest rate spread will be as follows:

Annual interest rate received:

From commercial loan portfolio	=	11.00%
From interest rate swap	=	6-month Libor
Total	=	11.00% + 6-month Libor

Annual interest rate paid:

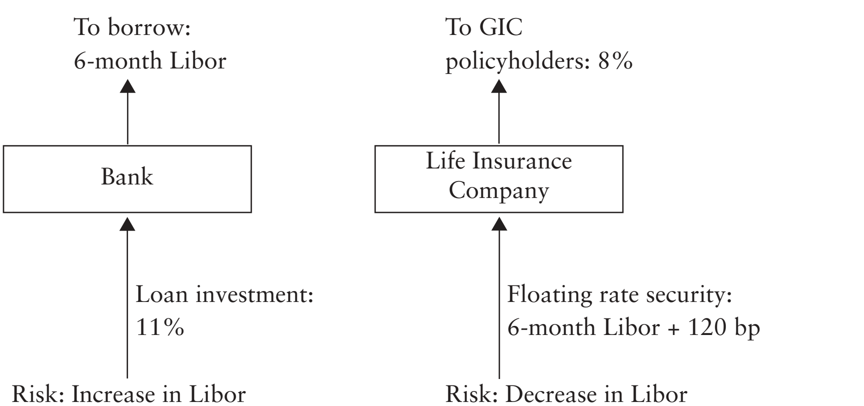
To borrow funds	=	6-month Libor
On interest rate swap	=	9.50%
Total	=	9.50% + 6-month Libor

Outcome:

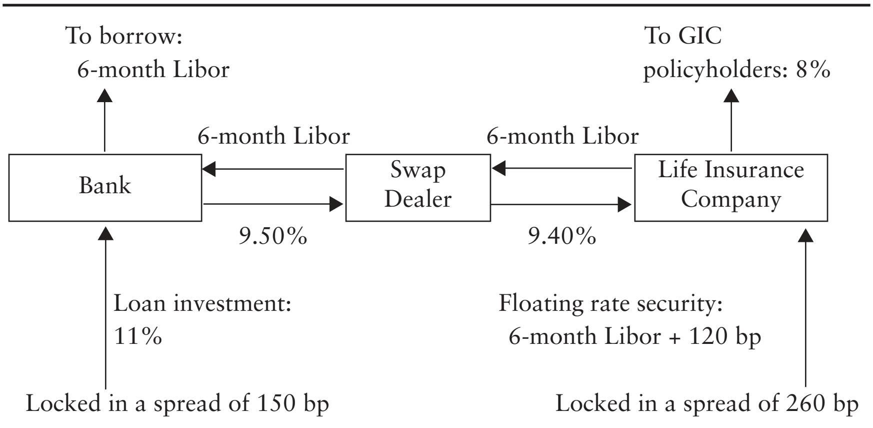
To be received	=	11.00% + 6-month Libor
To be paid	=	9.50% + 6-month Libor
Spread income	=	1.50% or 150 basis points

EXHIBIT 13.9 **Position of Bank and Life Insurance Company Before and After Swap**

Position before interest rate swap:



Position after interest rate swap:



Thus, whatever happens to 6-month Libor, the bank locks in a spread of 150 basis points.

Now let's look at the effect of the interest rate swap on the life insurance company:

Annual interest rate received:

From floating-rate instrument	=	1.20% + 6-month Libor
From interest rate swap	=	9.40%
Total	=	10.60% + 6-month Libor

Annual interest rate paid:

To GIC policyholders	=	8.00%
On interest rate swap	=	6-month Libor
Total	=	8.00% + 6-month Libor

Outcome:

To be received	=	10.60% + 6-month Libor
To be paid	=	8.00% + 6-month Libor
Spread income	=	2.60% or 260 basis points

Regardless of what happens to 6-month Libor, the life insurance company locks in a spread of 260 basis points.

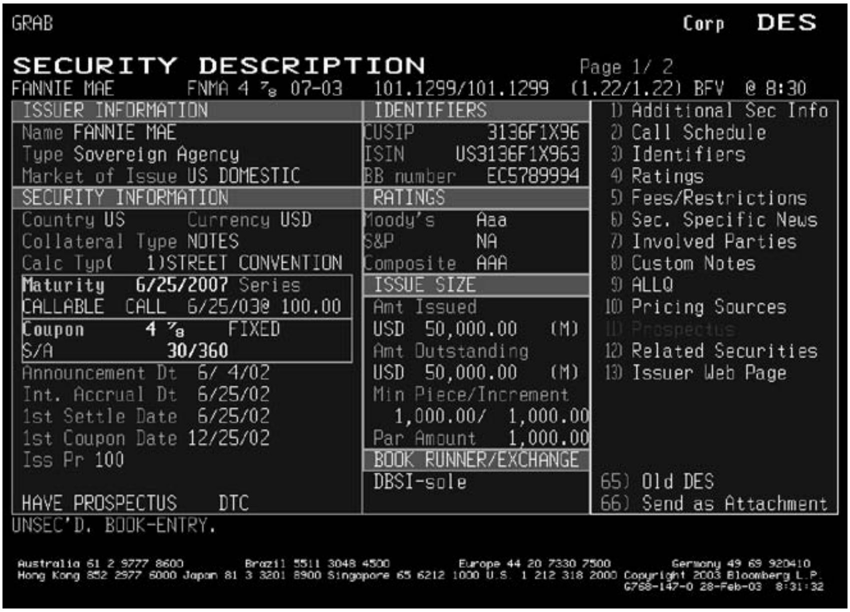
The interest rate swap has allowed each party to accomplish its asset/liability objective of locking in a spread.[7] It permits the two financial institutions to alter the cash flow characteristics of its assets: from fixed to floating in the case of the bank, and from floating to fixed in the case of the life insurance company. This type of transaction is referred to as an *asset swap*. (We'll have more to say about asset swaps later.) Another way the bank and the life insurance company could use the swap market would be to change the cash flow nature of their liabilities. Such a swap is called a *liability swap*.

Role of Swaptions

Suppose that a financial institution has a long position in Fannie Mae notes with a coupon of 4.875% due June 25, 2007. These notes are callable on one date only June 25, 2003 at par. Exhibit 13.10 presents Bloomberg's Security Description screen for these Fannie Mae notes. Suppose at the end of February 2003, the financial institution engages in an asset swap such they will take a long position in a swap contract (pay fixed/receive floating) with a swap rate of 2.91421% and a tenor that matches the remaining maturity of the Fannie Mae Notes (i.e., the swap contract expires on June 25, 2007). The combination of the notes and the swap contract effectively converts the notes' cash flows from fixed into floating. Simply put, the financial institution has created a synthetic Libor floater with a relatively sizable spread via the asset swap. This strategy's obvious risk is what happens if interest rate drop and Fannie Mae calls the issue on June 25, 2003? If this occurs, the financial institution must continue to pay fixed/receive floating on the swap but will no longer receive the coupon payments from the notes.

[7] Whether the size of the spread is adequate is not an issue in this illustration.

EXHIBIT 13.10 **Bloomberg Security Description Screen for a Fannie Mae Note**



Source: Bloomberg Financial Markets

To hedge this risk, the financial institution could purchase a receive fixed swaption which gives the buyer the right to establish a position in an interest rate swap of a particular tenor such that they will receive the fixed rate cash flows and pay the floating rate cash flows. The expiration date on the swaption should match the call date of the Fannie Mae notes—June 25, 2003. The tenor of the underlying swap should be four years beginning on June 25, 2003 and ending on June 25, 2007. Lastly, the strike rate of the swaption should be set equal to the existing swap rate of 2.91421%. If Fannie Mae calls the notes, the financial institution will simply exercise the swaption and establish a position in a swap whose cash flows exactly offset the cash flows of the existing swap. The asset swap is neutralized as a result. Conversely, if Fannie Mae does not call the notes, the financial institution will simply allow the swaption to expire and the asset swap will be in place until the notes mature on June 25, 2007.

Asset Swaps

As explained earlier, in an asset swap an investor used an interest rate swap to convert the character of a cash flow from fixed to floating or vice versa. A common use of an interest rate in an asset swap is for an investor to buy a credit-risky bond with a fixed rate and convert it to a floating rate. If the issuer of the bond defaults on the issue, the investor must continue to make payments to the dealer and is therefore still exposed to the credit risk of the issuer.

Let's now illustrate a basic asset swap. Suppose that an investor purchases $20 million par value of a 6.85%, 5-year telecom bond rated triple B at par value. The coupon payments are semiannual. At the same time, the investor enters into a 5-year interest rate swap with a dealer where the investor is the fixed-rate payer and the payments are made semiannually. Suppose that the swap rate is 6.00% and the investor receives 6-month Libor plus 45 basis points.

Let's look at the cash flow for the investor every six months for the next five years:

	Receives from telecom bonds:	6.85%
–	Pays to dealer on swap:	6.00%
+	Receives from dealer on swap:	6-month Libor
	Net received by investor:	0.85% + 6-month Libor

Thus, regardless of how interest rates change, if the telecom issuer does not default on the issue, the investor earns 85 basis points over 6-month Libor. Effectively, the investor has converted a fixed-rate triple B 5-year telecom bond into a 5-year floating-rate bond with a spread over 6-month Libor. Thus, the investor has created a synthetic floating-rate bond. The purpose of an asset swap is to due precisely that: Create a synthetic credit risky floating-rate security.

Asset Swap Structure (Package) Created by a Dealer

In our description of an asset swap the investor bought the credit-risky bond and entered into an interest rate swap with a dealer. Typically, an asset swap combines the sale of a credit-risky bond owned to a counterparty, at par and with no interest accrued, with an interest rate swap. This type asset swap structure or package is referred to as a *par asset swap*. If there is a default by the issuer of the credit-risky bond, the asset swap transaction is terminated and the defaulted bonds are returned to the investor plus or minus any mark-to-market on the asset swap transaction. Hence, the investor is still exposed to the issuer's credit risk.

The coupon on the bond in the par asset swap is paid in return for Libor, plus a spread if necessary. This spread is the asset-swap spread and is the price of the asset swap. In effect the asset swap allows investors that pay Libor-based funding to receive the asset-swap spread. This spread is a function of the credit risk of the underlying credit-risky bond. The asset-swap spread may be viewed as equivalent to the price payable on a credit default swap written on that asset.

To illustrate this asset swap structure, suppose that in our previous illustration the swap rate prevailing in the market is 6.30% rather than 6%. The investor owns the telecom bonds and sells them to a dealer at par with no accrued interest. The asset swap agreement between the dealer and the investor is as follows:

1. The term is five years.
2. The investor agrees to pay the dealer semiannually 6.30%.
3. The dealer agrees to pay the investor every six months 6-month Libor plus an asset-swap spread of 30 basis points.

Let's look at the cash flow for the investor every six months for the next five years in this asset swap structure:

	Receives from telecom bonds:	6.85%
–	Pays to dealer on swap:	6.30%
+	Receives from dealer on swap:	6-month Libor + 30 basis points
	Net received by investor:	0.85% + 6-month Libor

In our first illustration of an asset swap, the investor is creating a synthetic floater without a dealer. The investor owns the bonds. The only involvement of the dealer is as a counterparty to the interest rate swap. In the second structure, the dealer is the counterparty to the asset swap structure and the dealer owns the underlying credit-risky bonds. If there is a default, the dealer returns the bonds to the investor.

Variations of the Basic Asset Swap Structure

There are variations of the basic asset swap structure to remove unwanted noncredit structural features of the underlying credit risky bond. The simplest example of an asset swap variation to remove an unwanted noncredit structural feature is when the bond is callable. If the bond is callable, then the future cash flows of the bond are uncertain because the issuer can be called. Moreover, the issue is likely to be called if interest rates decline below the bonds coupon rate.

This problem can be handled in the case where the investor buys the bond and enters into an interest rate swap. The tenor of the interest rate swap would still be for the term of the bond. However, the investor would also enter into a swaption in which the investor has the right to effectively terminate the swap from the time of the first call date for the bond to the maturity date for the bond. In the swaption, since the investor is paying fixed and receiving floating, the swaption must be one in which the investor receives fixed and pays floating. Specifically, the investor will enter into a receive fixed swaption.

Asset Swap Classification as a Credit Derivative

In Chapter 16 we will discuss credit derivatives. Credit derivatives allow investors to manage the credit risk exposure of their portfolio or asset holdings, essentially by providing protection against a deterioration in credit quality of the borrowing entity. While an asset swap is not a true credit derivative, it is closely associated with the credit derivatives market because it explicitly sets out the price of credit as a spread over Libor. It allows the acquiring of credit risk while minimizing interest rate risk but it does not allow an investor to transfer credit risk. It is because of this shortcoming of an asset swap that the other types of derivative instruments and structured products that will be described in Chapter 16 were created.

USING CAPS AND FLOORS TO CONTROL RISK

Interest rate caps can be used by a liability manager to create a cap for funding costs. Combining a cap and a floor creates a collar for funding costs. Floors can be used by buyers of floating-rate instruments to set a floor on the periodic interest earned. To reduce the cost of a floor, a manager can sell a cap. By doing so, the manager limits the upside on the coupon rate of a floating rate instrument should rates rise, thereby creating a collar for the coupon interest on a floating-rate instrument.

To see how interest rate agreements can be used for asset/liability management, consider the problems faced by the commercial bank and the life insurance company we just discussed in demonstrating the use of an interest rate swap. The bank's objective is to lock in a spread over its cost of funds. Yet because it borrows short term, its cost of funds is uncertain. The bank may be able to purchase a cap, however, so that the cap rate plus the cost of purchasing the cap is less than the rate it is earning on its fixed-rate commercial loans. If short-term rates decline, the bank does not benefit from the cap, but its cost of funds declines. The cap therefore allows the bank to impose a ceiling on its cost of funds while retaining the opportunity to benefit from a decline in rates. This is consistent with the view of an interest rate cap as simply a package of options.

The bank can reduce the cost of purchasing the cap by selling a floor. In this case, the bank agrees to pay the buyer of the floor if the reference rate falls below the strike rate. The bank receives a fee for selling the floor, but it has sold off its opportunity to benefit from a decline in rates below the strike rate. By buying a cap and selling a floor, the bank has created a predetermined range for its cost of funds (i.e., a collar).

Recall the problem of the life insurance company that guarantees a 9% rate on a GIC for the next four years and is considering the purchase of an attractive floating-rate instrument in a private placement transaction. The risk that the company faces is that interest rates will fall so that it will not earn enough to realize the 9% guaranteed rate plus a spread. The life insurance company may be able to purchase a floor to set a lower bound on its investment return, yet retain the opportunity to benefit should rates increase. To reduce the cost of purchasing the floor, the life insurance company can sell an interest rate cap. By doing so, however, it gives up the opportunity of benefiting from an increase in the reference rate above the strike rate of the interest rate cap.

KEY POINTS

1. A macro risk control strategy is one used to control the interest rate risk of a portfolio without regard to the price movement of any individual bond comprising the portfolio.

2. A micro risk control strategy can be implemented to control the risk of an individual bond or a group of bonds with similar characteristics.

3. There are four preliminary steps that should be taken before a risk control strategy is initiated so that a manager can assess what a hedge strategy can and cannot accomplish.

4. The key factor to determine which derivative instrument or instruments to use is the degree of correlation between the rate underlying the derivative instrument and the rate that creates the risk that the manager seeks to control.

5. Buying an interest rate futures contract increases a portfolio's duration; selling an interest rate futures contract decreases a portfolio's duration.

6. The advantages of adjusting a portfolio's duration using futures rather than cash market instruments are transactions costs are lower, margin requirements are lower, and it is easier to sell short in the futures market.

7. The general principle in controlling interest rate risk with futures is to combine the dollar exposure of the current portfolio and that of a futures position so that it is equal to the target dollar exposure.

8. The number of futures contracts needed to achieve the target dollar duration depends on the current dollar duration of the portfolio without futures and the dollar duration per futures contract.

9. Hedging with futures calls for taking a futures position as a temporary substitute for transactions to be made in the cash market at a later date, with the expectation that any loss realized by the manager from one position (whether cash or futures) will be offset by a profit on the other position.

10. Hedging is a special case of controlling interest rate risk in which the target duration or target dollar duration is zero.

11. Cross hedging occurs when the bond to be hedged is not identical to the bond underlying the futures contract.

12. A short or sell hedge is used to protect against a decline in the cash price of a bond; a long or buy hedge is employed to protect against an increase in the cash price of a bond.

13. The manager should determine the target rate or target price, which is what is expected from the hedge.

14. The manager should estimate the hedge effectiveness, which indicates what percent of risk is eliminated by hedging.

15. The manager should estimate the residual hedging risk, which is the absolute level of risk in the hedged position and indicates how much risk remains after hedging.

16. The target rate, the hedge effectiveness, and the residual hedging risk determine the basic trade-off between risk and expected return and these statistics give the manager the information needed to decide whether to employ a hedge strategy.

17. The hedge ratio is the number of futures contracts needed for the hedge.

18. The basis is the difference between the spot price (or rate) and the futures price (or rate).

19. In general, when hedging to the delivery date of the futures contract, a manager locks in the futures rate or price.

20. Hedging with Treasury bond futures and Treasury note futures is complicated by the delivery options embedded in these contracts.

21. When a hedge is lifted prior to the delivery date, the effective rate (or price) that is obtained is much more likely to approximate the current spot rate than the futures rate the shorter the term of the hedge.

22. The proper target for a hedge that is to be lifted prior to the delivery date depends on the basis.

23. Basis risk refers only to the uncertainty associated with the target rate basis or target price basis.

24. Hedging substitutes basis risk for price risk.

25. Hedging non-Treasury securities with Treasury bond futures requires that the hedge ratio consider two relationships: (1) the cash price of the non-Treasury security and the cheapest-to-deliver issue and (2) the price of the cheapest-to-deliver issue and the futures price.

26. After a target is determined and a hedge is established, the hedge must be monitored during its life and evaluated after it is over and the sources of error in a hedge should be determined in order to gain insights that can be used to advantage in subsequent hedges.

27. Three popular hedge strategies are the protective put buying strategy, the covered call writing strategy, and the collar strategy.

28. A manager can use a protective put buying strategy—a combination of a long put option with a long position in a cash bond—to hedge against rising interest rates.

29. A covered call writing strategy involves selling call options against the bond portfolio.

30. A covered call writing strategy entails much more downside risk than buying a put to protect the value of the portfolio and many portfolio managers do not consider covered call writing a hedge.

31. It is not possible to say that the protective put strategy or the covered call writing strategy is necessarily the better or more correct options' hedge since it depends upon the manager's view of the market.

32. A collar strategy is a combination of a protective put strategy and a covered call writing strategy that eliminates part of the portfolio's downside risk by giving up part of its upside potential.

33. The best options contract to use depends upon the option price, liquidity, and correlation with the bond(s) to be hedged.

34. For a cross hedge, the manager will want to convert the strike price on the options that are actually bought or sold into an equivalent strike price for the actual bonds being hedged.

35. When using Treasury bond futures options, the hedge ratio is based on the relative dollar duration of the current portfolio, the cheapest-to-deliver issue, and the futures contract at the option expiration date, as well as the conversion factor for the cheapest-to-deliver issue.

36. An interest rate swap can be used to hedge interest rate risk by altering the cash flow characteristics of a portfolio of assets so as to match asset and liability cash flows.

37. A position in a swap can expose a position to greater interest rate risk if it is not coupled with a swaption.

38. An asset swap allows an investor to alter the cash flow character of an asset.

39. An asset swap can be created by an investor by buying a credit-risky bond and entering into an interest rate swap as the fixed-rate payer or by an investor selling a credit-risky bond purchased to a dealer and having the dealer create an asset swap package.

40. Interest rate caps can be used in liability management to create a cap for funding costs.

41. Combining a cap and a floor creates a collar for funding costs.

42. Floors can be used by buyers of floating-rate instruments to set a floor on the periodic interest earned and the sale of a cap can reduce the cost of a floor.

CHAPTER 14

Controlling Interest Rate Risk of an MBS Derivative Portfolio*

Investors in mortgage-backed securities (MBS) are exposed to level risk and yield curve risk. While we have demonstrated the yield curve risk for a bond portfolio, the value of an individual MBS is particularly vulnerable to changes in the shape of the yield curve. In this chapter, we will present a fairly simple approach to systematically measure and control the exposure of an MBS portfolio to changes in the level and slope of the yield curve. More specifically, we look at a portfolio of MBS derivative products. These products include collateralized mortgage obligations (CMOs) and stripped mortgage-backed securities (mortgage strips).[1]

> The objectives of this chapter are to:
> 1. Review the slope elasticity measure of yield curve risk.
> 2. Explain what is meant by positive and negative slope elasticity.
> 3. Demonstrate the importance of yield curve risk for an MBS.
> 4. Look at the yield curve risk for different types of MBS derivative products.
> 5. Look at the yield curve risk for different potential hedging instruments.
> 6. Demonstrate the steps that a manager can follow to measure and control level and yield curve risk exposures of an MBS portfolio.

[1] It is assumed that the reader is familiar with these products. For a description, see Frank J. Fabozzi and Chuck Ramsey, *Collateralized Mortgage Obligations: Structures and Analysis* (Hoboken, NJ: John Wiley & Sons, Inc., 1999).

*This chapter is adapted from Michael P. Schumacher, Daniel C. Dektar, and Frank J. Fabozzi, "Yield Curve Risk of CMO Bonds," Chapter 15 in Frank J. Fabozzi (ed.), *CMO Portfolio Management* (Summit, NJ: Frank J. Fabozzi Associates, 1994).

SLOPE ELASTICITY MEASURE OF YIELD CURVE RISK: A REVIEW

In Chapter 3, we explained that the effective duration and convexity of a bond or portfolio is a measure of its exposure to changes in the level of interest rates. In Chapter 4, we demonstrated that duration and convexity are inadequate measures of rate changes if the yield curve does not shift in a parallel fashion. We then described several approaches to the measurement of yield curve risk. The simplest approach is the slope elasticity measure which was defined in Chapter 4. We shall review this measure here.

The slope elasticity measure looks at the sensitivity of a position or portfolio to changes in the slope of the yield curve. The yield curve slope can be defined as the spread between a long-term and short-term on-the-run Treasury yield. In this chapter, we will use the 3-month Treasury bill yield and 30-year Treasury yield as the short-term and long-term yields, respectively. This is basically the longest and the shortest points on the Treasury yield curve. While this is not a perfect definition, it captures most of the effect of changes in yield curve slope.

Changes in the yield curve can be defined as follows: Half of any basis point change in the yield curve slope results from a change in the 3-month yield and half from a change in the 30-year yield. For example, with a 100-basis-point steepening of the yield curve, the assumption is that 50 basis points of that steepening come from a rise in the 30-year yield, and another 50 basis points come from a fall in the 3-month yield.

The slope elasticity is then defined as the approximate negative percentage change in a bond's price resulting from a 100-basis-point change in the slope of the curve. Slope elasticity is calculated as follows:

1. Increase and decrease the yield curve slope.
2. Calculate the price change for these two scenarios after adjusting for the price effect of a change in the level of yields.
3. Compare the prices to the initial or base price.

More specifically, the slope elasticity for each scenario is calculated as follows:

$$\frac{\text{Price effect of a change in slope/Base price}}{\text{Change in yield curve slope}}$$

The slope elasticity is then the average of the slope elasticity for the two scenarios.

A bond or portfolio that benefits when the yield curve flattens is said to have *positive slope elasticity*; a bond or portfolio that benefits when the yield curve steepens is said to have *negative slope elasticity.*

YIELD CURVE RISK AND ITS IMPORTANCE

The definition of yield curve risk follows naturally from that of slope elasticity. It is defined as the exposure of the bond to changes in the slope of the yield curve.

As an illustration, Exhibit 14.1 shows the yield curve slope based on Treasury rates as of December 31, 1992. Exhibit 14.2 shows the price

EXHIBIT 14.1 Yield Curve Slope Based on Treasury Rates as of 12/31/92

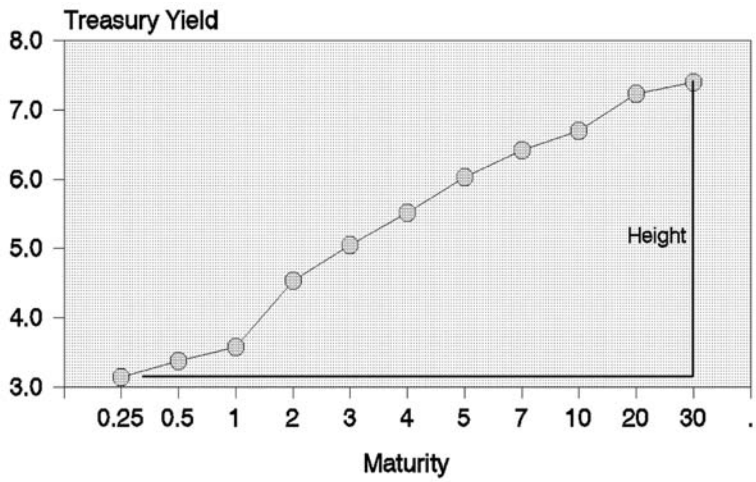
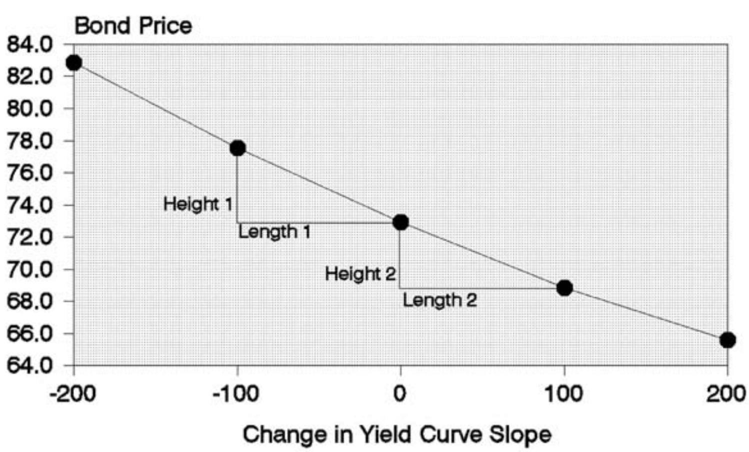


EXHIBIT 14.2 Yield Curve Risk for a Principal-Only Strip



behavior of a principal-only (PO) strip, given changes in the yield curve shown in Exhibit 14.1. As the curve flattens, the price of the PO increases substantially. As the yield curve steepens, the price of the PO declines.

This result is completely independent of changes in the level of interest rates. That is, we assume that the level of rates is fixed and therefore focus only on the effect of changes in the slope of the yield curve. Consequently, a portfolio manager who might hedge the effective duration of a PO strip position with interest rate futures will still face extremely significant exposure to changes in the slope of the yield curve.

This is important to remember when dealing with MBS derivatives because the structures are often complicated, and the cash flows may exhibit very odd patterns. While there is also yield curve risk for a passthrough, it is typically less significant than for mortgage derivatives.

Exhibit 14.3 illustrates the volatility of the yield curve slope from December 1983 to December 1992. In mid-1989 the yield curve was actually inverted. By 1992, the yield curve slope was more than 400 basis points, representing a remarkable steepening of the curve. While portfolio managers recognize that the yield curve slope has changed over time, what they may not realize is that this steepening has had an enormous effect on the value of some MBS derivatives.

For example, unhedged inverse floater positions typically benefit from both a decline in interest rates and a steepening of the yield curve, but the same position may be adversely affected if the yield curve flattens.

EXHIBIT 14.3 **Historical Yield Curve Slope: December 1983–December 1992**

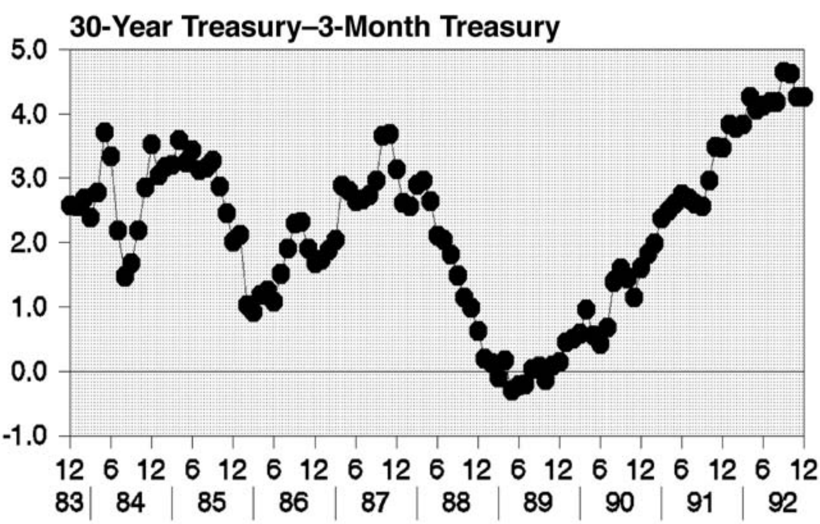
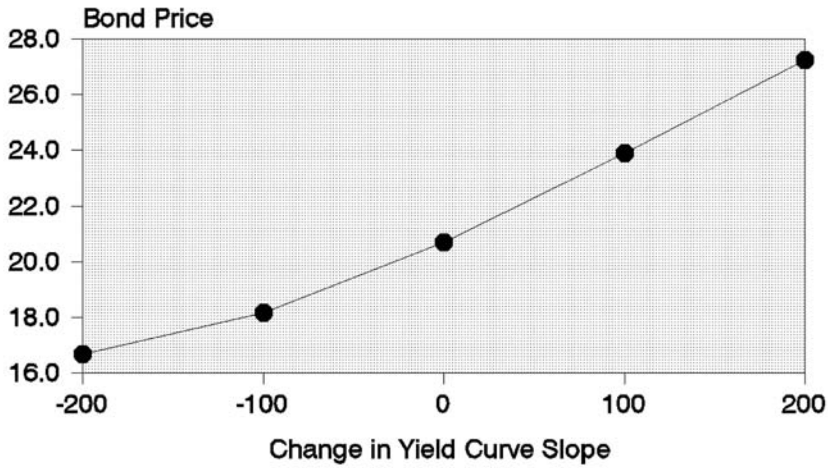


EXHIBIT 14.4 Yield Curve Risk of Interest-Only Strip



A graphic example of yield curve risk and its importance in the payoff pattern of an interest-only (IO) strip is shown in Exhibit 14.4. The particular IO shown in the exhibit is Trust 2, backed by FNMA 10% fixed-rate passthroughs. As the yield curve steepens, the IO appreciates significantly. If the yield curve flattens, the IO declines significantly. Basically, this pattern is the opposite of what we saw for the PO in Exhibit 14.2. This should not be surprising, as IOs and POs typically move in opposite directions with regard to parallel shifts in the yield curve. The same thing is true with regard to changes in the slope of the yield curve.

YIELD CURVE RISK FOR DIFFERENT MBS DERIVATIVES

Now that we have demonstrated the importance of yield curve risk, we examine the actual slope exposure for a variety of MBS derivatives. The analysis is based on projected prices under different yield curve scenarios using a valuation model. In this chapter, the analysis was performed using the Smith Breeden pricing model at the time. It should be noted that yield curve slope exposure is very structure-specific. Consequently, it is difficult to generalize about the slope exposure of MBS derivatives.

The approach we will use to determine the slope exposure of a particular MBS derivative is to assess the impact of changes in yield curve slope on three factors affecting the value of the bond: discount rates, projected prepayment rates (cash flows), and embedded caps and floors. The net slope exposure of the bond is essentially the sum of these three slope components.

The impact of a change in yield curve slope on the appropriate series of discount rates for an MBS derivative is usually clear-cut. For instance, a flattening yield curve implies discount rates on distant cash flows decrease, while discount rates on near cash flows increase. A security with a long average life benefits from this change in discount rates because its cash flows are weighted toward the long end of the yield curve. Conversely, a security with a relatively short average life suffers from the change in discount rates resulting from a flattening yield curve because its cash flows are weighted toward the short end of the yield curve.

The second factor affecting an MBS derivative's yield curve exposure is the impact of a change in yield curve slope on the bond's projected cash flows. As the yield curve flattens, forward rates decrease; consequently, anticipated prepayment rates increase, and the MBS derivative's expected life typically shortens. Not surprisingly, MBS derivatives priced below par usually benefit from an increase in projected prepayment rates, while bonds priced at a premium generally suffer when projected prepayment rates increase. The impact of a change in yield curve slope on future cash flows (via changing prepayment rates) can be very powerful, and this component of slope exposure often dominates the other two components.

Finally, the third factor we consider is the effect of a change in yield curve slope on the value of embedded options. Many MBS derivatives contain either explicit or implicit embedded options. For instance, most MBS derivatives are capped; hence, a manager who owns a MBS derivative is short an interest rate cap. The value of this cap varies with changes in yield curve slope and can have a significant influence on the overall yield curve exposure of the bond.

To analyze the change in value of an option embedded in an MBS derivative, we consider the behavior of a standard over-the-counter cap or floor, given the same change in the yield curve. We know that a flattening yield curve causes long-term forward rates to decrease. The value of a long-term cap will fall under this scenario because the decrease in long-term forward rates results in the cap being farther out of the money. Conversely, the value of a long-term interest rate floor increases as the yield curve flattens because the drop in forward rates makes the floor more in the money. The values of caps and floors change in opposite directions when the yield curve steepens: long-term caps increase in value, while long-term floors fall in value.

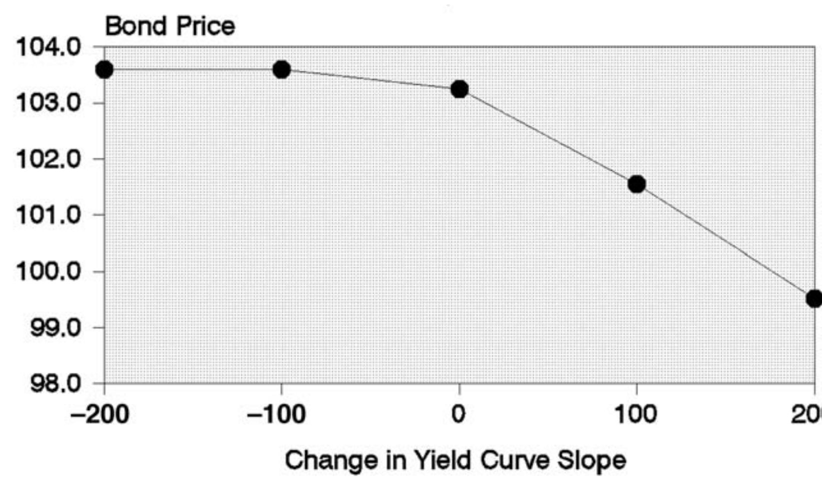
By analyzing these three components of slope sensitivity, we can determine the net slope exposure of an MBS derivative. A manager could gain additional insight by quantifying the effect of each of these factors independently, although that evaluation could prove quite complicated and is not necessary. This framework allows us to develop intuition for the likely impact of a change in yield curve slope on the value of a given MBS derivative.

Sequential-Pay Bonds

The first CMO bond to be analyzed for yield curve slope risk is a fixed-rate sequential pay bond—basically a plain vanilla CMO bond. The profile is shown in Exhibit 14.5. A short maturity sequential typically has very little slope risk. Most of these bonds are priced near par, so changes in projected prepayment rates have minimal impact on their value. These bonds are not explicitly capped, and do not have significant embedded options. Therefore, the factor that generally determines the bond's slope exposure is the impact of a change in yield curve slope on the bond's discount rates. As the yield curve flattens, short-term interest rates rise. Since the short sequential bond's cash flows are weighted toward the short end of the yield curve, the change in discount rates will reduce the value of the bond and can result in a small negative slope elasticity.

Changes in projected prepayment rates, however, will dominate the effect of changing discount rates and result in a positive (negative) slope elasticity if the sequential is priced significantly below (above) par. A long sequential bond is more likely than a short sequential to be priced significantly above or below par. If the bond is priced fairly close to par, it will benefit if the yield curve flattens. Again, the flattening yield curve

EXHIBIT 14.5 Yield Curve Risk of a Fixed-Rate Sequential



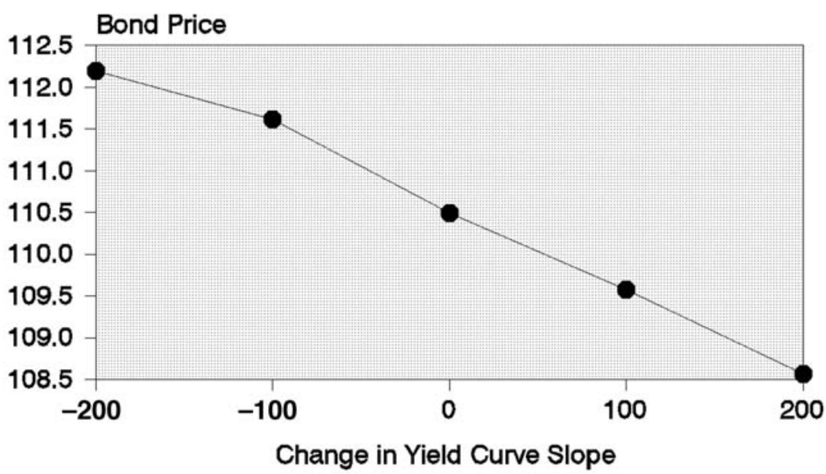
causes long-term discount rates to fall, and short-term discount rates to rise, thereby benefiting the long sequential, which by definition has cash flows weighted toward the long end of the yield curve.

An interesting feature of many CMO bonds is that slope exposure tends to be asymmetric. The short sequential bond is a good example of this effect. Exhibit 14.5 shows that the sequential bond benefits somewhat when the yield curve flattens 100 basis points. Yet an additional 100 basis points of flattening has little effect on the bond's value. The pattern for a steepening yield curve is quite different. As the yield curve steepens, the sequential bond extends, and it continues to extend over a relatively large range of yield curve slopes. Therefore, the bond loses substantially if the yield curve steepens, but benefits very little if the curve flattens.

PAC Bonds

Exhibit 14.6 shows the second type of CMO bond, a PAC bond. The PAC bond is very similar to a long sequential bond in terms of its yield curve slope risk. Both a long PAC bond and a long sequential bond benefit from relatively lower discount rates if the yield curve flattens. The major difference between the two bonds is that the PAC bond's cash flows are much more stable than the cash flows from the sequential bond because the bond is protected within a prepayment band. The PAC bond's cash flows are usually therefore much less sensitive than the sequential's cash flows to changes in projected prepayment rates. Conse-

EXHIBIT 14.6 **Yield Curve Risk of a Planned Amortization Class**



quently, the second component of yield curve slope exposure, the effect of changing prepayment rates, generally has little effect on a PAC bond.

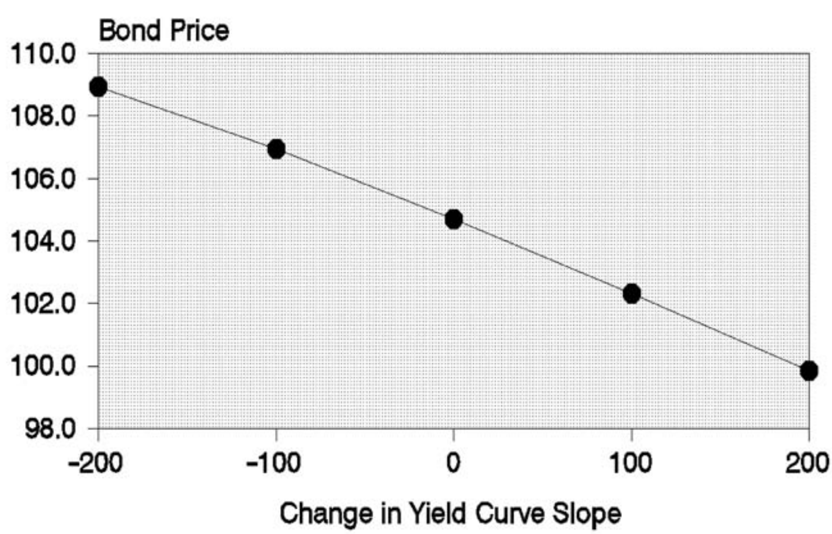
One caveat is that it is impossible to generalize accurately about the exposure of PAC bonds to large changes in slope since the value of a PAC bond is very dependent on its structure. For instance, the huge increase in prepayment rates in 1992 and early 1993 resulted in prepayment rates on many PAC bonds breaking the PAC band, thereby causing these bonds to behave as sequential bonds.

The example PAC bond in Exhibit 14.6 has fairly symmetric slope exposure. The slope elasticity is 0.6%. The bond's effective duration is approximately 4, which means this PAC bond's price will change by approximately 4% for a 100 basis point change in rates. Thus, in this case the slope elasticity is 15% of the effective duration. Although yield curve slope risk has a much smaller effect than changes in the actual level of rates, it can have an enormous impact on the value of a CMO portfolio.

VADM Bonds

The next CMO bond we examine is a very accurately defined maturity bond (VADM). The profile with respect to yield curve slope changes is shown in Exhibit 14.7. As the exhibit shows, the slope elasticity is very similar to that of a PAC (this is a 10-year VADM as opposed to the 7-year PAC in Exhibit 14.6). As with a PAC, a short average life VADM

EXHIBIT 14.7 Yield Curve Risk of a VADM Bond



has very low slope risk, while a longer VADM generally benefits if the yield curve flattens.

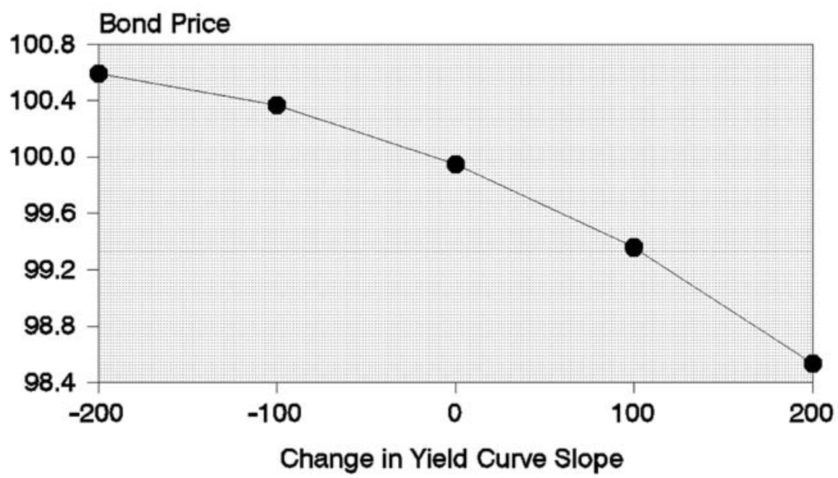
The main difference between a VADM and a PAC bond is that the VADM's cash flows are even better protected from changing prepayment rates than are the PAC bond's cash flows. The VADM receives its paydown from the interest accrual on a Z bond (zero coupon). On a continuum of prepayment sensitivity, sequential bonds are the most sensitive to prepayments, PACs are in the middle, and VADMs have the lowest prepayment sensitivity. The impact of low prepayment sensitivity on slope exposure is that the second component of slope risk, the effect of changing prepayment rates on the value of the bond, is usually quite small for a VADM.

The exception to this rule occurs if prepayment rates have increased or decreased markedly since the VADM was issued. The VADM is not perfectly protected from changing prepayment rates, and a large change in prepayments will affect the value of the bond. Consequently, a VADM's slope sensitivity is usually symmetric for small changes in yield curve slope, but becomes asymmetric for large changes in yield curve slope.

Pro Rata Libor Floater

A pro rata floater is a floating-rate class that pays down with the collateral. The coupon on a floater is usually capped; therefore, the investor is short a Libor cap. In essence, then, a Libor floater can be viewed as a pure floating-rate bond minus a Libor cap. Exhibit 14.8 shows the yield curve risk of a pro rata floater backed by fixed-rate collateral.

EXHIBIT 14.8 Yield Curve Risk of a Pro Rata Libor Floater



Only two of the three components determining net slope exposure are relevant for a pro rata Libor floater. The value of a floater is relatively unaffected by the impact on discount rates resulting from a change in yield curve slope, but changes in yield curve slope do affect prepayment rates and the value of embedded options.

The primary effect of a change in yield curve slope on a floater is through the value of embedded options. For example, as the yield curve flattens, forward Treasury and Libor rates decrease. A cap is simply a series of put options on forward Libor bond prices (or call options on forward Libor rates), so falling Libor rates reduce the value of the cap. The floater is short a Libor cap, so a flattening yield curve increases the value of the floater. Hence, the floater has positive slope elasticity.

The magnitude of the floater's slope elasticity is a function of the strike price of the embedded cap. The floater in Exhibit 14.8 has a high cap; consequently, its slope elasticity is not large, 0.4%. A floater with an at-the-money Libor cap would have a much larger slope elasticity than 0.4%. This pattern can be seen in Exhibit 14.8 for a 100-basis point flattening and a 100-basis point steepening.

The changes in the price of the floater for small changes in yield curve slope are fairly symmetric, but the floater does not benefit significantly for incremental yield curve flattening in excess of 100 basis points. If the curve steepens, however, the floater will lose considerably. When the yield curve steepens, the cap is coming closer to being in the money; in the parlance of option pricing theory, its delta is becoming more negative. Therefore, the floater's slope elasticity will become more positive, and its price decline as the yield curve steepens will accelerate.

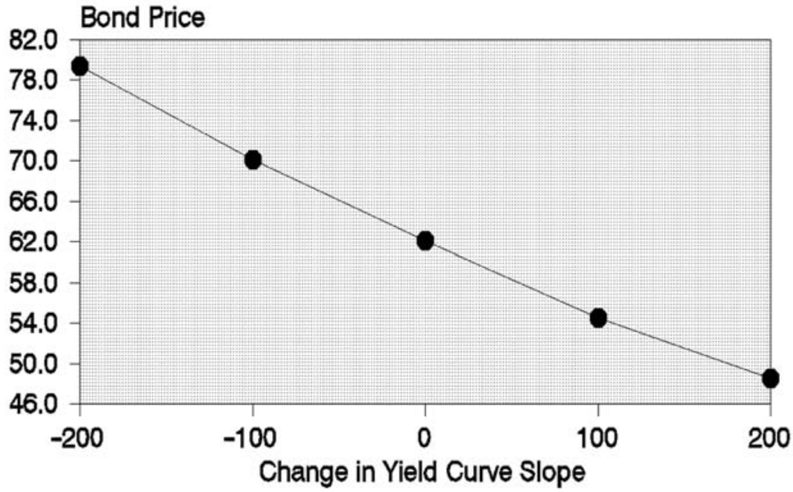
This graph looks quite similar to the profile of the price of a typical fixed-rate mortgage versus changes in the level of rates. So there is negative convexity in slope exposure, just as there is with respect to changes in the level of interest rates.

Inverse Floaters

Inverse floaters come in all shapes and sizes. An inverse floater and a floater can be combined to produce a fixed-rate tranche; so as the floater is short a cap, the inverse floater is long a cap. The long cap and the price at which an inverse floater trades effectively determine the bond's slope elasticity. At the time of this analysis, the inverse floater shown in Exhibit 14.9 was at a discount. As we go through this example, we explain how the slope elasticity would differ for a portfolio of premium inverse floaters.

The two most important factors to consider in evaluating the slope exposure of an inverse floater are its price (i.e., its tendency to benefit/suffer if prepayment rates increase) and the "delta" of its embedded long

EXHIBIT 14.9 Yield Curve Risk of a Discount Inverse Floater



Libor cap. As is the case with the capped pro rata floater, the cap embedded in the inverse floater decreases in value as the yield curve flattens.

The decline in the value of the cap benefits a floater, since it is short the cap, but reduces the price of the inverse floater since it is long the cap. The decline in the value of the cap would produce negative slope elasticity if no other effects were present. The inverse floater in Exhibit 14.9, however, has positive slope elasticity and actually benefits substantially as the yield curve flattens. This inverse floater's slope elasticity is positive rather than negative because it gains considerably from increasing prepayment rates. The price of this inverse floater is 62.13, so it is a deep discount bond.

An increase in prepayment rates will clearly benefit the holder of this bond, who will be repaid at par while the bond's price is far below par. A flattening yield curve causes projected prepayment rates to increase, thereby benefiting the inverse floater and overwhelming the impact of the change in value of the embedded long cap.

While the deep discount inverse floater in Exhibit 14.9 has positive slope elasticity, a premium inverse floater would almost certainly have negative slope elasticity. Suppose we are dealing with a premium inverse floater priced at 105. The investor who purchased this inverse floater would clearly be at a disadvantage if prepayment speeds increase. In this case, a flattening yield curve would hurt the investor, because the long cap position would become less valuable and also because a loss is incurred as the principal is prepaid more quickly.

As a result, a premium inverse floater has a negative slope elasticity, while a discount inverse floater has a positive slope elasticity—+9.6% for the discount inverse floater in Exhibit 14.9. Thus, it is important to analyze the yield curve risk exposure of each inverse floater.

Interest-Only Strip

Exhibit 14.10 shows the yield curve risk of a mortgage strip, a 10% IO strip. This instrument exhibits negative slope elasticity. Investors in IO strips suffer as prepayment rates increase because they do not receive any principal, and the stream of interest payments is shortened. We have seen that a flattening yield curve causes prepayment speeds to increase, while a steepening yield curve produces slower prepayment speeds. In this case, it is very clear why an IO strip has negative slope elasticity, unless it is backed by extremely high-premium collateral that is burned out. Consequently, IOs are a good hedge for a portfolio that has considerable positive slope elasticity (i.e., a portfolio that benefits if the curve flattens and loses if it steepens). IOs are one of the few MBS derivatives that a portfolio manager can use to counteract positive slope elasticity.

Principal-Only Strip

The PO strip is nearly the opposite of the IO strip. PO holders benefit as prepayment rates increase because they receive their principal more quickly. Consequently, a PO strip increases in value if the yield curve

EXHIBIT 14.10 **Yield Curve Risk of a 10% Interest-Only Strip**

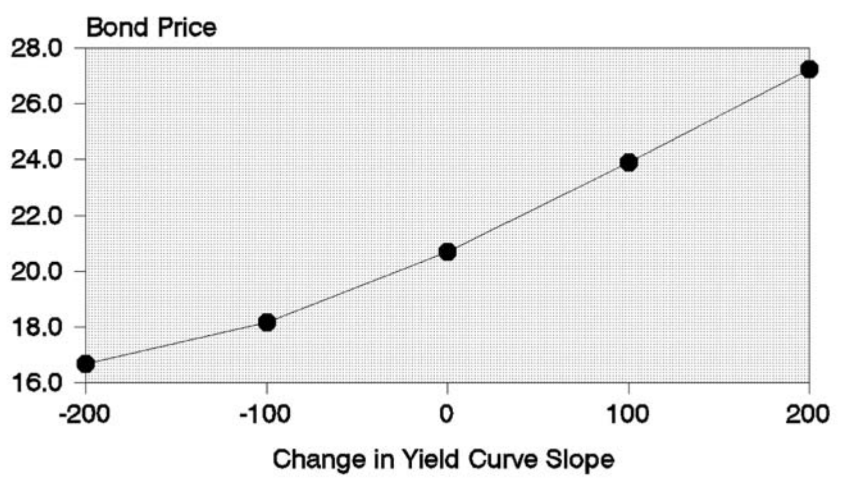
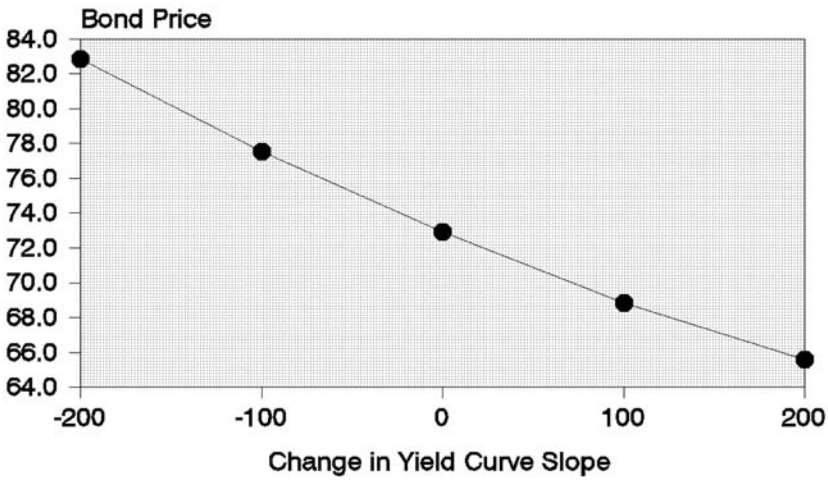


EXHIBIT 14.11 Yield Curve Risk of an 8% Principal-Only Strip



flattens, and decreases in value if the yield curve steepens. The yield curve risk profile shown in Exhibit 14.11 is for an 8% PO strip that has substantial positive slope elasticity.

Whether the curve in Exhibit 14.11 is shaped like this (positively convex) or is in fact bowed down (negatively convex) depends on the spread between the coupon of the collateral backing the PO and the current coupon mortgage rate. At the time of this analysis, the PO in the exhibit is backed by 8s selling at a discount. Had the PO used in the illustration been a PO backed by 11s (a premium), the curve would be negatively convex. Thus, the benefit of a flattening yield curve for a PO backed by high-coupon mortgages would be much less than it would be for a PO backed by low-coupon mortgages. Once again, yield curve risk is specific to the actual deal or structure from which the PO was created.

Exhibit 14.12 summarizes for various MBS derivatives the effective duration and the slope elasticity of the yield curve as of July 31, 1992.

MANAGING LEVEL AND SLOPE RISK INDEPENDENTLY

Given the exposure of each CMO bond in a portfolio, a manager should be able to make an informed decision about what kind of hedge to put on or how to manage that risk. For example, a portfolio with a substantial amount of POs benefits significantly if the yield curve flattens, but

EXHIBIT 14.12 **Effective Duration and Slope Elasticities for Several MBS Derivatives**

Type of CMO Bond	Effective Duration	Elasticity for Slope of Yield Curve (%)
Sequential	3.3	0.7
Libor Floater	0.9	0.4
Inverse Floater	30.7	9.6
PAC	3.6	0.6
VADM	5.7	1.7
IO Strip	–23.9	–11.5
PO Strip	13.1	4.7

suffers if the yield curve steepens. It would make sense to restructure the assets or implement hedges to reduce the portfolio's yield curve risk.

The key point is that yield curve slope risk and duration risk (i.e., exposure to parallel shifts in the yield curve) can, for the most part, be managed independently. The reason for this is that the correlation of changes in the level of rates and yield curve slope is very low. What if it were the case that whenever rates rose the yield curve got steeper, or when rates fell the yield curve flattened? In that case, changes in yield curve slope and changes in the level of rates would be highly correlated, and a manager would have to consider the effect or exposure of a portfolio to changes both in the level of rates and in slope simultaneously. A manager could not effectively separate the two effects.

The relationship of changes in the level of rates to changes in yield curve slope is an empirical question. To investigate this question, we calculated the historical correlation between changes in the slope of the curve and changes in the level of rates, as well as changes in the curvature of the yield curve. In this analysis the level of rates is defined as the average of the 6-month, 5-year, and 30-year Treasury yields, while the curvature of the yield curve is defined as the 5-year Treasury rate minus the average of the 6-month and 30-year Treasury yields.

The correlation results, shown in Exhibit 14.13, are based on monthly data. They would probably be somewhat higher if a longer differencing interval were used. The correlation of 0.12 between changes in the level of rates and changes in the slope of the curve is quite low. While it is not zero, it is low enough to give a manager comfort that yield curve slope exposure can be calculated and managed independently of parallel interest rate shifts. This is an important property because if this were not the case, a manager would have to implement an elaborate Monte Carlo model to assess the

EXHIBIT 14.13 Correlation Matrix for Changes in Level of Interest Rates, Yield Curve Slope, and Curvature: Monthly Rates from 12/82 through 12/92

Parameter	Level	Slope	Curvature
Level	1.0000		
Slope	0.1152	1.0000	
Curvature	0.4974	0.5540	1.0000

joint effect of level and slope exposures based on a correlation between the two factors. This would be a difficult and costly exercise.

MANAGING YIELD CURVE SLOPE RISK

We know how slope risk affects particular types of MBS derivatives. Now let's look at the problem of slope risk in the context of an MBS derivative portfolio. We also present an approach for measuring slope risk and how to hedge or manage this risk.

Determining the Slope Elasticity of Potential Risk Control Instruments

The first step is to determine the slope elasticity of the candidates for hedging yield curve risk exposure. The instruments most commonly used to hedge slope exposure are interest rate futures (Treasury bonds, 10-year Treasury notes, 5-year Treasury notes, and Eurodollar CDs), interest rate swaps, yield curve options, and caps and floors. A customized risk control vehicle can be created by a commercial bank or an investment bank, but interest rate futures typically do a very good job of controlling slope exposure.

Slope Exposure of Interest Rate Futures

Interest rate futures contracts have different types of yield curve risk, but their yield curve exposures are intuitively obvious. Treasury bond futures are on the long end of the yield curve, so they benefit when the yield curve flattens (long rates decline relative to short rates). Treasury bond futures therefore have large positive slope elasticity.

Ten-year Treasury note futures also benefit when the yield curve flattens. They have positive slope elasticity, but it's less than the slope elasticity for Treasury bonds. Five-year Treasury note futures are largely unaffected by changes in yield curve slope since five years is approximately the center of the yield curve. Given the way slope is defined in this chapter (the 30-year Treasury rate minus the 3-month Treasury bill rate), 5-year Treasury note futures have a very small, positive slope elasticity.

Eurodollar futures are on the short end of the yield curve, so a long position in Eurodollar futures benefits when short rates decline. A decline in short rates corresponds to a steepening yield curve; hence, Eurodollar futures have a negative slope elasticity. Treasury bill futures are also on the short end of the yield curve and have negative slope elasticity.

Exhibit 14.14 shows the yield curve slope exposure for all but the Treasury bill futures contract. The steepness (or slope) of the line corresponds to the slope elasticity. The line corresponding to the Treasury bond futures contract is the steepest, which means it has the highest slope elasticity.

Thus, a manager who owns an MBS derivative with positive slope elasticity could establish a position that would gain if the curve flattens, simply by shorting bonds. As can be seen from Exhibit 14.14, as well as Exhibit 14.15, which shows both the yield curve slope elasticity and effective duration, futures contracts have slope sensitivities that differ in magnitude as well as direction. This means that a manager should be able to manage the risk of an MBS derivative portfolio with these contracts.

EXHIBIT 14.14 **Yield Curve Exposure of Interest Rate Futures**

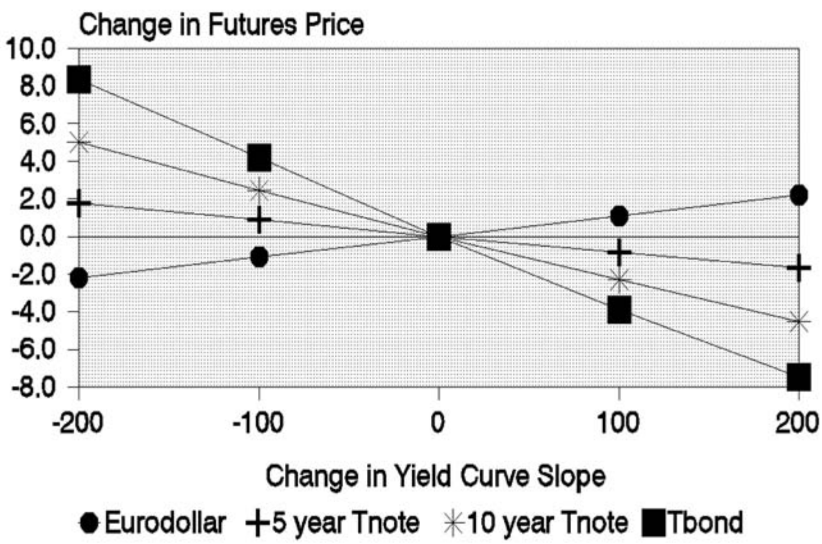


EXHIBIT 14.15 Sample Effective Duration and Slope Elasticities for Interest Rate Futures

Futures Contract	Effective Duration	Elasticity for Slope of Yield Curve (%)
Treasury bond	9.7	3.6
10-year T-note	6.3	2.1
5-year T-note	3.8	0.8
Eurodollar	0.3	–0.2
Treasury bill	0.3	–0.2

Slope Exposure of Interest Rate Swaps

An interest rate swap can have either positive or negative slope elasticity, depending on the maturity of the swap. A swap in which the investor receives a fixed rate and makes floating-rate payments is called a "long" swap. A long swap is very similar to a long position in a Treasury note or bond coupled with a short position in Eurodollar futures. A long swap generally has positive slope elasticity if the maturity of the swap is greater than five years; otherwise it has negative slope elasticity.

Slope Exposure of Yield Curve Options

Yield curve options can be structured in numerous ways. This flexibility provides a manager with many ways in which to control slope risk, but also makes it difficult to generalize in describing the slope elasticity of these options.

Slope Exposure of Caps and Floors

As explained in Chapter 12, an interest rate cap is an agreement in which one party receives payments from a counterparty if the underlying reference rate, usually 3-month Libor, exceeds the cap. We have discussed caps in the context of CMO floaters and inverse floaters. A cap is a series of call options on the reference rate. Equivalently, a 3-month Libor cap is a series of put options on a strip of Eurodollar futures contracts. If the yield curve flattens, forward rates decrease, and Eurodollar futures prices increase. The put options are then farther out of the money and decrease in value. Therefore, a flattening yield curve reduces the value of the cap, and interest rate caps have negative slope elasticity. Conversely, interest rate floors benefit if the yield curve flattens; floors have positive slope elasticity.

Sample Analysis

Given the elasticity of each of the various risk control instruments, we can now demonstrate the steps that a manager can follow to measure and control yield curve risk exposure. The first step involves defining a set of yield curves that the manager is going to use to analyze the slope exposure of the portfolio. A Monte Carlo model could generate thousands of interest rate paths, each of which would be a yield curve to be used in the analysis.

The second step is to determine the value of every MBS derivative and risk control instrument using every yield curve. The third step is to calculate the effective duration and slope elasticity of the yield curve for each bond.

The fourth step is to compute the value of the portfolio for each yield curve. This is done by multiplying the face amount of each bond in the portfolio by its price for each yield curve scenario. The sum of all the market values gives the market value of the portfolio.

The fifth step is to calculate the slope exposure of the portfolio. Basically, this is done by examining the market value of the portfolio for each yield curve and determining how much of the change in the market value is due to a change in the yield curve slope. Given this slope exposure, the sixth step involves determining what kind of hedge should be employed in order to achieve the desired level of exposure. The last step is checking to see that the proposed hedge would actually achieve the goal established.

Below we illustrate these seven steps.

Step 1: The first step is to define the set of yield curves to be used. While Monte Carlo analysis is the preferred way to define the set of yield curves, it is more difficult to implement than simply specifying a set. We use the following five yield curves in our analysis: (1) today's yield curve; (2) a 200-basis-point steepening; (3) a 100-basis-point steepening; (4) a 100-basis-point flattening; and (5) a 200-basis-point flattening.

There is nothing difficult about constructing these yield curves. As we indicate at the outset of the chapter, we assume half the change in the yield curve slope comes from a change in the 3-month rate and half from a change in the 30-year rate. Thus, for a 200-basis point steepening of the curve, the assumption is that 100 basis points of that steepening come from the 30-year rate (i.e., the 30-year rate goes up 100 basis points), and the second 100 basis points come from the 3-month rate (i.e, the 3-month rate goes down 100 basis points).

For changes at other points on the yield curve, however, a methodology must be employed to determine how the rate changes. The procedure we use to construct the full yield curve is based on regressions of long-term changes in rates.[2] For example, suppose that the prevailing

[2] Regression analysis is explained in Chapter 6.

yield (i.e., the yield based on today's yield curve) is 3.27% for the 3-month rate, 5.82% for the 5-year rate, and 7.46% for the 30-year rate. A yield curve steepening of 200 basis points means that the 30-year rate goes up 100 basis points to 8.46% and the 3-month rate goes down 100 basis points to 2.27%.

Suppose further that using multiple regression analysis it is found that the coefficient between changes in the 5-year rate and changes in the 3-month rate is 0.21 and between the 5-year rate and the 30-year rate is 0.79. The change in the 5-year rate is determined as follows:

$$0.21(\text{Change in 3-month rate in bp}) + 0.79(\text{Change in 30-year rate in bp})$$

In our illustration, the change in the 5-year rate for a 200-basis-point steepening of the yield curve would be:

$$0.21(-100) + 0.79(+100) = 58 \text{ basis points}$$

Since the prevailing 5-year rate is 5.82%, the 5-year rate after the yield curve steepening of 200 basis points is 6.40%.

A general formula can be used to determine the change in basis points for any intermediate maturity on the yield curve for any slope change:

$$b_{\text{3-month}} \times \text{Change}_{\text{3-month}} + b_{\text{30-year}} \times \text{Change}_{\text{30-year}}$$

where

$b_{\text{3-month}}$ = regression coefficient for the 3-month rate
$b_{\text{30-year}}$ = regression coefficient for the 30-year rate
$\text{Change}_{\text{3-month}}$ = change (in basis points) of the 3-month rate
$\text{Change}_{\text{30-year}}$ = change (in basis points) of the 30-year rate

The yield for the intermediate maturity is then the prevailing yield plus the change computed using the formula. Exhibit 14.16 illustrates this approach for the yield curves assumed in our analysis.

Step 2: The second step is to value the MBS derivatives in the portfolio and the risk control instruments for every yield curve. Derivation of the market prices for each yield curve must be obtained from a good valuation model. Exhibit 14.17 shows the estimated market prices for seven MBS derivatives and four interest rate futures contracts for each yield curve. The estimated market prices were based on the Smith Breeden pricing model at the time.

EXHIBIT 14.16 Determining the Yield Curve

	Assumed Yield Curves			
Type	Steepen	Steepen	Flatten	Flatten
Net change (in bp)	200	100	–100	–200
3-month (in bp)	–100	–50	50	100
30-year (in bp)	100	50	–50	–100

Regression Current Coefficient				Yield Curve Slope Changes			
				Steepen	Steepen	Flatten	Steepen
Maturity	Yield	3 mo	30 yr	200 bp	100 bp	100 bp	200 bp
0.25	3.27	1.00	0.00	2.27	2.77	3.77	4.27
0.50	3.37	0.87	0.13	2.63	3.00	3.74	4.11
1	3.61	0.70	0.30	3.21	3.41	3.81	4.01
2	4.38	0.46	0.54	4.46	4.42	4.34	4.30
3	4.85	0.38	0.62	5.09	4.97	4.73	4.61
4	5.32	0.29	0.71	5.74	5.53	5.11	4.90
5	5.82	0.21	0.79	6.40	6.11	5.53	5.24
7	6.26	0.11	0.89	7.04	6.65	5.87	5.48
10	6.71	0.08	0.92	7.55	7.13	6.29	5.87
20	7.36	0.00	1.00	8.36	7.86	6.86	6.36
30	7.46	0.00	1.00	8.46	7.96	6.96	6.46

Step 3: The third step is to calculate the slope elasticity of each bond. To illustrate this procedure, we use the inverse floater shown in Exhibit 14.17. The base value, that is, the market price at today's yield curve, is 62.13. The effective duration is 30.7% (see Exhibit 14.12).

The procedure we use to construct these yield curves produces slight changes in the level of rates as we vary the yield curve slope. For example, in the case where the yield curve steepened 100 basis points, there was also a 10-basis-point increase in the level of rates; there was a 9-basis-point decrease in the level of rates when the yield curve flattened 100 basis points.

The main virtue of our methodology for measuring and controlling yield curve slope exposure is its simplicity, for which we have sacrificed some precision. The procedure we use to construct the "twisted" yield curves is an instance in which the simple approach is imprecise—it tends to produce slight changes in the level of interest rates when we change

EXHIBIT 14.17 Determining the Value of the MBS Derivatives and Risk Control Instruments for Every Assumed Yield Curve
MBS derivative prices

MBS Derivative	Current	Steepen 200	Steepen 100	Flatten 100	Flatten 200
Sequential	103.24	99.51	101.55	103.59	103.60
Floater	99.95	98.53	99.36	100.37	100.59
Inverse floater	62.13	48.57	54.51	70.12	79.40
PAC	110.50	108.57	109.58	111.61	112.19
VADM	104.69	99.84	102.31	106.95	108.93
IO	20.70	27.24	23.89	18.17	16.69
PO	72.93	65.60	68.83	77.54	82.85

Futures prices

Futures Contract	Current	Steepen 200	Steepen 100	Flatten 100	Flatten 200
Eurodollar	96.48	97.48	96.98	95.98	95.48
5-year T-note	108.30	105.81	107.05	109.57	110.86
10-year T-note	107.75	101.91	104.78	110.84	114.04
T-bond	104.84	95.41	99.95	110.01	115.14

the yield curve slope to achieve a desired yield curve slope. A more complicated, iterative procedure could produce the desired change in yield curve slope without affecting the level of rates, but it would be cumbersome to implement.

In any case, when we compare the market value of an MBS derivative using twisted yield curves to the market value of that bond using the current yield curve, we need to recognize that some portion of the change in price is due to a small change in the level of interest rates. We need to subtract the portion of the price change in the level of rates from the overall price change to isolate the price impact of the change in yield curve slope.

The deep discount inverse floater illustrates this procedure. The base price of the inverse floater is 62.13. The price decreases to 54.51 if the yield curve slope increases by 100 basis points (and the level of rates increases by ten basis points). Conversely, the price rises to 70.12 if the yield curve slope declines by 100 basis points (and the level of rates drops by 9 basis points).

To isolate the price effect of a change in the level and a change in the slope, we first determine the price effect of a change in the level, which can be found as follows:

$$-\text{Base price} \times \text{Change in yield level} \times \text{Effective duration}/100$$

For example, for the inverse floater, the effect of an increase in the level of 10 basis points is

$$-62.13 \times 0.10 \times 30.7/100 = -1.91$$

and for a decrease in the level of 9 basis points is

$$-62.13 \times (-0.09) \times 30.7/100 = 1.72$$

Given the new price and the price effect due to a change in the level, the effect due to a change in slope can be calculated as follows:

$$\text{New price} - \text{Price effect of a change in level} - \text{Base price}$$

If the change in the level is 10 basis points and the slope change is 100 basis points, the resulting price is 54.51. The price effect of the change in slope is

$$54.51 - (-1.91) - 62.13 = -5.71$$

A 100-basis-point yield curve flattening combined with a 9-point-basis point reduction in the level of rates results in a price of 70.12. The price effect of the change in slope is

$$70.12 - 1.72 - 62.13 = 6.27$$

The slope elasticity for a scenario is:

$$\text{Slope elasticity} = \frac{(\text{Price effect of a change in slope}/\text{Base price})}{\text{Change in yield curve slope}}$$

The slope elasticity for the two scenarios is then

$$\frac{(5.71/62.13)}{1.00} = 0.092 = 9.2\%$$

$$\frac{(-6.27/62.13)}{-1.00} = 0.101 = 10.1\%$$

The average slope elasticity is then 9.6%.

The general procedure is presented in Exhibit 14.18.

Step 4: The fourth step is to compute the value of the portfolio for each yield curve. Exhibit 14.19 illustrates this step using two hypothetical MBS derivative portfolios. The total market value for each portfolio is $100 million. Notice that Portfolio #1 has only inverse floaters and

EXHIBIT 14.18 **Calculating the Average Slope Elasticity**

Scenario 1: +100bp change in slope of curve

New price (estimated based on +100-basis-point change in slope and corresponding level)

Price effect due to change in:

$$\text{Level} = -\text{Base price} \times \text{Change in yield level} \times \text{Effective duration}/100$$

$$\text{Slope} = \text{New price} - \text{Price effect of change in level} - \text{Base price}$$

Slope elasticity in Scenario 1:

$$\text{Slope elasticity} = \frac{(\text{Price effect of a change in slope}/\text{Base price})}{\text{Change in yield curve slope}}$$

Scenario 2: –100bp change in slope of curve

New price (estimated based on –100-basis-point change in slope and corresponding level)

Price effect due to change in:

$$\text{Level} = -\text{Base price} \times \text{Change in yield level} \times \text{Effective duration}/100$$

$$\text{Slope} = \text{New price} - \text{Price effect of change in level} - \text{Base price}$$

Slope elasticity in Scenario 2:

$$\text{Slope elasticity} = \frac{(\text{Price effect of a change in slope}/\text{Base price})}{\text{Change in yield curve slope}}$$

Average slope elasticity

$$\frac{\text{Slope elasticity in Scenario 1} + \text{Slope elasticity in Scenario 2}}{2}$$

EXHIBIT 14.19 **Calculating the Value for Two Hypothetical Portfolios and the Slope Exposure of the Portfolio**

	Portfolio #1 (in Millions of $)			Portfolio #2 (in Millions of $)		
MBS Derivative	Market	Level $ Move	Slope $ Move	Market	Level $ Move	Slope $ Move
Sequential	0	0	0	5,000	164	33
Floater	0	0	0	0	0	0
Inverse Floater	43,789	13,428	4,201	17,662	5,416	1,695
PAC	0	0	0	20,000	711	115
VADM	0	0	0	25,000	1,435	415
IO	56,211	–13,428	–6,468	32,338	–7,725	–3,721
PO	0	0	0	0	0	0
Total	100,000	0	–2,267	100,000	0	–1,462

IO strips. As can be seen from column 3, the effective duration for this portfolio is zero: The effective duration of the inverse floaters offsets the negative effective duration of the IO strips.

Step 5: Calculate the dollar slope exposure of the portfolio. The dollar slope exposure is the sum of the dollar slope exposures of the individual MBS derivatives. The dollar slope exposure for a given MBS derivatives is defined as the market value invested in that bond multiplied by the bond's slope elasticity.

In this example, the slope exposure for any individual bond is less than the level exposure for that bond. Notice that Portfolio #1, which has an effective duration of zero, is hedged with respect to the level of rates but not changes in the slope of the yield curve. The dollar slope exposure for this portfolio is such that the inverse floater gains $4.2 million if the yield curve flattens 100 basis points, but the IO strip loses $6.5 million—a net loss of $2.3 million.

Step 6: In this step, the hedge position is determined based on the desired exposure to level and slope risks. Focusing on Portfolio #1, the effective duration is zero, but there is slope exposure of $2.3 million if the yield curve flattens by 100 basis points. This means that if the manager wants to eliminate this exposure, it will be necessary to find a risk control instrument or combination of instruments that will gain $2.3 million if the yield curve flattens.

One possibility is to use interest rate futures. Recall from our earlier discussion of interest rate futures that Treasury bond futures and 10-year Treasury note futures benefit if the yield curve flattens. The 5-year note is not affected significantly, but shorter contracts benefit if the yield curve steepens.

Consider this hedge strategy: go long Treasury bond futures and short an appropriate number of Eurodollar futures so that the dollar duration of the combination of long bonds and short Eurodollar futures is zero. This can be done by first calculating the dollar move of both the Treasury bond futures contract and the Eurodollar futures contract, and then finding the hedge ratio that produces a zero duration position in futures.

While the effective duration of this futures position is zero, the slope elasticity is positive. If the yield curve flattens, the long Treasury bond futures position would gain, and the short Eurodollar futures position would also gain. Since both legs of the futures trade gain if the yield curve flattens, the aggregate position clearly benefits from a flattening of the yield curve. We refer to this position as a "T-bond/Eurodollar futures unit."

The number of T-bond/Eurodollar futures units is found by determining the slope exposure for each contract. The following procedure is used to determine the number of futures contracts in the unit given the slope exposure:

1. Construct a zero effective duration T-bond/Eurodollar futures position by determining the hedge ratio, indicating the number of Eurodollar futures contracts for each Treasury bond futures contract as follows:

$$\text{Hedge ratio} = -\frac{\text{Dollar duration of Treasury bond futures}}{\text{Dollar duration of Eurodollar futures}}$$

In our illustration, the dollar duration is \$10,180 for the Treasury bond futures and \$2,500 for the Eurodollar futures. Therefore, the hedge ratio is –4.072 (–\$10,180/\$2,500). Thus, for each Treasury bond futures contract purchased, 4.072 Eurodollar futures contracts will be sold.

2. Calculate the slope exposure (in dollars) of one unit of the zero effective duration position as follows:

$$\begin{array}{l}\text{Dollar slope elasticity of Treasury bond futures position}\\ +\text{Dollar slope elasticity of Eurodollar futures position} \times \text{Hedge ratio}\end{array}$$

In our illustration, the dollar slope elasticity is \$3,719 for a long Treasury bond futures contract and –\$1,492 for a short Eurodollar futures position. Therefore,

$$\$3{,}719 + (-4.072) \times (-\$1{,}492) = \$9{,}794$$

3. Determine the number of zero effective duration T-bond/Eurodollar futures units needed as follows:

$$-\frac{\text{Slope exposure of portfolio}}{\text{Slope exposure of 1 unit of zero effective duration position}}$$

In our illustration, since the slope exposure of Portfolio #1 is \$2,266,784, then

$$-\frac{\$2{,}266{,}784}{\$9{,}794} = 231.45$$

4. Determine the number of Eurodollar futures contracts to short for each Treasury bond futures contract bought. This is found by multiplying the number of units found in the previous calculation by the hedge ratio. In our illustration, since 231 T-bond/Eurodollar futures units are needed, 231 Treasury bond futures will be purchased and 943 (231 times 4.072) Eurodollar futures will be sold. Rounding these values, the hedge position will include a long position in 231 Treasury bond futures and a short position in 943 Eurodollar futures.

For Portfolio #2 in Exhibit 14.19, the number of T-bond/Eurodollar futures units needed is 149, consisting of a long position in 149 Treasury bond futures and a short position in 608 Eurodollar futures.

5. Check that the level exposure of the hedge position is zero.

In our illustration, since the dollar duration is \$10,180 per Treasury bond futures contract and the dollar duration of the Eurodollar futures contract is \$2,500, the dollar duration of the hedged position is

$$231 \times \$10{,}180 + 943 \times (-\$2{,}500) = \$5{,}920$$

The difference is approximately zero, the difference resulting from the rounding of the number of futures contracts.

6. Check that the slope exposure of the initial portfolio hedged with the T-bond/Eurodollar units is zero.

In our illustration, the dollar slope exposure for Portfolio #1 is \$2,266,784. The dollar slope exposure for the hedged position is:

$$231 \times \$3{,}719 + 943 \times \$1{,}492 = \$2{,}266{,}045$$

Therefore, the dollar slope exposure of the hedged portfolio is essentially zero.

7. Check to make sure the hedged portfolio has the target slope exposure. Exhibit 14.20 demonstrates this for Portfolio #1 and Exhibit 14.21 for Portfolio #2.

A natural question is whether the hedged portfolio works better than the unhedged portfolio in terms of the target slope exposure. Looking at Exhibit 14.20, this can be seen for Portfolio #1 by comparing the row showing the unhedged portfolio results with the last line in the exhibit showing the hedged portfolio results. The portfolio begins with a market value of $100 million. If the yield curve flattens by 100 basis points, there is a loss of approximately $1.2 million if it is unhedged but a gain of about $1.1 million if hedged. If the yield curve steepens 100 basis points, there would be a gain of about $3.3 million if unhedged but a gain of just under $1 million if hedged. Thus, the hedged portfolio is relatively insulated for changes in yield curve slope.

Notice, however, that for a larger change in yield curve slope, such as 200 basis points, the exposure becomes a little stranger. The reason

EXHIBIT 14.20 **Verification that Hedged Portfolio #1 Has the Expected Sensitivity**

	Change in Yield Curve Parameters (in bp)				
Level	–19	–9	0	10	20
Slope	–200	–100	0	100	200
	Market Value of Assets (in $ 000)				
Sequential	0	0	0	0	0
Floater	0	0	0	0	0
Inverse floater	55,961	49,417	43,789	38,418	34,230
PAC	0	0	0	0	0
VADM	0	0	0	0	0
IO	45,321	49,340	56,211	64,873	73,970
PO	0	0	0	0	0
Total: Unhedged	101,283	98,757	100,000	103,290	108,200
	Impact of Hedges (in $ 000)				
T-bond futures	2,383	1,197	0	–1,132	–2,184
Eurodollar futures	2,357	1,178	0	–1,178	–2,357
Total: Hedges	4,739	2,375	0	–2,310	–4,540
Total: Hedged	106,022	101,132	100,000	100,980	103,660

EXHIBIT 14.21 Verification that Hedged Portfolio #2 Has the Expected Sensitivity

	Change in Yield Curve Parameters (in bp)				
Level	–19	–9	0	10	20
Slope	–200	–100	0	100	200
	Market Value of Assets (in $ 000)				
Sequential	5,017	5,017	5,000	4,918	4,819
Floater	0	0	0	0	0
Inverse floater	22,571	19,932	17,662	15,495	13,806
PAC	20,307	20,202	20,000	19,834	19,651
VADM	26,012	25,541	25,000	24,432	23,845
IO	26,073	28,386	32,338	37,321	42,555
PO	0	0	0	0	0
Total: Unhedged	99,982	99,077	100,000	102,001	104,674
	Impact of Hedges (in $ 000)				
T bond futures	1,537	772	0	–730	–1,409
Eurodollar futures	1,520	760	0	–760	–1,520
Total: Hedges	3,057	1,532	0	–1,490	–2,929
Total: Hedged	103,039	100,609	100,000	105,511	101,745

for this behavior is that Portfolio #1 contains inverse floaters matched with interest-only strips, and the slope exposure of these instruments tends to change substantially as the yield curve slope changes. In other words, there is considerable convexity in the slope exposure. A portfolio manager must be aware of this, and must rebalance the portfolio as market conditions change, so as to maintain the desired slope exposure. This is no different from the rebalancing required to maintain a target level exposure (effective duration).

While our focus in this illustration has been on completely hedging yield curve slope exposure, the same approach can be used to position a portfolio to benefit from an anticipated change in the yield curve slope.

KEY POINTS

1. Duration and convexity can be used to measure the level risk exposure of an MBS portfolio.

2. A simple approach to quantify the yield curve risk of an MBS portfolio is the slope elasticity measure.

3. The yield curve slope can be defined as the spread between the long-term Treasury (i.e., the 30-year on-the-run issue) and the short-term Treasury (i.e., the 3-month on-the-run issue).

4. Changes in the yield curve can be defined as follows: Half of any basis point change in the yield curve slope results from a change in the 3-month yield and half from a change in the 30-year yield.

5. The slope elasticity is defined as the approximate negative percentage change in a bond's price resulting from a 100-basis-point change in the slope of the curve.

6. A bond or portfolio that benefits when the yield curve flattens is said to have positive slope elasticity; a bond or a portfolio that benefits when the yield curve steepens is said to have negative slope elasticity.

7. Yield curve risk is defined as the exposure of the bond to changes in the slope of the yield curve.

8. CMO and mortgage strips (IOs and POs) are particularly sensitive to changes in the yield curve.

9. It is difficult to generalize about the slope exposure of individual MBS derivatives because the exposure is specific to the actual deal or structure from which the bond was created.

10. To examine the slope exposure of a particular MBS derivative, the impact of changes in discount rates, projected prepayment rates (cash flows), and embedded caps and floors on the bond's value must be assessed.

11. The net slope exposure of an MBS derivative is the sum of the three slope components.

12. An interesting feature of many MBS derivatives is that slope exposure tends to be asymmetric.

13. Only two of the three components determining net slope exposure are relevant for a pro rata Libor floater since the value of a floater is relatively unaffected by the impact on discount rates resulting from a change in yield curve slope, but changes in yield curve slope do affect prepayment rates and the value of embedded options.

14. The primary effect of a change in yield curve slope on a floater is through the value of embedded options.

15. The two most important factors to consider in evaluating the slope exposure of an inverse floater are its price (i.e., its tendency to benefit/suffer if prepayment rates increase) and the "delta" of its embedded long Libor cap.

16. IOs are a good hedge for a portfolio that has considerable positive slope elasticity (i.e., a portfolio that benefits if the curve flattens and loses if it steepens).

17. An IO strip is one of the few bonds that a portfolio manager can use to counteract positive slope elasticity.

18. A PO strip increases in value if the yield curve flattens, and decreases in value if the yield curve steepens.

19. Given the exposure of each MBS derivative in a portfolio, a manager should be able to make an informed decision about what kind of hedge to put on or how to manage that risk.

20. Yield curve slope risk and duration risk can, for the most part, be managed independently because the correlation of changes in the level of rates and yield curve slope is very low.

21. The slope exposure of potential hedging instruments must be estimated in order to control yield curve risk.

22. A portfolio manager who has a long position in an MBS derivative with positive slope elasticity could establish a position that would gain if the curve flattens, simply by shorting bonds.

23. Both the yield curve slope elasticity and effective duration of futures contracts have slope sensitivities that differ in magnitude as well as direction and therefore a portfolio manager should be able to manage the risk of an MBS derivative portfolio with these contracts.

24. An interest rate swap can have either positive or negative slope elasticity, depending on the maturity of the swap.

25. Since yield curve options can be structured in numerous ways, a portfolio manager has flexibility in controlling slope risk.

26. A flattening yield curve reduces the value of an interest rate cap, and therefore a cap has negative slope elasticity; an interest rate floor benefits if the yield curve flattens and therefore has positive slope elasticity.

CHAPTER 15

Credit Risk and Credit Value-at-Risk

In the previous chapters in this book, the focus has been on interest rate risk. In this chapter and the three to follow, we turn our attention to credit risk. Credit risk emerged as a significant risk management issue during the 1990s. In increasingly competitive markets, banks and broker/dealers began taking on greater credit risk in this period. For instance, consider the following developments:

The objectives of this chapter are to:

1. Introduce the various types of credit risk–credit default risk and credit spread risk.
2. Describe credit ratings and their role.
3. Describe a rating transition table and how it can be used to assess the risk of a change in credit rating of an issuer or a bond issue.
4. Describe default rates, recovery rates, and default loss rate.
5. Introduce the concept of modeling credit risk and how credit returns exhibit different patterns than market returns that reflect only interest rate risk.
6. Describe the different methodologies used in two credit risk measurement models, CreditMetrics and CreditRisk+.
7. Explain the concepts for calculating a credit value-at-risk (credit VaR).
8. Discuss applications of credit VaR.
9. Introduce how to integrate credit risk and interest rate risk using VaR.
10. Explain how tracking error due to quality risk and tracking error due to nonsystematic risk can be used to assess the exposure of a bond portfolio to credit risk.

- Credit spreads tightened during the late 1990s onwards, to the point where blue chip companies such as General Electric or Ford were being offered syndicated loans for as little as 10–12 basis points over Libor. To maintain margin or increased return on capital, banks increased lending to lower rated corporates.
- The growth in the use of complex financial instruments such as credit derivatives led to the need for more sophisticated analysis and awareness of the risks presented by these instruments.
- Investors were finding fewer opportunities in interest rate and currency markets, and moved towards yield enhancement through extending and trading credit-risky bonds in the cash market or synthetically via credit default swaps across lower-rated and emerging market assets.
- The rapid expansion of high-yield and emerging market sectors, again lower-rated assets.

The growth in credit exposures and rise of complex instruments have led to a need for more sophisticated risk management techniques. Moreover, as documented later in this chapter, default rates for high-yield corporate bonds have been at historical highs.

In this chapter, we explain what is meant by credit risk and methodologies for measuring credit risk. In the next chapter, we describe the various types of credit derivatives that can be used to manage exposure to credit risk. In Chapter 16, we provide the basic elements to understand how credit derivatives are valued. Finally, in Chapter 17 we demonstrate how to manage credit risk by creating structured products that use credit derivatives.

CREDIT RISK

There are two main types of credit risk that a portfolio or position is exposed to. They are credit default risk and credit spread risk.

Credit Default Risk

Credit default risk is the risk that an issuer of debt (obligor) is unable to meet its financial obligations. Where an obligor defaults, an investor generally incurs a loss equal to the amount owed by the obligor less any recovery amount which the investor recovers as a result of foreclosure, liquidation or restructuring of the defaulted obligor. All portfolios with credit exposure exhibit credit default risk.

The magnitude of credit default risk is described by a firm's *credit rating*. The three ratings agencies in the United States—Fitch Ratings, Moody's, and Standard & Poor's—undertake a formal analysis of the borrower, after which a rating is announced. The issues considered in the analysis include:

- The financial position of the firm itself, for example, its balance sheet position and anticipated cash flows and revenues.
- Other firm-specific issues such as the quality of the management and succession planning.
- An assessment of the firm's ability to meet scheduled interest and principal payments, both in its domestic and foreign currencies.
- The outlook for the industry as whole, and competition within it.
- General assessments for the domestic economy.

The credit ratings are summarized in Exhibit 15.1. Bonds rated triple B or higher are referred to as *investment grade bonds*. Bonds rated below triple B are referred to as *noninvestment grade bonds*, or more popularly *high-yield bonds* or *junk bonds*.

We'll have more to say about credit ratings shortly.

Credit Spread Risk

The *credit spread* is the excess premium over the government or risk-free rate required by the market for taking on a certain assumed credit exposure. Exhibit 15.2 shows the credit spread in January 2003 for industrial corporate bonds with different ratings (AAA, A, and BBB). The benchmark is the on-the-run or "active" U.S. Treasury issue for the given maturity. Notice that the higher the credit rating, the smaller the credit spread.

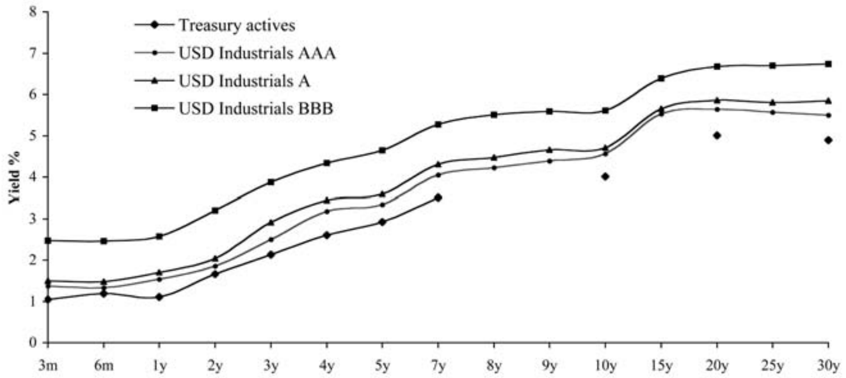
Credit spread risk is the risk of financial loss resulting from changes in the level of credit spreads used in the marking-to-market of a fixed income product. It is exhibited by a portfolio for which the credit spread is traded and marked. Changes in observed credit spreads affect the value of the portfolio and can lead to losses for traders or underperformance for portfolio managers.

An estimate of this risk is spread duration, a measure discussed in Chapter 3. For credit-risky bonds, spread duration is the approximate percentage change in the bond's price for a 100-basis-point increase in the credit spread (holding the Treasury rate constant). For example, a spread duration of 2.5 means that for a 100-basis-point increase in the credit spread, the bond's price will decline by approximately 2.5%. The spread duration for a portfolio is found by computing a market weighted average of the spread duration for each bond. The same is true for a bond market index. Note, however, that the spread duration reported for a bond market index is not the same as the spread duration

EXHIBIT 15.1 Corporate Bond Credit Ratings

Fitch	Moody's	S&P	Summary Description
Investment Grade			
AAA	Aaa	AAA	Gilt edged, prime, maximum safety, lowest risk, and when sovereign borrower considered "default-free"
AA+	Aa1	AA+	
AA	Aa2	AA	High-grade, high-credit quality
AA-	Aa3	AA-	
A+	A1	A+	
A	A2	A	Upper-medium grade
A-	A3	A-	
BBB+	Baa1	BBB+	
BBB	Baa2	BBB	Lower-medium grade
BBB-	Baa3	BBB-	
Speculative Grade			
BB+	Ba1	BB+	
BB	Ba2	BB	Low grade; speculative
BB-	Ba3	BB-	
B+	B1		
B	B	B	Highly speculative
B-	B3		
Predominantly speculative, Substantial Risk or in Default			
CCC+		CCC+	
CCC	Caa	CCC	Substantial risk, in poor standing
CC	Ca	CC	May be in default, very speculative
C	C	C	Extremely speculative
		CI	Income bonds—no interest being paid
DDD			
DD			Default
D		D	

EXHIBIT 15.2 U.S. Dollar Bond Yield Curves, January 2003



Source: Bloomberg Financial Markets

for estimating the credit spread risk of an index. For example, on April 17, 2003, the spread duration reported for the Lehman Brothers Aggregate Bond Index was 3. However, the spread duration for the index is computed by Lehman Brothers based on all non-Treasury securities. Some of these sectors offer a spread to Treasuries that encompasses more than just credit risk. For example, the mortgage sector in the index offers a spread due to prepayment risk. The same is true for some subsectors within the ABS sector. Lehman Brothers does have a Credit Sector for the index. For that sector, the spread duration reflects the exposure to credit spreads in general. It was 1.48 on April 17, 2003 and is interpreted as follows: If credit spreads increase by 100 basis points, the approximate decline in the value of the index will be 1.48%.

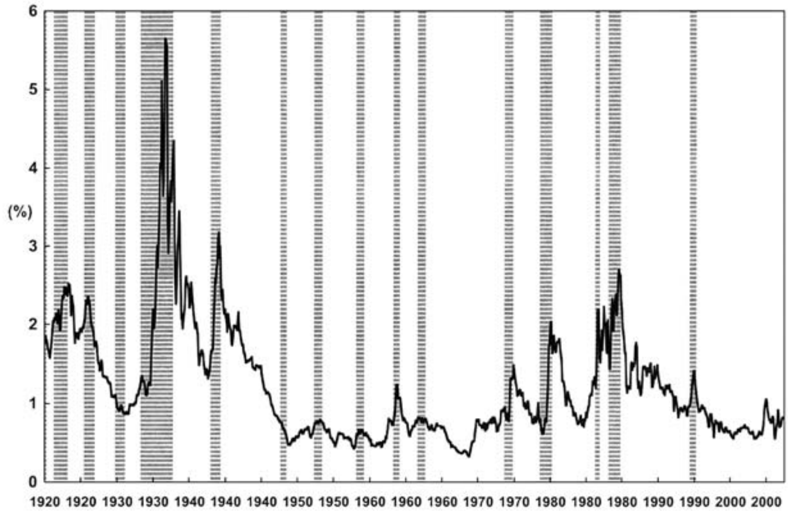
To understand this risk it is necessary to understand the fundamental factors that affect credit spreads. The fundamental factors that affect credit spreads can be classified as macro and micro.

Macro Fundamentals[1]

The ability of a corporation to meet its obligations on its debt depends on its expected cash flows. During prosperous economic times, investors expect that corporate cash flows will improve. In contrast, in an economic recession, investors expect that corporate cash flows will deteriorate, making it more difficult to satisfy its bond obligations. Consequently, it is reasonable to assume that credit spreads are tied to the business cycle.

[1] For a more detailed discussion of macro fundamental factors that affect credit spreads, see Chapter 10 in Leland E. Crabbe and Frank J. Fabozzi, *Managing a Corporate Bond Portfolio* (Hoboken, NJ: John Wiley & Sons, 2002).

EXHIBIT 15.3 Yield Spread Between Baa and Aaa Bonds



Source: Exhibit 1 in Chapter 10 of Leland E. Crabbe and Frank J. Fabozzi, *Managing a Corporate Bond Portfolio* (Hoboken, NJ: John Wiley & Sons, 2002).

The empirical evidence supports the view that the economic cycle has an effect on credit spreads. Exhibit 15.3 shows the yield spread between Baa rated and Aaa rated corporate bonds over business cycles dating back to 1919. Using the National Bureau of Economic Research's definition of economic cycles, economic recessions are shaded in the exhibit.[2] The evidence suggests that, in general, spreads tightened during the early stages of economic expansion, and spreads widened sharply during economic recessions.

Market participants tend to be forward looking and therefore credit spreads react to anticipated changes in the economic cycle. For example, typically credit spreads begin to widen before the official end of an economic expansion. Consequently credit spreads can change based on an anticipated change in the economic cycle that does not materialize.

Anticipating changes in economic cycles is therefore important in assessing an adverse change in credit spreads. There has been extensive research by economists to identify economic indicators that lead economic cycles, referred to as "leading economic indicators." Exhibit 15.4 shows the ten U.S. leading economic indicators used by The Conference

[2] See Geoffrey H. Moore, "Measures of Recession and Expansion," Chapter 7 in Frank J. Fabozzi and Harry Greenfield, *The Handbook of Economic and Financial Measures* (Homewood, IL: Dow Jones-Irwin, 1984).

EXHIBIT 15.4 The Conference Board's Components of the Leading Index the United States

Leading Economic Indicator	Factor
Average weekly hours, manufacturing	0.1946
Average weekly initial claims for unemployment insurance	0.0268
Manufacturers' new orders, consumer goods and materials	0.0504
Vendor performance, slower deliveries diffusion index	0.0296
Manufacturers' new orders, nondefense capital goods	0.0139
Building permits, new private housing units	0.0205
Stock prices, 500 common stocks	0.0309
Money supply, M2	0.2775
Interest rate spread, 10-year Treasury bonds less federal funds	0.3364
Index of consumer expectations	0.0193

Source: The Conference Board, www.tcb-indicators.org/us/LatestReleases/

Board.[3] From the ten leading economic indicators a leading index is constructed. The weighting used for each leading economic indicator to obtain the leading index is shown in the exhibit.

Moreover, some industries within the economy exhibit strong economic cycle patterns. As a result, credit spreads for industries can be expected to be affected by economic cycles. For example, the auto industry is more adversely impacted by a recession than other industries such as consumer staples.

Micro Fundamentals

At the micro level, the analysis of a potential change in the credit spread focuses on the fundamental factors that have changed the individual corporation's ability to meet its debt obligations. These are the factors that the rating agencies use to assess the credit default risk of a corporation.

Rating agencies monitor the bonds and issuers that they have rated. A rating agency may announce that it is reviewing a particular credit rating, and may go further and state that the outcome of the review may result in a downgrade (i.e., a lower credit rating being assigned) or upgrade (i.e., a higher credit rating being assigned). When this announcement is made by a rating agency, the issue or issuer is said to be under "credit watch." The actual or anticipated downgrading of an issue or issuer results in an increase in the credit spread. This form of credit spread risk is referred to as *downgrade risk*.

[3] The Conference Board constructs a leading index for other countries—Germany, Japan, Australia, France, Spain, and Korea.

CREDIT RATINGS AND DEFAULT RISK

Investors in securities accept the risk that the issuer will default on coupon payments or fail to repay the principal in full on the maturity date. Generally credit risk is greater for securities with a long maturity, as there is a longer period for the issuer potentially to default. For example if company issues 10-year bonds, investors cannot be certain that the company will still exist in ten years' time. It may have failed and gone into liquidation some time before that. That said, there is also risk attached to short-dated debt securities, indeed there have been instances of default by issuers of commercial paper, which is a very short-term instrument.

The prospectus or offer document for an issue provides investors with some information about the issuer so that some credit analysis can be performed on the issuer before the bonds are placed. The information in the offer documents enables investors themselves to perform their own credit analysis by studying this information before deciding whether or not to invest. Credit assessments take up time however and also require the specialist skills of credit analysts. Large institutional investors do in fact employ such specialists to carry out credit analysis, however often it is too costly and time-consuming to assess every issuer in every debt market. Therefore investors commonly employ two other methods when making a decision on the credit risk of debt securities:

- Name recognition.
- Formal credit ratings.

Name recognition is when the investor relies on the good name and reputation of the issuer and accepts that the issuer is of such good financial standing, or sufficient financial standing, that a default on interest and principal payments is highly unlikely. An investor may feel this way about say, Microsoft or British Petroleum plc. However the experience of Barings in 1995 suggested to many investors that it may not be wise to rely on name recognition alone in today's marketplace. The tradition and reputation behind the Barings name allowed the bank to borrow at Libor or occasionally at sub-Libor interest rates in the money markets, which put it on a par with the highest-quality banks in terms of credit rating. However name recognition needs to be augmented by other methods to reduce the risk against unforeseen events, as happened with Barings.

A credit rating is a formal opinion given by a rating agency of the default risk faced by investing in a particular issue of debt securities. For long-term debt obligations, a credit rating is a forward-looking assessment of the probability of default and the relative magnitude of the loss should a default occur. For short-term debt obligations, a credit rating a forward-looking assessment of the probability of default.

Formal Credit Ratings

Credit ratings are provided by the specialist agencies. On receipt of a formal request, the credit rating agencies will carry out a rating exercise on a specific issue of debt capital. The request for a rating comes from the organization planning the issue of bonds. Although ratings are provided for the benefit of investors, the issuer must bear the cost. However it is in the issuer's interest to request a rating as it raises the profile of the bonds, and investors may refuse to buy paper that is not accompanied with a recognized rating.

Although the rating exercise involves a credit analysis of the issuer, the rating is applied to a specific debt issue. This means that in theory the credit rating is applied not to an organization itself, but to specific debt securities that the organization has issued or is planning to issue. In practice it is common for the market to refer to the creditworthiness of organizations themselves in terms of the rating of their debt. A highly-rated company such as Rabobank is therefore referred to as a "triple-A rated" company, although it is the bank's debt issues that are rated as triple-A.

The rating for an issue is kept constantly under review and if the credit quality of the issuer declines or improves, the rating will be changed accordingly. An agency may announce in advance that it is reviewing a particular credit rating, and may go further and state that the review is a precursor to a possible downgrade or upgrade. This announcement is referred to as putting the issue under credit watch. The outcome of a credit watch is in most cases likely to be a rating downgrade, however the review may reaffirm the current rating or possibly upgrade it.

During the credit watch phase the agency will advise investors to use the current rating with caution. When an agency announces that an issue is under credit watch, the price of the bonds may fall in the market as investors look to sell out of their holdings. This upward movement in yield will be more pronounced if an actual downgrade results. For example, in October 1992 the government of Canada was placed under credit watch and subsequently lost its triple-A credit rating. As a result, there was an immediate and sharp sell off in Canadian government eurobonds, before the rating agencies had announced the actual results of their credit review.

Credit ratings vary among agencies. Separate categories are used by each agency for short-term debt (with original maturity of 12 months or less) and long-term debt of over one year original maturity. Exhibit 15.1 shows the long-term debt ratings. It is also usual to distinguish between higher "investment grade" ratings where the credit risk is low and lower quality "speculative grade" ratings, where the credit risk is greater. High-yield bonds are speculative-grade bonds and are generally rated no higher than double-B, although some issuers have been upgraded to tripe-B in recent years and a triple-B rating is still occasionally awarded to a high-yield bond.

Credit Rating Changes Over Time: Rating Transition Table

To see how ratings change over time, the rating agencies publish periodically this information in the form of a table. This table is called a *rating transition table* or *rating migration table*. The table is useful for investors to assess potential downgrades and upgrades. A rating transition matrix is available for different transition periods.

Exhibit 15.5 shows a hypothetical rating transition matrix for a 1-year period. The first column shows the ratings at the start of the year and the first column shows the rating at the end of the year. Let's interpret one of the numbers. Look at the cell where the rating at the beginning of the year is AA and the rating at the end of the year is AA. This cell represents the percentage of issues rated AA at the beginning of the year that did not change their rating over the year. That is, there were no downgrades or upgrades. As can be seen, 92.75% of the issues rated AA at the start of the year were rated AA at the end of the year. Now look at cell where the rating at the beginning of the year is AA and at the end of the year is A. This shows the percentage of issues rated AA at the beginning of the year that were downgraded to A by the end of the year. In our hypothetical 1-year rating transition matrix, this percentage

EXHIBIT 15.5 **Hypothetical One-Year Rating Transition Table**

Rating at start of year	Rating at End of Year								
	AAA	AA	A	BBB	BB	B	CCC	D	Total
AAA	93.20	6.00	0.60	0.12	0.08	0.00	0.00	0.00	100
AA	1.60	92.75	5.07	0.36	0.11	0.07	0.03	0.01	100
A	0.18	2.65	91.91	4.80	0.37	0.02	0.02	0.05	100
BBB	0.04	0.30	5.20	87.70	5.70	0.70	0.16	0.20	100
BB	0.03	0.11	0.61	6.80	81.65	7.10	2.60	1.10	100
B	0.01	0.09	0.55	0.88	7.90	75.67	8.70	6.20	100
CCC	0.00	0.01	0.31	0.84	2.30	8.10	62.54	25.90	100

is 5.07%. One can view this figure as a probability. It is the probability that an issue rated AA will be downgraded to A by the end of the year. A rating transition matrix also shows the potential for upgrades.

Again, using Exhibit 15.5 look at the row that shows issues rated AA at the beginning of the year. Looking at the cell shown in the column AAA rating at the end of the year, there is the figure 1.60%. This figure represents the percentage of issues rated AA at the beginning of the year that were upgraded to AAA by the end of the year.

In general the following hold for actual rating transition matrices. First, the probability of a downgrade is much higher than for an upgrade for investment-grade bonds. Second, the longer the transition period, the lower the probability that an issuer will retain its original rating. That is, a one-year rating transition matrix will have a lower probability of a downgrade for a particular rating than a five-year rating transition matrix for that same rating.

Default and Recovery Statistics

There is a good deal of research published on default rates by both rating agencies and academicians.[4] From an investment perspective, default rates

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[4] See, for example, Edward I. Altman, "Measuring Corporate Bond Mortality and Performance," *Journal of Finance*, September 1989, pp. 909–922; Edward I. Altman, "Research Update: Mortality Rates and Losses, Bond Rating Drift," unpublished study prepared for a workshop sponsored by Merrill Lynch Merchant Banking Group, High Yield Sales and Trading, 1989; Edward I. Altman and Scott A. Nammacher, *Investing in Junk Bonds* (New York: John Wiley & Sons, Inc., 1987); Paul Asquith, David W. Mullins, Jr., and Eric D. Wolff, "Original Issue High Yield Bonds: Aging Analysis of Defaults, Exchanges, and Calls," *Journal of Finance,* September 1989, pp. 923–952; Marshall Blume and Donald Keim, "Risk and Return Characteristics of Lower-Grade Bonds 1977–1987," Working Paper (8-89), Rodney L. White Center for Financial Research, Wharton School, University of Pennsylvania, 1989; Marshall Blume and Donald Keim, "Realized Returns and Defaults on Lower-Grade Bonds," Rodney L. White Center for Financial Research, Wharton School, University of Pennsylvania, 1989; Bond Investors Association, "Bond Investors Association Issues Definitive Corporate Default Statistics," press release dated August 15, 1989; Gregory T. Hradsky and Robert D. Long, "High Yield Default Losses and the Return Performance of Bankrupt Debt," *Financial Analysts Journal,* July–August 1989, pp. 38–49; "Historical Default Rates of Corporate Bond Issuers 1970–1988," *Moody's Special Report*, July 1989 (New York: Moody's Investors Service); "High-Yield Bond Default Rates," *Standard & Poor's Creditweek*, August 7, 1989, pp. 21–23; David Wyss, Christopher Probyn, and Robert de Angelis, "The Impact of Recession on High-Yield Bonds," DRI-McGraw-Hill (Washington, D.C.: Alliance for Capital Access, 1989); and the 1984–1989 issues of *High Yield Market Report: Financing America's Futures* (New York and Beverly Hills: Drexel Burnham Lambert, Incorporated).

by themselves are not of paramount significance: It is perfectly possible for a portfolio of corporate bonds to suffer defaults and to outperform Treasuries at the same time, provided the yield spread of the portfolio is sufficiently high to offset the losses from default. Furthermore, because holders of defaulted bonds typically recover a percentage of the face amount of their investment, the default loss rate can be substantially lower than the default rate. The default loss rate is defined is defined as follows:

$$\text{Default loss rate} = \text{Default rate} \times (100\% - \text{Recovery rate})$$

For instance, a default rate of 5% and a recovery rate of 30% means a default loss rate of only 3.5% (70% × 5%). Therefore, focusing exclusively on default rates merely highlights the worst possible outcome that a diversified portfolio of corporate bonds would suffer, assuming all defaulted bonds would be totally worthless.

The studies by Edward Altman and his colleagues on default rates and default loss rates are the most commonly followed by market participants. The default rates and default loss rates are updated periodically.[5] Exhibit 15.6 provides information about defaults or restructuring under distressed conditions from 1978–2002 for high-yield bonds in the United States and Canada. The information shown is the par value outstanding for the year, the amount defaulted, and the default rate. The annual default rate reported in the exhibit is measured by the par value of the high-yield corporate bonds that have defaulted in a given calendar year divided by the total par value outstanding of high-yield corporate bonds during the year. The weighted average default rate for the entire period was 5.49%.

One can see the increased with credit risk in recent years by looking at the default rates in 2001 and 2002. The default rate of 12.8% in 2002 which was greater than the default rate in 2001 (9.8%), the previous record default rate in the 1978–2001 period (1991, 10.3%), and the weighted average default rate for the 1978–2002 period (5.49%).

The last column in Exhibit 15.6 provides the historical default loss rate realized by investors in high-yield corporate bonds from 1978 to 2002. The methodology for computing the default loss rate is as follows. First, the default loss of principal is computed by multiplying the default rate for the year by the average loss of principal. The average loss of principal is computed by first determining the recovery per \$100 of par value. The recovery per \$100 of par value uses the weighted average price of all issues after default. The difference between par value of \$100 and the

[5] The most recent statistics at the time of this writing are reported in Michael T. Kender and Gabriella Petrucci, *Altman Report on Defaults and Returns on High Yield Bonds: 2002 in Review and Market Outlook*, Salomon Smith Barney, February 5, 2003.

EXHIBIT 15.6 **Historical Default Rates and Default Loss Rates for High-Yield Corporate Bonds (Dollars in Millions): 1978–2002[a]**

Year	Par Value Outstanding[a]	Par Value of Default	Default Rate (%)	Weighted Price After Default	Weighted Coupon (%)	Default Loss (%)
2002	$757,000	$96,858	12.79	25.3	9.37	10.15[b]
2001	649,000	63,609	9.80	25.5	9.18	7.76
2000	597,200	30,295	5.07	26.4	8.54	3.95
1999	567,400	23,532	4.15	27.9	10.55	3.21
1998	465,500	7,464	1.60	35.9	9.46	1.10
1997	335,400	4,200	1.25	54.2	11.87	0.65
1996	271,000	3,336	1.23	51.9	8.92	0.65
1995	240,000	4,551	1.90	40.6	11.83	1.24
1994	235,000	3,418	1.45	39.4	10.25	0.96
1993	206,907	2,287	1.11	56.6	12.98	0.56
1992	163,000	5,545	3.40	50.1	12.32	1.91
1991	183,600	18,862	10.27	36.0	11.59	7.16
1990	181,000	18,354	10.14	23.4	12.94	8.42
1989	189,258	8,110	4.29	38.3	13.40	2.93
1988	148,187	3,944	2.66	43.6	11.91	1.66
1987	129,557	7,486	5.78	75.9	12.07	1.74
1986	90,243	3,156	3.50	34.5	10.61	2.48
1985	58,088	992	1.71	45.9	13.69	1.04
1984	40,939	344	0.84	48.6	12.23	0.48
1983	27,492	301	1.09	55.7	10.11	0.54
1982	18,109	577	3.19	38.6	9.61	2.11
1981	17,115	27	0.16	72.0	15.75	0.15
1980	14,935	224	1.50	21.1	8.43	1.25
1979	10,356	20	0.19	31.0	10.63	0.14
1978	8,946	119	1.33	60.0	8.38	0.59
Arithmetic Average, 1978–2002			3.62	42.3	11.06	2.51
Weighted Average, 1978–2002			5.49			4.10

Source: Figure 25, p. 29 in Michael T. Kender and Gabriella Petrucci, *Altman Report on Defaults and Returns on High Yield Bonds: 2002 in Review and Market Outlook*, Salomon Smith Barney, February 5, 2003.
[a] Excludes defaulted issues.
[b] Default loss rate adjusted for fallen angels is 9.256% in 2002.

recovery of principal is the default loss of principal. Next the default loss of coupon is computed. This is found by multiplying the default rate by the weighted average coupon rate divided by two (because the coupon payments are semiannual). The default loss rate is then the sum of the default loss of principal and the default loss of coupon.

The weighted average default loss rate for the entire period was 4.10%. This indicates that the weighted average recovery rate is 95.9%. In the last two years in the study period, the weighted average default rate was considerably higher that the average rate.

INTRODUCTION TO CREDIT VALUE-AT-RISK

Credit risk VaR methodologies take a portfolio approach to credit risk analysis. This means that

- Credit risks to each obligor across the portfolio are restated on an equivalent basis and aggregated in order to be treated consistently, regardless of the underlying asset class.
- Correlations of credit quality moves across obligors are taken into account.

This method allows for portfolio effects—the benefits of diversification and risks of concentration—to be quantified.

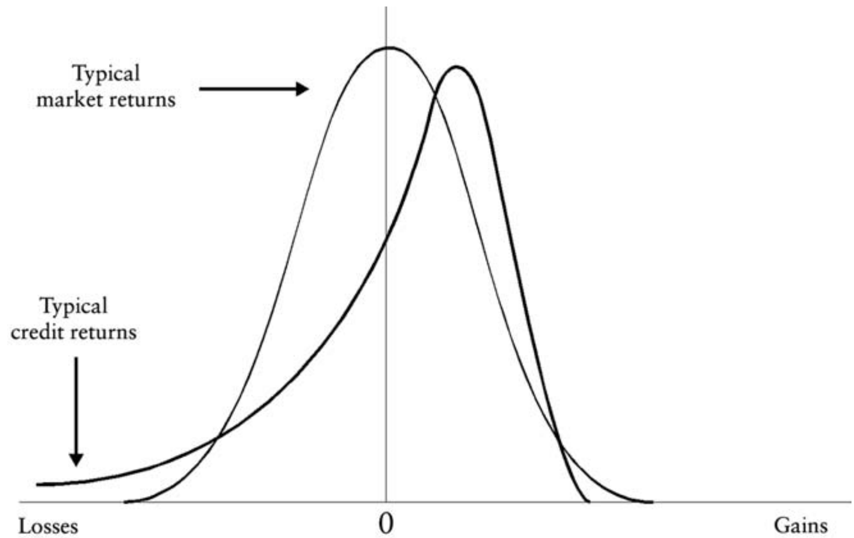
The portfolio risk of an exposure is determined by four factors:

- Size of the exposure
- Maturity of the exposure
- Probability of default of the obligor
- Systematic or concentration risk of the obligor

Credit VaR, like interest rate VaR discussed in Chapter 8, considers (credit) risk in a mark-to-market framework. It arises from changes in value due to credit events, that is changes in obligor credit quality including defaults, upgrades, and downgrades.

Nevertheless credit risk is different in nature from interest rate risk. Typically market return distributions are assumed to be relatively symmetrical and approximated by normal distributions. In credit portfolios, value changes will be relatively small upon minor upgrades or downgrades, but can be substantial upon default. This remote probability of large losses produces skewed distributions with heavy downside tails that differ from the more normally distributed returns assumed for interest rate VaR models. This is shown in Exhibit 15.7.

EXHIBIT 15.7 Comparison of Distribution of Market Returns and Credit Returns



This difference in risk profiles does not prevent us from assessing risk on a comparable basis. Analytical method interest rate VaR models consider a time horizon and estimate value-at-risk across a distribution of estimated market outcomes. Credit VaR models similarly look to a horizon and construct a distribution of value given different estimated credit outcomes.

When modeling credit risk the two main measures of risk are

- *Distribution of loss: Obtaining distributions of loss that may arise from the current portfolio*. This considers the question of what the expected loss is for a given confidence level.
- *Identifying extreme or catastrophic outcomes*. This is addressed through the use of scenario analysis and concentration limits.

To simplify modeling, no assumptions are made about the causes of default. Mathematical techniques used in the insurance industry are used to model the event of an obligor default.

Time Horizon

The choice of time horizon will not be shorter than the time frame over which risk-mitigating actions can be taken. Credit Suisse First Boston, who introduced the CreditRisk+ model shortly after *CreditMetrics* was introduced, suggests two alternatives:

- A constant time horizon such as one year
- A hold-to-maturity time horizon

The constant time horizon is similar to the CreditMetrics approach and also to that used for market risk measures. It is more suitable for trading desks. The hold-to-maturity approach is used by entities such as commercial bank asset/liability management (ALM) desks.

Data Inputs

Modeling credit risk requires certain data inputs. For example, CreditRisk+ uses the following:

- Credit exposures
- Obligor default rates
- Obligor default rate volatilities
- Recovery rates

These data requirements present some difficulties. There is a lack of comprehensive default and correlation data and assumptions need to be made at certain times. The most accessible data are compiled by the credit ratings agencies such as Moody's.

We now consider two methodologies used for measuring credit value-at-risk, the CreditMetrics model and the CreditRisk+ model.

CREDITMETRICS™

One purpose of a risk management system is to direct and prioritize actions. When considering risk-mitigating actions there are various features of risk worth targeting, including obligors having

- The largest absolute exposure
- The largest percentage level of risk (volatility)
- The largest absolute amount of risk

A CreditMetrics-type methodology helps to identify these areas and allow the risk manager to prioritize risk-mitigating action. CreditMetrics is JP Morgan's portfolio model for analyzing credit risk, providing an estimate of value-at-risk due to credit events caused by upgrades, downgrades, and default. A software package known as *CreditManager* is available that allows users to implement the CreditMetrics methodology.[6]

[6] The JP Morgan department behind CreditMetrics was split into a separate corporate entity, known as Riskmetrics, during 2001.

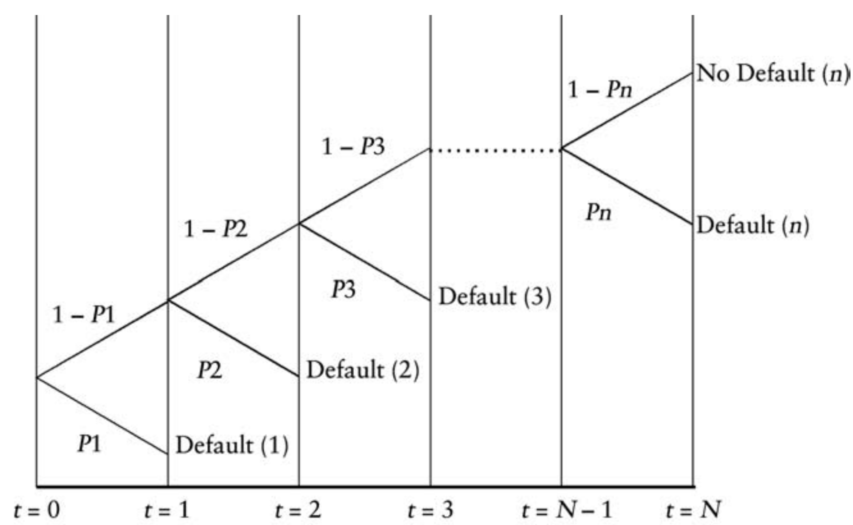
Methodology

There are two main frameworks in use for quantifying credit risk. One approach considers only two states: default and no default. This model constructs a binomial tree of default versus no default outcomes until maturity. This approach is shown at Exhibit 15.8.

The other approach, sometimes called the RAROC (Risk Adjusted Return on Capital) approach holds that risk is the observed volatility of corporate bond values within each credit rating category, maturity band, and industry grouping. The idea is to track a benchmark corporate bond (or index) which has observable pricing. The resulting estimate of volatility of value is then used to proxy the volatility of the exposure (or portfolio) under analysis.

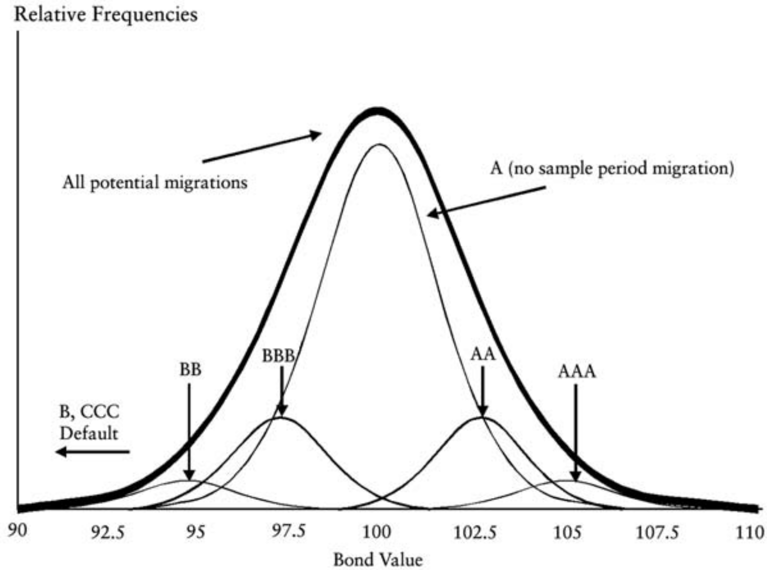
The CreditMetrics™ methodology sits between these two approaches. The model estimates portfolio VaR at the risk horizon due to credit events that include upgrades and downgrades, rather than just defaults. Thus it adopts a mark-to-market framework. As shown in Exhibit 15.9, bonds within each credit rating category have volatility of value due to day-to-day credit spread fluctuations. The exhibit shows the loss distributions for bonds of varying credit quality. CreditMetrics assumes that all credit migrations have been realized, weighting each by a migration likelihood.

EXHIBIT 15.8 **A Binomial Model of Credit Risk**



Source: JP Morgan, *RiskMetrics* Technical document, 1997. Reproduced with permission.

EXHIBIT 15.9 Distribution of Credit Returns by Rating



Source: JP Morgan, *RiskMetrics* Technical document, 1997. Reproduced with permission.

Time Horizon

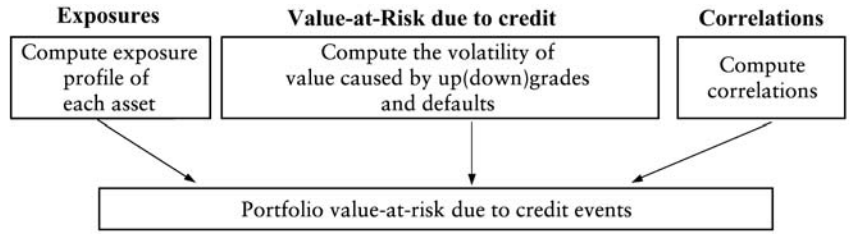
CreditMetrics™ adopts a one-year risk horizon. The justification given in its technical document[7] is that this is because much academic and credit agency data are stated on an annual basis. This is a convenient convention similar to the use of annualized interest rates in the financial markets. The risk horizon is adequate as long as it is not shorter than the time required to perform risk mitigating actions. Users must therefore adopt their risk management and risk adjustments procedures with this in mind.

The steps involved in CreditMetrics measurement methodology are shown in Exhibit 15.10, described by JP Morgan as its analytical "roadmap." The elements in each step are:

Exposures:
User portfolio
Market volatilities
Exposure distributions

[7] JPMorgan, *Introduction to CreditMetrics™*, JPMorgan & Co., 1997.

EXHIBIT 15.10 Analytics Road Map for CreditMetrics



Source: JP Morgan, *RiskMetrics* Technical document, 1997. Reproduced with permission.

VaR due to credit events:
Credit rating
Credit spreads
Rating change likelihood
Recovery rate in default
Present value bond revaluation
Standard deviation of value due to credit quality changes

Correlations:
Ratings series
Models (e.g., correlations)
Joint credit rating changes

Calculating the Credit VaR

CreditMetrics methodology assesses individual and portfolio VaR due to credit in three steps:

Step 1: It establishes the exposure profile of each obligor in a portfolio.

Step 2: It computes the volatility in value of each instrument caused by possible upgrade, downgrade, and default.

Step 3: Taking into account correlations between each of these events it combines the volatility of the individual instruments to give an aggregate portfolio risk.

Step 1—Exposure Profiles

CreditMetrics incorporates the exposure of instruments such as bonds (fixed or floating rate) as well as other loan commitments and market driven instruments such as swaps. The exposure is stated on an equivalent basis for all products. The products covered include:

- Receivables (or trade credit)
- Bonds and loans
- Loan commitments
- Letters of credit
- Market driven instruments

Step 2—Volatility of Each Exposure from Up(down)grades and Defaults

The levels of likelihood are attributed to each possible credit event of upgrade, downgrade, and default. The probability that an obligor will change over a given time horizon to another rating is calculated. Each change (migration) results in an estimated change in value (derived from credit spread data and in default, recovery rates). Each value outcome is weighted by its likelihood to create a distribution of value across each credit state, from which each asset's expected value and volatility (standard deviation) of value are calculated.

There are three steps to calculating the volatility of value in a credit exposure:

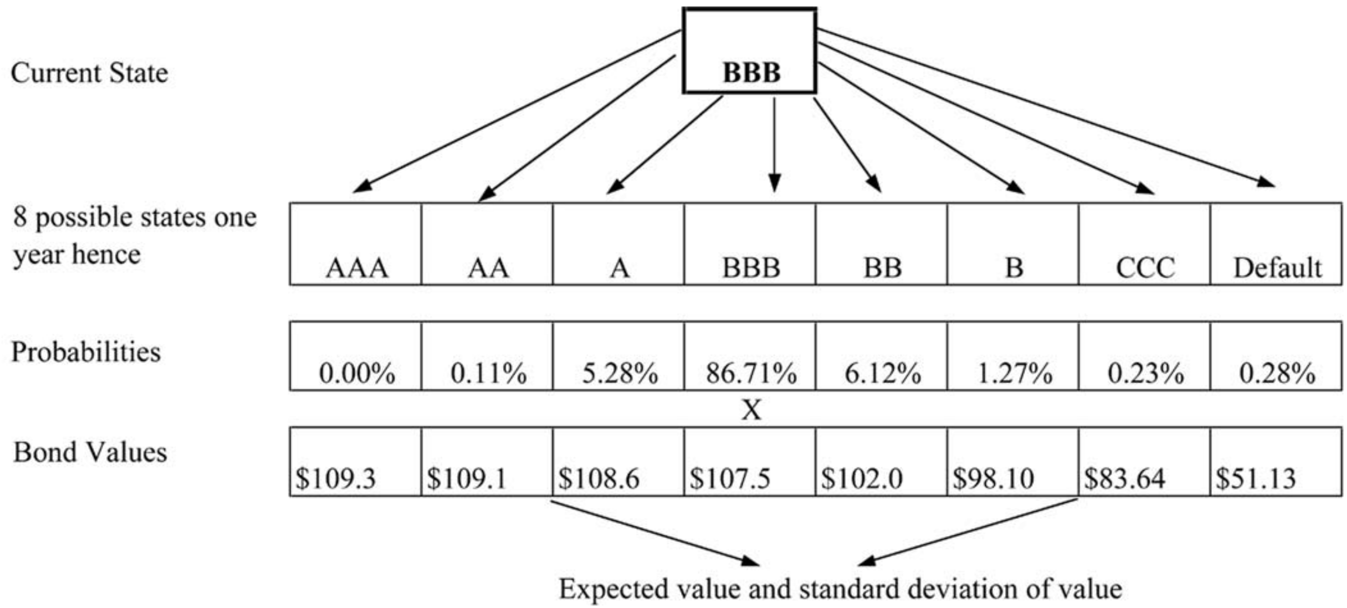
- The senior unsecured credit rating of the issuer determines the chance of either defaulting or migrating to any other possible credit quality state in the risk horizon.
- Revaluation at the risk horizon can be by either (1) the seniority of the exposure, which determines its recovery rate in case of default or (2) the forward zero coupon curve (spot curve) for each credit rating category which determines the revaluation upon up(down)grade.
- The probabilities from the two steps above are combined to calculate volatility of value due to credit quality changes.

An example of calculating the probability step is illustrated in Exhibit 15.11. The probabilities of all possible credit events on an instrument's value must be established first. Given this data, the volatility of value due to credit quality changes for this one position can be calculated. The process is shown in Exhibit 15.11.

Step 3—Correlations

Individual value distributions for each exposure are combined to give a portfolio result. To calculate the portfolio value from the volatility of individual asset values requires estimates of correlation in credit quality changes. CreditMetrics™ itself allows for different approaches to estimating correlations including a simple constant correlation. This

EXHIBIT 15.11 Constructing the Distribution Value for a BBB-Rated Bond



Source: JP Morgan, *RiskMetrics* Technical document, 1997. Reproduced with permission.

is because of frequent difficulty in obtaining directly observed credit quality correlations from historical data.

CreditManager™

CreditManager is the software implementation of CreditMetrics as developed by JP Morgan. It is a PC-based application that measures and analyzes credit risk in a portfolio context. It measures the VaR exposure due to credit events across a portfolio, and also quantifies concentration risks and the benefits of diversification by incorporating correlations (following the methodology utilized by CreditMetrics). The CreditManager application provides a framework for portfolio credit risk management that can be implemented "off-the-shelf" by virtually any institution. It uses the following:

- Obligor credit quality database: details of obligor credit ratings, transition and default probabilities, industries, and countries.
- Portfolio exposure database, containing exposure details for the following asset types: loans, bonds, letters of credit, total return swaps, credit default swaps,[8] interest rate and currency swaps and other market instruments.
- Frequently updated market data: including yield curves, spreads, transition probabilities, and default probabilities.
- Flexible risk analyses with user-defined parameters supporting VaR analysis, marginal risk, risk concentrations, event risk, and correlation analysis.
- Stress testing scenarios, applying user-defined movements to correlations, spreads, recovery rates, transition and default probabilities.
- Customized reports and charts.

CreditManager data sources include Dow Jones, Moody's, Reuters, and Standard and Poor's. By using the software package, risk managers can analyze and manage credit portfolios based on virtually any variable, from the simplest end of the spectrum—single position or obligor—to more complex groupings containing a range of industry and country obligors and credit ratings.

Generally this quantitative measure is employed as part of an overall risk management framework that retains traditional, qualitative methods.

CreditMetrics can be a useful tool for risk managers seeking to apply VaR methodology to credit risk. The model enables risk managers to apply portfolio theory and VaR methodology to credit risk. It has

[8] Total return swaps and credit default swaps are explained in Chapter 16.

several applications including prioritizing and evaluating investment decisions and perhaps most important, setting risk-based exposure limits. Ultimately the model's sponsors claim its use can aid maximizing shareholder value based on risk-based capital allocation. This should then result in increased liquidity in credit markets, the use of a marking-to-market approach to credit positions, and closer interweaving of regulatory and economic capital.

CREDITRISK+

CreditRisk+ was developed by Credit Suisse First Boston (CSFB) and can, in theory, handle all instruments that give rise to credit exposure including bonds, loans commitments, letters of credit, and derivative instruments. We provide a brief description of its methodology here.

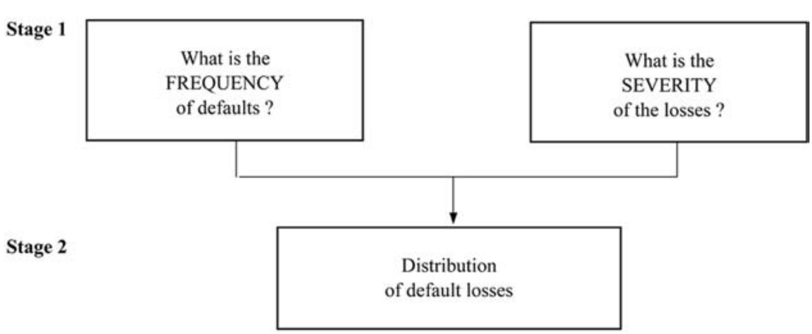
Modeling Process

CreditRisk+ uses a two-stage modeling process as illustrated in Exhibit 15.12.

CreditRisk+ considers the distribution of the number of default events in a time period such as one year, within a portfolio of obligors having a range of different annual probabilities of default.

The annual probability of default of each obligor can be determined by its credit rating and then mapping between default rates and credit ratings. A default rate can then be assigned to each obligor (an example of what this would look like is shown in Exhibit 15.13). Default rate volatilities can be observed from historic volatilities.

EXHIBIT 15.12 CreditRisk+ Modeling Process



Source: Credit Suisse First Boston, *CreditRisk+*, 1998. Reproduced with permission.

EXHIBIT 15.13 Hypothetical One Year Default Rates (%)

Credit Rating	One Year Default Rate (%)
Aaa	0.00
Aa	0.03
A	0.01
Baa	0.12
Ba	1.36
B	7.27

Correlation and Background Factors

Default correlation impacts the variability of default losses from a portfolio of credit exposures. CreditRisk+ incorporates the effects of default correlations by using default rate volatilities and sector analysis.

Unsurprisingly enough, it is not possible to forecast the exact occurrence of any one default or the total number of defaults. Often there are background factors that may cause the incidence of default events to be correlated, even though there is no causal link between them. For example an economy in recession may give rise to an unusually large number of defaults in one particular month, which would increase the default rates above their average level. CreditRisk+ models the effect of background factors by using default rate volatilities rather than by using default correlations as a direct input. Both distributions give rise to loss distributions with fat tails.

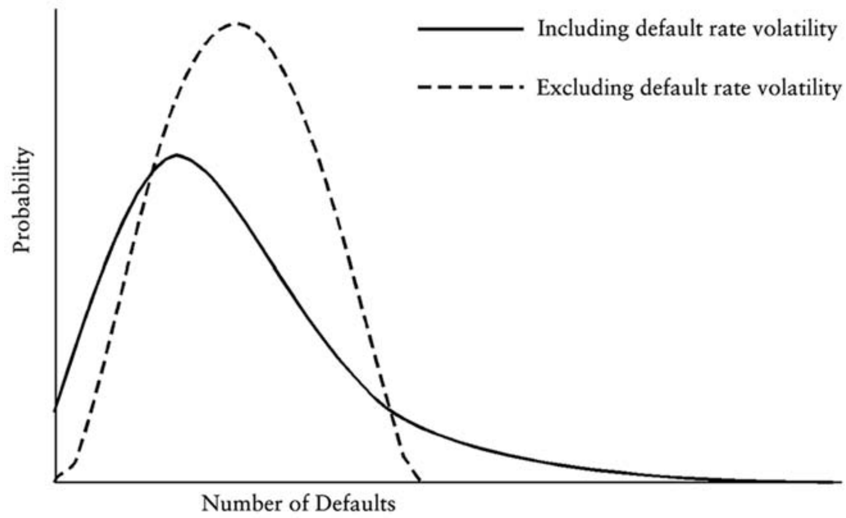
Concentration

As noted above, there are background factors that affect the level of default rates. For this reason it is useful to capture the effect of concentration in particular countries or sectors. CreditRisk+ uses a sector analysis to allow for concentration. Exposures are broken down into an obligor-specific element independent of other exposures, as well as nonspecific elements that are sensitive to particular factors such as countries or sectors.

Distribution of the Number of Default Events

CreditRisk+ models the underlying default rates by specifying a default and a default rate volatility. This aims to take account of the variation in default rates. The effect of using volatility is illustrated in Exhibit 15.14, which shows the distribution of default rates generated by the model when default rate volatility is varied. The distribution becomes skewed to the right when volatility is increased.

EXHIBIT 15.14 CreditRisk+ Distribution of Default Events



Source: Credit Suisse First Boston, *CreditRisk+*, 1998. Reproduced with permission.

This is an important result and demonstrates the increased risk represented by an extreme number of default events. By varying the volatility in this way, CreditRisk+ is attempting to model real-world shock much in the same way that interest rate risk VaR models aim to allow for the fact that market returns do not follow exact normal distributions, as shown by the incidence of market crashes.

Application Software

CSFB has released software that allows the CreditRisk+ model to be run on Microsoft Excel® as a spreadsheet calculator. The user inputs the portfolio static data into a blank template and the model calculates the credit exposure. Obligor exposure can be analyzed on the basis of all exposures being part of the same sector, alternatively up to eight different sectors (government, countries, industry, and so on) can be analyzed. The spreadsheet template allows the user to include up to 4,000 obligors in the static data. An example portfolio of 25 obligors and default rates and default rate volatilities (assigned via a sample of credit ratings) is included with the spreadsheet.

The user's static data for the portfolio will therefore include details of each obligor, the size of the exposure, the sector for that obligor (if

EXHIBIT 15.15 Example Default Rate Data

Credit Rating	Mean Default Rate (%)	Standard Deviation (%)
A+	1.50	0.75
A	1.60	0.80
A–	3.00	1.50
BBB+	5.00	2.50
BBB	7.50	3.75
BBB–	10.00	5.00
BB	15.00	7.50
B	30.00	15.00

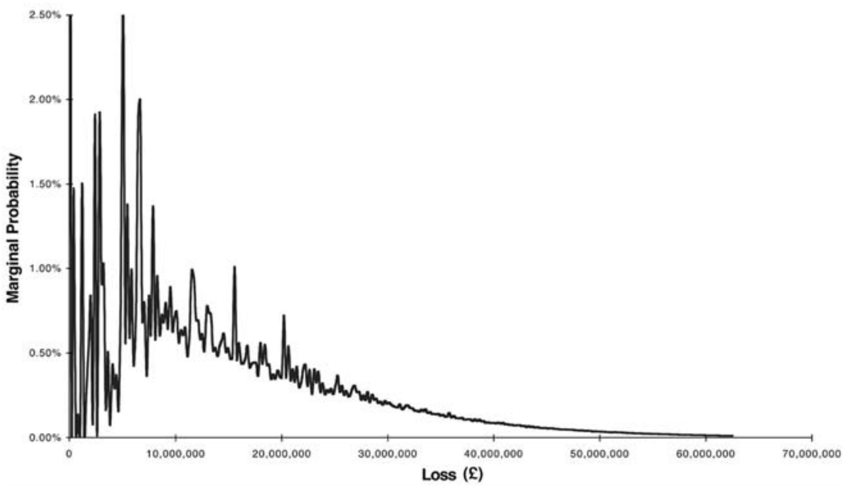
EXHIBIT 15.16 Example Obligor Details

Name	Exposure (£)	Rating	Mean Default Rate (%)	Default Rate Standard Deviation (%)	Sector Split General Economy (%)
Co name	358,475	B	30.00	15.00	100
Co (2)	1,089,819	B	30.00	15.00	100
Co (3)	1,799,710	BBB–	10.00	5.00	100
Co (4)	1,933,116	BB	15.00	7.50	100
Co (5)	2,317,327	BB	15.00	7.50	100
Co (6)	2,410,929	BB	15.00	7.50	100
Co (7)	2,652,184	B	30.00	15.00	100
Co (8)	2,957,685	BB	15.00	7.50	100
Co (9)	3,137,989	BBB+	5.00	2.50	100
Co (10)	3,204,044	BBB+	5.00	2.50	100

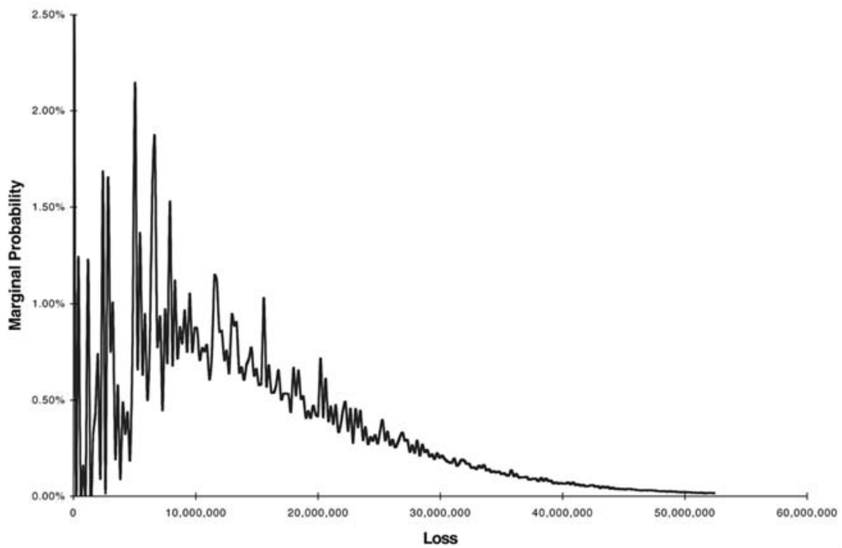
not all in a single sector), and default rates. An example of static data is given in Exhibits 15.15 and 15.16.

An example credit loss distribution calculated by the model is shown in Exhibit 15.17. This shows the distribution for the basic analysis for a portfolio at the simplest level of assumption; all obligors are assigned to a single sector. The full loss distribution over a one-year time horizon is calculated together with percentiles of the loss distribution (not shown here), which assess the relative risk for different levels of loss. The model can calculate distributions for a portfolio with obligors grouped across different sectors, as well as the distribution for a portfolio analyzed over a “hold to maturity” time horizon.

EXHIBIT 15.17 Illustration of Credit Loss Distribution (Single Sector Obligor Portfolio)



Credit Loss Distribution



Summary of CreditRisk+ Model

CreditRisk+ captures the main characteristics of credit default events. In this model, credit default events are rare and occur in a random manner with observed default rates varying from year to year. The model's approach attempts to reflect this by making no assumptions about the timing or causes of these events and by incorporating a default rate volatility. It also takes a portfolio approach and uses sector analysis to allow for concentration risk.

CreditRisk+ is capable of handling large exposure portfolios. The low data requirements and minimum assumptions make the model comparatively easy to implement for firms.

However the model is limited to two states of the world: default or nondefault. This means it is not as flexible as CreditMetrics, for example, and ultimately therefore not modeling the full exposure that a credit portfolio would be subject to.

EXPOSURE LIMITS

Within bank trading desks, credit risk limits are often based on intuitive, but arbitrary, exposure amounts. This is not a logical approach because resulting decisions are not risk-driven. Limits should ideally be set with the help of a quantitative analytical framework.

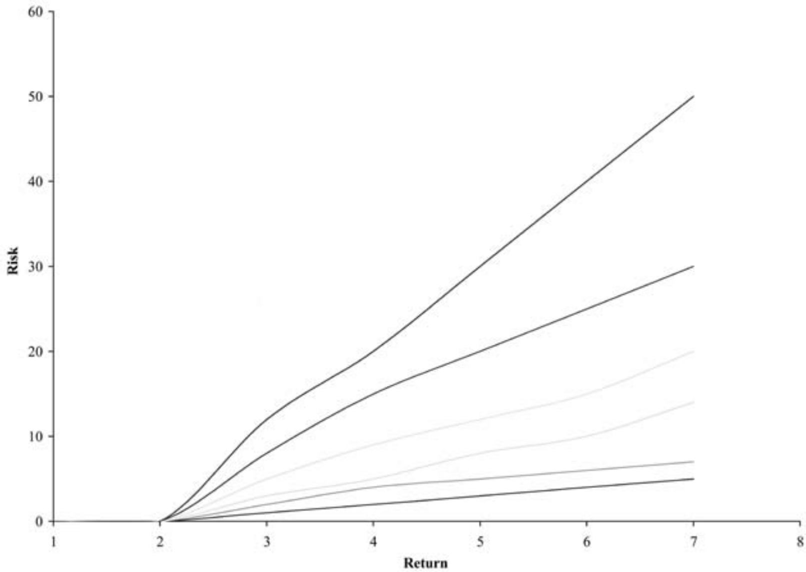
Risk statistics used as the basis of VaR methodology can be applied to limit setting. Ideally such a quantitative approach should be used as an aid to business judgment and not as a stand-alone limit setting tool.

A credit committee considering limit setting can use several statistics such as marginal risk (i.e., the risk over and above the risk expected in normal operations) and standard deviation or percentile levels. Exhibit 15.18 illustrates how marginal risk statistics can be used to make credit limits sensitive to the trade-off between risk and return. The lines on Exhibit 15.18 represent risk/return trade-offs for different credit ratings, all the way from AAA to BBB. The exhibit shows how marginal contribution to portfolio risk increases geometrically with exposure size of an individual obligor, noticeably so for weaker credits. To maintain a constant balance between risk and return, proportionately more return is required with each increment of exposure to an individual obligor.

Standard Credit Limit Setting

In order to equalize a firm's risk appetite between obligors as a means of diversifying its portfolio, a credit limit system could aim to have a large

EXHIBIT 15.18 Size of Total Exposure to Obligor—Risk/Return Profile



number of exposures with equal expected losses. The expected loss for each obligor can be calculated as

$$\text{Default rate} \times (\text{Exposure amount} - \text{Expected recovery})$$

This means that individual credit limits should be set at levels that are inversely proportional to the default rate corresponding to the obligor rating.

Concentration Limits

Concentration limits identified by CreditRisk+ type methodologies have the effect of trying to limit the loss from identified scenarios and is used for managing "tail" risk.

INTEGRATING THE CREDIT RISK AND INTEREST RATE RISK FUNCTIONS

It is logical for banks to integrate credit risk and interest rate risk management for the following reasons:

- The need for comparability between returns on interest rate and credit risk.
- The convergence of risk measurement methodologies.
- The transactional interaction between credit and interest rate risk.
- The emergence of hybrid credit and interest rate risk product structures.

The objective is for returns on capital to be comparable for businesses involved in credit and interest rate risk, which will assist with strategic allocation of capital.

To illustrate, assume that at the time of annual planning a bank's lending manager says his department can earn $10 million over the year if it can increase their loan book by $600 million, while the trading manager says they can also make $10 million if the position limits are increased by $40 million.

Assuming that due to capital restriction only one option can be chosen. Which should it be? The ideal choice is the one giving the higher return on capital, but the bank needs to work out how much capital is required for each alternative. This is a quantitative issue that calls for the application of similar statistical and analytical methods to measure both credit and interest rate risk, if one is comparing like with like.

With regard to the loan issue in the example above the expected return is the mean of the distribution of possible returns. Since the revenue side of a loan (that is, the spread) is known with certainty, the area of concern is the expected credit loss rate. This is the mean of the distribution of possible loss rates, estimated from historic data based on losses experienced with similar quality credits.

In the context of market price risk the common denominator measure of risk is volatility (the statistical standard deviation of the distribution of possible future price movements). To apply this to credit risk the decision maker therefore needs to take into account the standard deviation of the distribution of possible future credit loss rates, thereby comparing like with like.

As VaR has been adopted as an interest rate risk measurement tool, the methodologies behind it were steadily applied to the next step along the risk continuum, that of credit risk. Market events, such as bank trading losses in emerging markets and the meltdown of the Long Term Capital hedge fund in the summer of 1998, have illustrated the interplay between credit risk and interest rate risk. The ability to measure interest rate and credit risk in an integrated model would allow for a more complete picture of the underlying risk exposure. (We would add that adequate senior management understanding and awareness of a third type of risk—liquidity risk—would almost complete the risk measurement picture.)

Interest rate risk VaR measures can adopt one of the different methodologies available; in all of them there is a requirement for the estimation of the distribution of portfolio returns at the end of a holding period. This distribution can be assumed to be normal, which allows for analytical solutions to be developed. The distribution may also be estimated using historical returns. Finally a Monte Carlo simulation can be used to create a distribution based on the assumption of certain stochastic processes for the underlying variables. The choice of methodology is often dependent on the characteristics of the underlying portfolio plus other factors. For example, risk managers may wish to consider the degree of leptokurtosis in the underlying asset returns distribution (see Chapter 5), the availability of historical data or the need to specify a more sophisticated stochastic process for the underlying assets. The general consensus is that Monte Carlo simulation, while the most computer-intensive methodology, is the most flexible in terms of specifying an integrated market and credit model.

As discussed earlier, credit risk measurement models generally fall into two categories. The first category includes models that specify an underlying process for the default process. In these models firms are assumed to move from one credit rating to another with specified probabilities. Default is one of the potential states that a firm could move to. The CreditMetrics model is of this type. The second type of model requires the specification of a stochastic process for firm value. Here default occurs when the value of the firm reaches an externally specified barrier. In both models, when the firm reaches default, the credit exposure is impacted by the recovery rate. Again, market consensus would seem to indicate that the second type of methodology, the firm value model, most easily allows for development of an integrated model that is linked not only through correlation but also the impact of common stochastic variables.

TRACKING ERROR AND CREDIT RISK EXPOSURE

In Chapter 4 we discussed tracking error—how it is computed and the quantification of a portfolio to the factors that affect tracking error. We also made a distinction between backward-looking tracking error and forward-looking tracking error. The former tells a portfolio manager what the exposure of a portfolio was over some investment period while the latter quantifies the future exposure to risk factors.

Total tracking error consists of systematic risk—forces that affect all securities in the benchmark—and nonsystematic risk—risk that is not attributable to the systematic risk factors. Systematic risk factors include term structure risk factors and non-term structure risk factors. Our focus in Chapter 4 was on the former. One of the risk factors included in non-term structure risk factors is quality risk. Quality risk for a portfolio is gauged in terms of the credit rating exposure of the portfolio holdings relative to the benchmark.

Therefore, forward-looking tracking error is a measure of the exposure of a portfolio to the credit risk relative to the benchmark. For example, in Exhibit 4.9 in Chapter 4, a 57-bond portfolio is shown. The benchmark for the portfolio is the Lehman Brothers Aggregate Bond Index. The total tracking was estimated to be 52 basis points. Exhibit 15.19 shows the credit rating distribution of the portfolio versus the benchmark. The distribution is in terms of contribution to duration. (Lehman Brothers used the term "adjusted duration" rather than effective or option-adjusted duration.) The forward-looking tracking error attributable to the deviation from the credit rating distribution of the benchmark (i.e., the tracking error due to quality risk) was estimated to be 5.8 basis points. Thus, the tracking error due to term structure risk factors of 36.3 basis points plus the tracking error due to quality risk of

EXHIBIT 15.19 Analysis of Quality Risk

	Portfolio			Benchmark			Difference	
Quality	% of Portf.	Adj. Dur.	Cntrb. to Adj. Dur.	% of Portf.	Adj. Dur.	Cntrb. to Adj. Dur.	% of Portf.	Cntrb. to Adj. Dur.
Aaa+	34.72	5.72	1.99	47.32	5.41	2.56	–12.60	–0.57
MBS	27.04	1.51	0.41	30.67	1.37	0.42	–3.62	–0.01
Aaa	1.00	6.76	0.07	2.33	4.84	0.11	–1.33	–0.05
Aa	5.54	5.67	0.31	4.19	5.32	0.22	1.35	0.09
A	17.82	7.65	1.36	9.09	6.23	0.57	8.73	0.80
Baa	13.89	4.92	0.68	6.42	6.28	0.40	7.47	0.28
Ba	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
B	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Caa	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Ca or lower	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
NR	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Totals	100.00		4.82	100.00		4.29	0.00	0.54

Source: Exhibit 5 in Lev Dynkin, Jay Hyman, and Wei Wu, "Multi-Factor Risk Models and Their Applications," *Professional Perspectives on Fixed Income Portfolio Management*, Volume 3 (2001), pp. 101–145.

5.8 basis points is expected to produce 42.1 of the 52 basis points of tracking error for the portfolio.

Let's now turn to nonsystematic tracking error. Nonsystematic risk factors are unique risks associated with a particular issuer, *issuer-specific risk*, and those associated with a particular issue, *issue-specific risk*. These two forms of nonsystematic risk factors are related to credit risk as discussed earlier in this chapter. This risk exists because a portfolio manager is not able to buy all the issues included in an bond market index. Thus, there will be a considerable mismatch between exposure to specific issuers and specific issues of a given issuer. For example, suppose in early 2001 a portfolio manager had an overweight of exposure to Enron. The overweight did not necessarily reflect an intended overweight because of the view that Enron's credit might improve. Rather, it was the result of the portfolio construction process in selecting an issue that reflected the risk factors to obtain a target forward-looking tracking error for the portfolio sought by the manager. Because of an exposure to this one credit, Enron, there would have been considerable backward-looking tracking error due to nonsystematic risk factors.

For the 57-bond portfolio, the forward-looking tracking error due to systematic risk factors was 45 basis points. Since the total tracking for the portfolio was 52 basis points, the tracking error due to nonsystematic risk factors was 7 basis points.

KEY POINTS

1. There are two main forms of credit risk: credit spread risk and credit default risk.

2. Credit default risk is the risk that an issuer of debt (obligor) is unable to meet its financial obligations.

3. The credit spread is the excess premium over the government or risk-free rate required by the market for taking on a certain assumed credit exposure.

4. Credit spread risk is the risk of financial loss or underperformance resulting from changes in the level of credit spreads used in the marking-to-market of a product.

5. At the macro level, the empirical evidence suggests that economic cycle affects credit spreads—in general, spreads tighten during the early stages of an economic expansion, and spreads widen sharply during an economic recession.

6. At the micro level, the analysis of a potential change in the credit spread focuses on the fundamental factors that have changed the individual issuer's ability to meet its debt obligations.

7. Downgrade risk is the risk that an issue will be downgraded, resulting in an increase in the credit spread.

8. For long-term debt obligations, a credit rating is a forward-looking assessment of the probability of default and the relative magnitude of the loss should a default occur. For short-term debt obligations, a credit rating is a forward-looking assessment of the probability of default.

9. Focusing on default rates on high-yield corporate bonds does not provide sufficient insight into the risks of investing in this sector of the bond market.

10. An investor in high-yield corporate bonds must look at both the default rate and the recovery rate.

11. The default loss rate is defined as the product of the default rate and (100% – recovery rate).

12. Modeling credit risk takes into account the skewed distribution pattern of credit returns and credit loss patterns.

13. The most commonly used measures of credit value-at-risk use a portfolio approach to risk measurement. The portfolio risk exposure is determined by: (1) size of exposure, (2) maturity of exposure, (3) probability of default of the obligor, and (4) systematic or concentration risk of the obligor.

14. In quantifying credit risk there are two frameworks to adopt: (1) default and non-default and (2) the risk-adjusted return on capital approach.

15. CreditMetrics uses the variance-covariance and portfolio approaches: it estimates portfolio VaR due to credit events (downgrades and defaults).

16. CreditRisk+ uses the default and nondefault approach.

17. Applications of credit VaR include: prioritizing risk-reducing actions (including targeting largest absolute exposure, largest percentage level of risk and volatility, and largest absolute amount of risk) and setting exposure limits and concentration limits.

18. Reasons for integrating the interest rate risk and credit risk functions include: (1) the need for comparability between returns on credit risk and interest rate risk, (2) the transactional interaction between credit and interest risk, and (3) the emergence of hybrid credit and interest risk structures.

19. Forward-looking tracking error due to quality risk and due to nonsystematic risk (issuer-specific risk and issue-specific risk) can be used to estimate the exposure of a bond portfolio relative to a benchmark.

CHAPTER 16

Credit Derivatives: Instruments and Applications

Credit derivatives allow investors to manage the credit risk exposure of their portfolios or asset holdings, essentially by providing insurance against a deterioration in credit quality of the borrowing entity.[1] If there is a technical default by the borrower[2] or an actual default on the loan itself, and the bond is marked down in price, the losses suffered by the investor can be recouped in part or in full through the payout made by the credit derivative.

The objectives of this chapter are to:

1. Explain and describe what credit derivatives are.
2. Explain how credit risk can be managed and hedged using credit derivatives.
3. Explain what credit default swaps, total return swaps, credit-linked notes, and credit options are.
4. Define credit derivative mechanics, including the concept of credit event, physical settlement, and cash settlement.
5. Describe the conditions under which banks and financial institutions use credit derivatives.
6. Present an overview of the main applications of credit derivatives for commercial banks and portfolio managers.

[1] The simplest credit derivative works exactly like an insurance policy, with regular premiums paid by the protection-buyer to the protection-seller, and a payout in the event of a specified credit event.

[2] A technical default is a delay in timely payment of the coupon, or nonpayment of the coupon altogether.

CREDIT RISK AND CREDIT DERIVATIVES

Credit derivatives are financial contracts designed to reduce or eliminate credit risk exposure by providing insurance against losses suffered due to *credit events*. A payout from a credit derivative is triggered by a credit event. As banks define default in different ways, the terms under which a credit derivative is executed usually include a specification of what constitutes a credit event.

The principle behind credit derivatives is straightforward. Investors desire exposure to nondefault free debt because of the higher returns this offers. However, such exposure brings with it concomitant credit risk. This risk can be managed with credit derivatives. At the same time, the exposure itself can be taken on synthetically if, for instance, there are compelling reasons why a cash market position cannot be established. The flexibility of credit derivatives provides users a number of advantages and because they are over-the-counter (OTC) products, they can be designed to meet specific user requirements.

In this chapter, we focus on credit derivatives as instruments that may be used to manage risk exposure inherent in a corporate or non-AAA sovereign bond portfolio. They may also be used to manage the credit risk of commercial loan books. The intense competition amongst commercial banks, combined with rapid disintermediation, has meant that banks have been forced to evaluate their lending policy, with a view to improving profitability and return on capital. The use of credit derivatives assists banks with restructuring their businesses, because they allow banks to repackage and parcel out credit risk, while retaining assets on balance sheet (when required) and thus maintain client relationships.

Because credit derivatives isolate certain aspects of credit risk from the underlying loan or bond and transfer them to another entity, it becomes possible to separate the ownership and management of credit risk from the other features of ownership associated with the assets in question. This means that illiquid assets such as bank loans, and illiquid bonds, can have their credit risk exposures transferred. The bank owning the assets can protect against credit loss even if it cannot transfer the assets themselves.

The same principles carry over to the credit risk exposures of portfolio managers. For fixed-income portfolio managers some of the advantages of credit derivatives include the following:

- They can be customized to meet the specific requirements of the entity buying the risk protection, as opposed to the liquidity or term of the underlying reference asset.
- They can be "sold short" without risk of a liquidity or delivery squeeze, as it is a specific credit risk that is being traded. In the cash

market it is not possible to "sell short" a bank loan for example, but a credit derivative can be used to establish synthetically the economic effect of such a position.
- They theoretically isolate credit risk from other factors such as client relationships and interest rate risk, thereby introducing a formal pricing mechanism to price credit issues only. This means a market can develop in credit only, allowing more efficient pricing, and it becomes possible to model a term structure of credit rates.
- They are off-balance sheet instruments[3] and as such incorporate tremendous flexibility and leverage, exactly like other financial derivatives. For instance, bank loans are not particularly attractive investments for certain investors because of the administration required in managing and servicing a loan portfolio. However an exposure to bank loans and their associated return can be achieved by say, a total return swap (one type of credit derivative discussed later) while simultaneously avoiding the administrative costs of actually owning the assets. Hence credit derivatives allow investors access to specific credits while allowing banks access to further distribution for bank loan credit risk.

Bond portfolio managers can also use credit derivatives to increase the liquidity of their portfolios, gain from the relative value arising from credit pricing anomalies, and enhance portfolio returns. Some key applications are summarized later in the chapter.

CREDIT EVENT

The occurrence of a specified credit event will trigger payment of the default payment by the seller of protection to the buyer of protection. Contracts specify physical or cash settlement. In *physical settlement*, the protection buyer transfers to the protection seller the deliverable obligation (usually the reference asset or assets), with the total principal outstanding equal to the nominal amount specified in the default swap contract. The protection seller simultaneously pays to the buyer 100% of the nominal amount. In *cash settlement*, the protection seller hands to the buyer the difference between the nominal amount of the default swap and the *final value* for the same nominal amount of the reference asset. This final value is usually determined by means of a poll of dealer banks.

The following may be specified as credit events in the legal documentation between counterparties:

[3] When credit derivatives are embedded in certain fixed-income products, such as structured notes and credit-linked notes, they are then off-balance sheet but part of a structure that may have on-balance sheet elements.

- Downgrade in S&P and/or Moody's credit rating below a specified minimum level.
- Financial or debt restructuring, for example occasioned under administration or as required under U.S. bankruptcy protection.
- Bankruptcy or insolvency of the reference asset obligor.
- Default on payment obligations such as bond coupon and continued nonpayment after a specified time period.
- Technical default, for example the nonpayment of interest or coupon when it falls due.
- A change in credit spread payable by the obligor above a specified maximum level.

The 1999 International Swap and Derivatives Association (ISDA) credit default swap documentation specifies bankruptcy, failure to pay, obligation default, debt moratorium, and restructuring to be credit events. Note that it does not specify a rating downgrade to be a credit event.[4]

The precise definition of "restructuring" is open to debate and has resulted in legal disputes between protection buyers and sellers. Prior to issuing its 1999 definitions, ISDA had specified restructuring as an event or events that resulted in making the terms of the reference obligation "materially less favorable" to the creditor (or protection seller) from an economic perspective. This definition is open to more than one interpretation and caused controversy when determining if a credit event had occurred. The 2001 ISDA definitions specified more precise conditions, including any action that resulted in a reduction in the amount of principal. In the European market, restructuring is generally retained as a credit event in contract documentation, but in the U.S. market it is less common to see it included. Instead, U.S. contract documentation tends to include as a credit event a form of "modified restructuring," the impact of which is to limit the options available to the protection buyer as to the type of assets it could deliver in a physically-settled contract.

CREDIT DERIVATIVE INSTRUMENTS

Credit derivative instruments enable participants in the financial market to trade in credit as an asset, as they isolate and transfer credit risk. They also enable the market to separate funding considerations from credit risk. A number of instruments come under the category of credit derivatives. In this section we consider the most commonly encountered

[4] The ISDA definitions from 1999 and restructuring supplement from 2001 are available at www.ISDA.org.

credit derivative instruments. Irrespective of the particular instrument under consideration, all credit derivatives can be described under the following characteristics:

- The *reference entity*, which is the asset or name on which credit protection is being bought and sold.
- The credit event, or events, which indicate that the reference entity is experiencing or about to experience financial difficulty and which act as trigger events for payments under the credit derivative contract.
- The settlement mechanism for the contract, whether cash settled or physically settled.
- When there is physical settlement, the deliverable obligation that the protection buyer delivers to the protection seller on the occurrence of a trigger event.

Credit derivatives are grouped into funded and unfunded instruments. In a *funded credit derivative*, typified by a credit-linked note (CLN), the investor in the note is the credit-protection seller and is making an upfront payment to the protection buyer when it buys the note. Thus, the protection buyer is the issuer of the note. If no credit event occurs during the life of the note, the redemption value of the note is paid to the investor at the maturity date. If a credit event does occur, then at the maturity date a value less than par will be paid out to the investor. This value will be reduced by the nominal value of the reference asset that the CLN is linked to. The exact process will differ according to whether cash settlement or physical settlement has been specified for the note. We will consider this later.

In an *unfunded credit derivative*, typified by a credit default swap, the protection seller does not make an upfront payment to the protection buyer. Credit default swaps have a number of applications and are used extensively for flow trading of single reference name credit risks or, in *portfolio swap* form, for trading a basket of reference credits. Credit default swaps and CLNs are used in structured products, in various combinations, and their flexibility has been behind the growth and wide application of the synthetic collateralized debt obligation and other credit hybrid products.

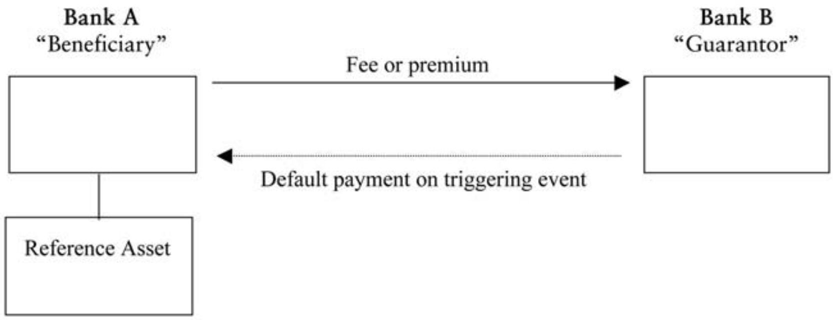
We now consider the key credit derivative instruments.

Credit Default Swap

The most common credit derivative is the *credit default swap,* also referred to as *credit swap* and *default swap*.[5] This is a bilateral contract in

[5] The authors prefer the first term, but the other two terms are common. "Credit swap" does not, we feel adequately describe the actual purpose of the instrument.

EXHIBIT 16.1 Credit Default Swap



which a periodic fixed fee or a one-time premium is paid to a *protection seller*, in return for which the seller will make a payment on the occurrence of a specified credit event. The fee is usually quoted as a basis point multiplier of the nominal value. It is usually paid quarterly in arrears.

The swap can refer to a single asset, known as the *reference asset*, *reference entity*, or *underlying asset*, or a basket of assets. The default payment can be paid in whatever way suits the protection buyer or both counterparties. For example it may be linked to the change in price of the reference asset or another specified asset, it may be fixed at a predetermined recovery rate, or it may be in the form of actual delivery of the reference asset at a specified price. The basic structure of a credit default swap is illustrated in Exhibit 16.1.

The credit default swap enables one party to transfer its credit risk exposure to another party. Banks may use default swaps to trade sovereign and corporate credit spreads without trading the actual assets themselves; for example someone who has gone long a credit default swap (the protection buyer) will gain if the reference asset obligor suffers a rating downgrade or defaults, and can sell the default swap at a profit if he can find a buyer counterparty.[6] This is because the cost of protection on the reference asset will have increased as a result of

[6] Be careful with terminology here. To "go long" an instrument generally is to purchase it. In the cash market, going long the bond means one is buying the bond and so receiving coupon; the buyer has therefore taken on credit risk exposure to the issuer. In a credit default swap, going long is to buy the swap, but the buyer is purchasing protection and therefore paying the premium; the buyer has no credit exposure on the reference entity and has in effect "gone short" on the reference entity (the equivalent of shorting a bond in the cash market and paying coupon). So buying a credit default swap is frequently referred to in the market as "shorting" the reference entity.

the credit event. The original buyer of the credit default swap need never have owned a bond issued by the reference asset obligor.

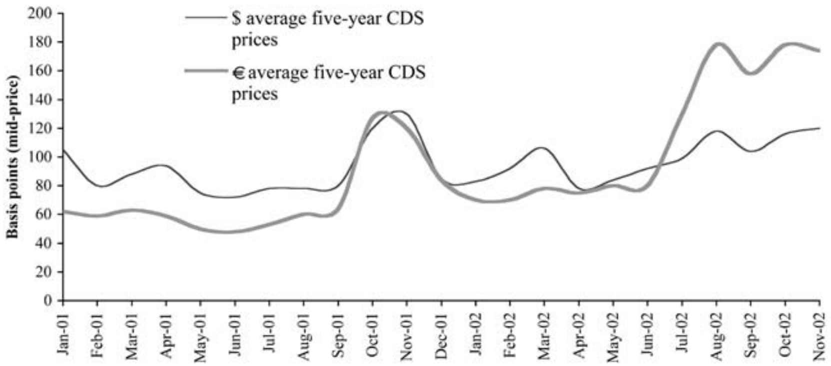
The maturity of the credit default swap does not have to match the maturity of the reference asset and often does not. On occurrence of a credit event, the swap contract is terminated and a settlement payment made by the protection seller or guarantor to the protection buyer. This termination value is calculated at the time of the credit event, and the exact procedure that is followed to calculate the termination value will depend on the settlement terms specified in the contract. This will be either cash settlement or physical settlement.

- *Cash settlement:* the contract may specify a predetermined payout value on occurrence of a credit event. This may be the nominal value of the swap contract. Such a swap is known in some markets as a *digital credit derivative*. Alternatively, the termination payment can be calculated as the difference between the nominal value of the reference asset and its market value at the time of the credit event. This arrangement is more common with cash-settled contracts.[7]
- *Physical settlement:* on occurrence of a credit event, the buyer delivers the reference asset to the seller, in return for which the seller pays the face value of the delivered asset to the buyer. The contract may specify a number of alternative assets that the buyer can deliver. These are known as *deliverable obligations*. This may apply when a swap has been entered into on a reference entity rather than a specific obligation (such as a particular bond) issued by that entity. Where more than one deliverable obligation is specified, the protection buyer will invariably deliver the asset that is the cheapest on the list of eligible assets. This gives rise to the concept of the *cheapest-to-deliver*, as encountered with government bond futures contracts (see Chapter 9), and is in effect an embedded option afforded the protection buyer.

In theory, the value of protection is identical irrespective of which settlement option is selected. However under physical settlement the protection seller can gain if there is a recovery value that can be extracted from the defaulted asset; or its value may rise as the fortunes of the issuer improve. Swap market-making banks often prefer cash settlement as there is less administration associated with it. It is also more suitable when the swap is used as part of a synthetic structured product, because such vehicles may not be set up to take delivery of physical assets. Another advan-

[7] Determining the market value of the reference asset at the time of the credit event may be problematic: the issuer of the asset may well be in default or administration (state of bankruptcy). An independent third-party *Calculation Agent* is usually employed to make the termination payment calculation.

EXHIBIT 16.2 **Investment-Grade Credit Default Swap (CDS) Levels**



Source: Bloomberg Financial Markets

tage of cash settlement is that it does not expose the protection buyer to any risks should there not be any deliverable assets in the market, for instance due to shortage of liquidity in the market—were this to happen, the buyer may find the value of its settlement payment reduced.

Nevertheless physical settlement is widely used because counterparties wish to avoid the difficulties associated with determining the market value of the reference asset under cash settlement. Physical settlement also permits the protection seller to take part in the creditor negotiations with the reference entity's administrators, which may result in improved terms for them as holders of the asset.

For illustrative purposes, Exhibit 16.2 shows investment-grade credit default swap levels during 2001 and 2002 for U.S. dollar and euro reference assets (average levels taken).

Credit Default Swap Example

XYZ Corp. credit spreads are currently trading at 120 basis points (bps) over Treasuries for 5-year maturities and 195 bps over for 10-year maturities. A portfolio manager hedges a $10 million holding of a 10-year bond by purchasing the following credit default swap, written on the 5-year bond:

Term	5 years
Reference credit	XYZ Corp. 5-year bond
Credit event	The business day following occurrence of specified credit event
Default payment	Nominal value of bond × [100% – Price of bond a a percent of par after credit event]
Swap premium	3.35%

This hedge protects for the first five years of the holding, and in the event of XYZ's credit spread widening, will increase in value. It may then be sold before expiry at a profit. The 10-year bond also earns 75 bps over the 5-year bond for the portfolio manager.

Assume now that midway into the life of the credit default swap there is a technical default on the XYZ Corp. 5-year bond, such that its price now stands at $28. Under the terms of the credit default swap the protection buyer delivers the bond to the seller, who pays out $7.2 million to the buyer, as shown below:

$$\begin{aligned}\text{Default payment} &= \$10{,}000{,}000 \times (100\% - 28\%) \\ &= \$7{,}200{,}000\end{aligned}$$

Credit-Linked Note

Credit-linked notes (CLN) exist in a number of forms, but all of them contain a link between the return they pay and the credit-related performance of the underlying asset. A standard credit-linked note is a security, usually issued by an investment-graded entity, that has an interest payment and fixed maturity structure similar to a vanilla bond. The performance of the note however, including the maturity value, is linked to the performance of a specified underlying asset or assets as well as that of the issuing entity. CLNs are usually issued at par. They are often used by borrowers to hedge against credit risk, and by investors to enhance the yield received on their holdings. Hence, the issuer of the note is the protection buyer and the buyer of the note is the protection seller.

Essentially CLNs are hybrid instruments that combine a credit derivative with a vanilla bond. The credit-linked note pays regular coupons, however the credit derivative element is usually set to allow the issuer to decrease the principal amount if a credit event occurs.

For example, consider an issuer of credit cards that wants to fund its (credit card) loan portfolio via an issue of debt. In order to reduce the credit risk of the loans, it issues a 2-year CLN. The principal amount of the bond is 100% as usual, and it pays a coupon of 7.50%, which is 200 basis points above the 2-year benchmark. If, however, the incidence of bad debt amongst credit card holders exceeds 10%, then the terms state that note holders will only receive back $85 per $100 nominal. The credit card issuer has in effect purchased a credit option that lowers its liability in the event that it suffers from a specified credit event, which in this case is an above-expected incidence of bad debts. The credit card issuer has issued the credit-linked note to reduce its credit exposure, in the form of this particular type of credit insurance. If the incidence of bad debts is low, the note is redeemed at par. However if there a high incidence of such debt, the issuer will only have to repay a part of its loan liability.

Investors may wish to purchase the CLN because the coupon paid on it will be above what the credit card issuer would pay on a vanilla bond it issued, and higher than other comparable investments in the market. In addition such notes are usually priced below par at issue. Assuming the notes are eventually redeemed at par, investors will also have realized a substantial capital gain.

As with credit default swaps, CLNs may be specified under cash settlement or physical settlement. Specifically,

- Under cash settlement, if a credit event has occurred, on maturity the protection seller receives the difference between the value of the initial purchase proceeds and the value of the reference asset at the time of the credit event (Exhibit 16.3 illustrates a cash-settled credit-linked note).

EXHIBIT 16.3 Credit-Linked Note

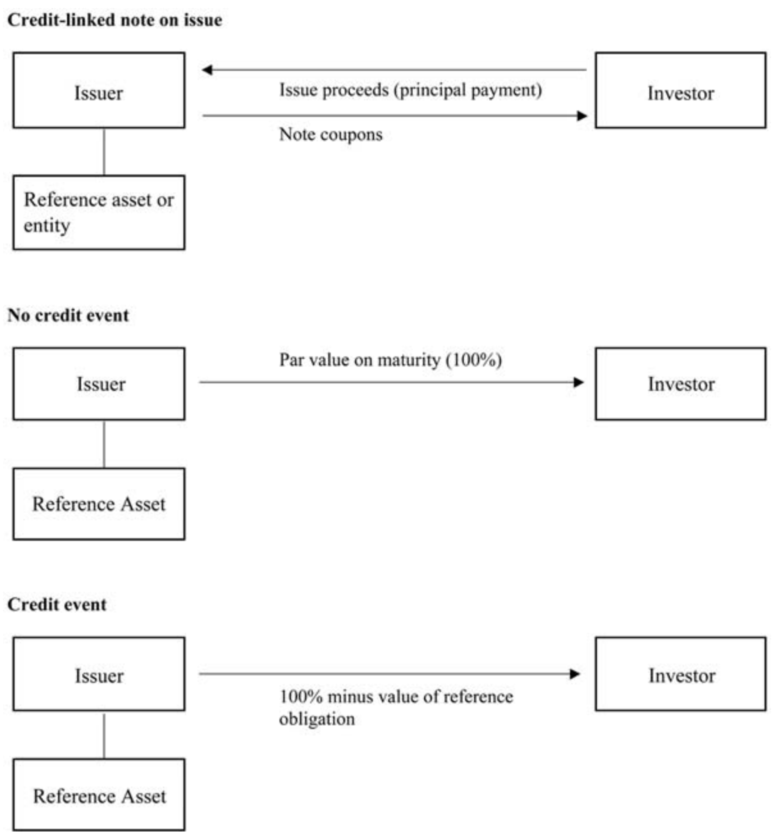
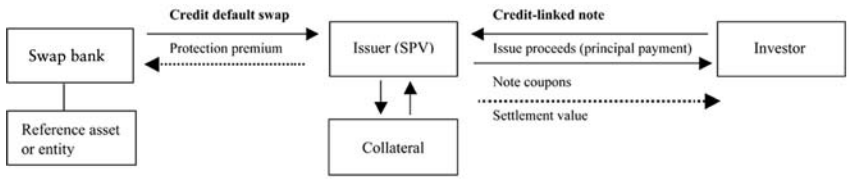


EXHIBIT 16.4 **CLN and Credit Default Swap Structure on Single-Reference Name**



- Under physical settlement, on occurrence of a credit event, at maturity the protection buyer delivers the reference asset or an asset among a list of deliverable assets, and the protection seller receives the value of the original purchase proceeds minus the value of the asset that has been delivered.

Structured products may combine both CLNs and credit default swaps, to meet issuer and investor requirements. For instance, Exhibit 16.4 shows a credit structure designed to provide a higher return for an investor on comparable risk to the cash market. An issuing entity is set up in the form of a special purpose vehicle (SPV) which issues CLNs to the market. The structure is engineered so that the SPV has a neutral position on a reference asset. It has bought protection on a single-reference name by issuing a funded credit derivative, the CLN, and simultaneously sold protection on this name by selling a credit default swap on this name. The proceeds of the CLN are invested in risk-free collateral such as Treasury bills. The coupon on the CLN will be a spread over Libor. It is backed by the collateral account and the fee generated by the SPV in selling protection with the credit default swap. Investors in the CLN will have exposure to the reference asset or entity, and the repayment of the note is linked to the performance of the reference entity. If a credit event occurs, the maturity date of the CLN is brought forward and the note is settled as par minus the value of the reference asset or entity.

Total Return Swap

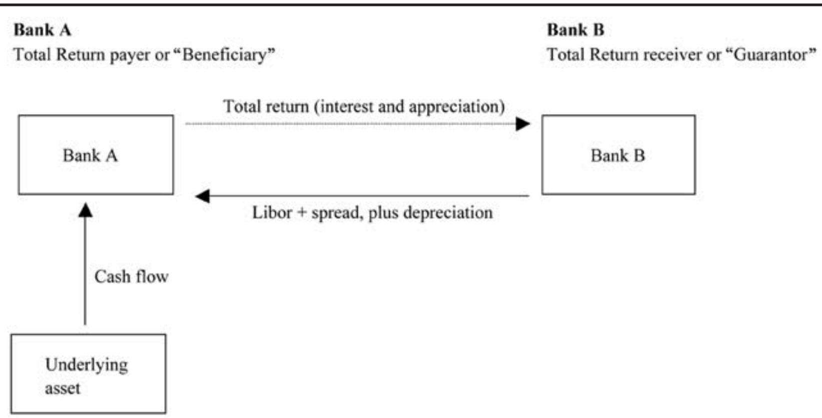
A *total return swap* (TR swap), sometimes known as a *total rate of return swap* or *TR swap*, is an agreement between two parties that exchanges the total return from a financial asset between them. This is designed to transfer the credit risk from one party to the other. It is one of the principal instruments used by banks and other financial institutions to manage their credit risk exposure, and as such is a credit derivative. One definition of a TR swap states that it is a swap agreement in which the *total return* of a bank loan or credit-sensitive security is exchanged for some other cash flow, usually tied to Libor or some other loan or credit-sensitive security.

In some versions of a TR swap the actual underlying asset is sold to the counterparty, with a corresponding swap transaction agreed alongside; in other versions there is no physical change of ownership of the underlying asset. The TR swap trade itself can be to any maturity term; that is, it need not match the maturity of the underlying security. In a TR swap the total return from the underlying asset is paid over to the counterparty in return for a fixed or floating cash flow. This makes it slightly different to other credit derivatives, as the payments between counterparties to a TR swap are connected to changes in the market value of the underlying asset, as well as changes resulting from the occurrence of a credit event.

Illustration of a Total Return Swap

Exhibit 16.5 illustrates a generic TR swap. The two counterparties are labelled as banks, but the party termed "Bank A" can be another financial institution, including cash-rich fixed income portfolio managers such as insurance companies and hedge funds. In the exhibit, Bank A has contracted to pay the "total return" on a specified reference asset, while simultaneously receiving a Libor-based return from Bank B. The reference or underlying asset can be a bank loan such as a corporate loan or a sovereign or corporate bond. The total return payments from Bank A include the interest payments on the underlying loan as well as any appreciation in the market value of the asset. Bank B will pay the Libor-based return; it will also pay any difference if there is a depreciation in the price of the asset.

EXHIBIT 16.5 Total Return Swap



The economic effect is as if Bank B owned the underlying asset; as such TR swaps are synthetic loans or securities. A significant feature is that Bank A will usually hold the underlying asset on its balance sheet, so that if this asset was originally on Bank B's balance sheet, this is a means by which the latter can have the asset removed from its balance sheet for the term of the TR swap.[8] If we assume Bank A has access to Libor funding, it will receive a spread on this from Bank B. Under the terms of the swap, Bank B will pay the difference between the initial market value and any depreciation, so it is sometimes termed the "guarantor" while Bank A is the "beneficiary."

The total return on the underlying asset is the interest payments and any change in the market value if there is capital appreciation. The value of an appreciation may be cash settled, or alternatively there may be physical delivery of the reference asset on maturity of the swap, in return for a payment of the initial asset value by the total return "receiver." The maturity of the TR swap need not be identical to that of the reference asset, and in fact it is rare for it to do so.

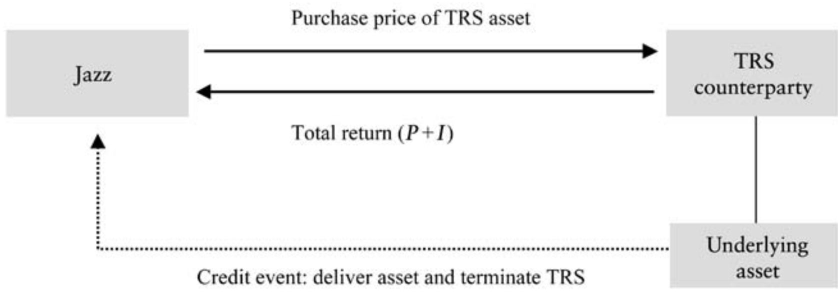
The swap element of the trade will usually pay on a quarterly or semiannual basis, with the underlying asset being revalued or marked-to-market on the refixing dates. The asset price is usually obtained from an independent third party source such as Bloomberg or Reuters, or as the average of a range of market quotes. If the *obligor* of the reference asset defaults, the swap may be terminated immediately, with a net present value payment changing hands according to what this value is, or it may be continued with each party making appreciation or depreciation payments as appropriate. This second option is only available if there is a market for the asset, which is unlikely in the case of a bank loan. If the swap is terminated, each counterparty will be liable to the other for accrued interest plus any appreciation or depreciation of the asset. Commonly under the terms of the trade, the guarantor bank has the option to purchase the underlying asset from the beneficiary bank, and then dealing directly with the loan defaulter.

The Total Return Swap and the Synthetic CDO

A variation on the generic TR swap has been used in structured credit products such as synthetic collateralized debt obligations (CDOs). An example of this is the Jazz I CDO B.V., which is a vehicle that can trade in cash bonds as well as credit default swaps and total return swaps. It has been called a "hybrid CDO" for this reason. In the Jazz structure, the TR swap is a funded credit derivative because the market price of the reference asset is paid upfront by the Jazz special purpose vehicle to the swap coun-

[8] Although it is common for the receiver of the Libor-based payments to have the reference asset on its balance sheet, this is not always the case.

EXHIBIT 16.6 **Total Return Swap as used in Jazz I CDO BV**



terparty. In return the swap counterparty pays the principal and interest on the reference asset to Jazz CDO. The Jazz CDO has therefore purchased the reference asset synthetically. On occurrence of a credit event, the swap counterparty delivers the asset to the CDO and the TR swap is terminated. Because these are funded credit derivatives, a liquidity facility is needed by the vehicle, which it will draw on whenever it purchases a TR swap. This facility is provided by the arranging bank to the structure.

The TR swap arrangement in the Jazz structure is shown at Exhibit 16.6.

Creating a Synthetic Repo

There are a number of reasons why portfolio managers may wish to enter into TR swap arrangements. One of these is to reduce or remove credit risk. Using TR swaps as a credit derivative instrument, a party can remove exposure to an asset without having to sell it. In a vanilla TR swap the total return payer retains rights to the reference asset, although in some cases servicing and voting rights may be transferred. The total return receiver gains an exposure to the reference asset without having to pay out the cash proceeds that would be required to purchase it. As the maturity of the swap rarely matches that of the asset, the swap receiver may gain from the positive funding or *carry* that derives from being able to roll over short-term funding of a longer-term asset.[9] The total return payer on the other hand benefits from protection against interest rate and credit risk for a specified period of time, without having to liquidate the asset itself. At the maturity of the swap the total return payer may reinvest the asset if it continues to own it, or it may sell the asset in the open market. Thus the instrument may be considered a *synthetic repo*.

[9] This assumes a positively sloping yield curve.

A TR swap agreement entered into as a credit derivative is a means by which banks can take on unfunded off-balance sheet credit exposure. Higher-rated banks that have access to London interbank bid rate (Libid) funding can benefit by funding on-balance sheet assets that are credit protected through a credit derivative such as a TR swap, assuming the net spread of asset income over credit protection premium is positive.

A TR swap conducted as a synthetic repo is usually undertaken to effect the temporary removal of assets from the balance sheet. This may be desired for a number of reasons, for example if the institution is due to be analyzed by credit rating agencies, or if the annual external audit is due shortly. Another reason a bank may wish to temporarily remove lower credit-quality assets from its balance sheet is if it is in danger of breaching capital limits in between the quarterly return periods. In this case, as the return period approaches, lower quality assets may be removed from the balance sheet by means of a TR swap, which is set to mature after the return period has passed.

Credit Options

Credit options are also OTC financial contracts. However, there is often confusion in the credit derivatives market about what market participant refer to as a credit option. These options can be classified based on the factor that will trigger or determine whether or not there is a payoff to the option. Accordingly, they can be classified as credit default options and credit spread options. In turn, the latter can be categorized based on the underlying, as will be discussed below. In general, the credit options market is nowhere near as large as the market for credit default swaps.

Credit Default Options

In a credit default option, the payoff triggers (activates) if a credit event occurs. These options are binary credit options—the option seller will pay out a fixed sum if a credit event by the financial obligation or financial entity triggers (activates) the payout Therefore, a binary option represents two states of the world: no credit event or credit event. A binary credit option could also be triggered by a rating downgrade.

Credit Spread Options[10]

A credit spread option is an option whose value/payoff depends on the change in credit spreads for a reference obligation. It is critical in dis-

[10] The discussion in this section is adapted from Mark J.P. Anson and Frank J. Fabozzi, "Credit Derivatives for Bond Portfolio Management," in Frank J. Fabozzi (ed.), *Fixed Income Readings for the Chartered Financial Analyst Program: Second Edition* (New Hope, PA: Frank J. Fabozzi Associates, 2004).

cussion credit spread options to define what the underlying is. The underlying can be either

1. A reference obligation which is a credit-risky bond with a fixed credit spread.
2. The level of the credit spread for a reference obligation.

Underlying is a Reference Obligation with a Fixed Credit Spread When the underlying is a reference obligation with a fixed credit spread, then a credit spread option is defined as follows:

> *Credit spread put option:* An option that grants the option buyer the right, but not the obligation, to sell a reference obligation at a price that is determined by a strike credit spread over a referenced benchmark at the exercise date.
>
> *Credit spread call option:* An option that grants the option buyer the right, but not the obligation, to buy a reference obligation at a price that is determined by a strike credit spread over a referenced benchmark at the exercise date.

A credit spread option can have any exercise style: European (only exercisable at the expiration date); American (exercisable at any time prior to and including the exercise date); or Bermuda (exercisable only on specified dates by the exercise date.

The price for the reference obligation (i.e., the credit-risky bond) is determined by specifying a strike credit spread over the referenced benchmark, typically a default-free government security. For example, suppose that the reference obligation is an 8% 10-year bond selling to yield 8%. The price of this bond is 100. Suppose further that the referenced benchmark is a same-maturity U.S. Treasury bond that is selling to yield 6%. Then the current credit spread is 200 basis points. Assume that a strike credit spread of 300 basis points is specified and that the option expires in six months. At the end of six months, suppose that the 9.5-year Treasury rate is 6.5%. Since the strike credit spread is 300 basis points, then the yield used to compute the strike price for the reference obligation is 9.5% (the Treasury rate of 6.5% plus the strike credit spread of 300 basis points). The price of a 9.5-year 8% coupon bond selling to yield 9.5% is $90.75 per $100 par value.

The payoff at the expiration date would then depend on the market price for the reference obligation. For example, suppose that at the end of six months, the reference obligation is trading at 82.59. This is a yield of 11% and therefore a credit spread of a 450 basis points over the

9.5-year Treasury yield of 6.5%. For a credit spread put option, the buyer can sell the reference obligation selling at 82.59 for the strike price of 90.75. The payoff from exercising is 8.16. This payoff is reduced by the cost of the option. For a credit spread call option, the buyer will not exercise the option and will allow it to expire worthless. There is a loss equal to the cost of the option.

There is one problem with using a credit spread option in which the underlying is a reference obligation with a fixed credit spread. The payoff is dependent upon the value of the reference obligation's price, which is affected by both the change in the level of interest rates (as measured by the referenced benchmark) and the change in the credit spread. For example, suppose in our illustration that the 9.5-year Treasury at the exercise date is 4.5% (instead of 6.5%) and the credit spread increases to 450 basis points. This means that the reference obligation is trading at 9% (4.5% plus 450 basis points). Since it is an 8% coupon bond with 9.5-years to maturity selling at 9%, the price is 93.70. In this case, the credit spread put option would have a payoff of zero because the price of the reference obligation is 93.70 and the strike price is 90.74. Thus, there was no protection against credit spread risk because interest rates for the referenced benchmark fell enough to offset the increase in the credit spread.

Notice the following payoff before taking into account the option cost when the underlying for a credit spread option is the reference obligation with a fixed credit spread:

Type of option	**Positive payoff if at expiration**
Put	Credit spread at expiration > Strike credit spread
Call	Credit spread at expiration < Strike credit spread

Consequently, to protect against credit spread risk, an investor can buy a credit spread put option where the underlying is a reference obligation with a fixed credit spread.

Underlying is a Credit Spread on a Reference Obligation When the underlying for a credit spread option is the credit spread for a reference obligation over a referenced benchmark, then the payoff of a call and a put option are as follows:

Credit spread call option:

$$\text{Payoff} = (\text{Credit spread at exercise} - \text{Strike credit spread}) \times \text{Notional amount} \times \text{Risk factor}$$

Credit spread put option:

$$\text{Payoff} = (\text{Strike credit spread} - \text{Credit spread at exercise}) \times \text{Notional amount} \times \text{Risk factor}$$

The strike credit spread (in decimal form) is fixed at the outset of the option. The credit spread at exercise (in decimal form) is the credit spread over a referenced benchmark at the exercise date.

The risk factor is equal to:

$$\text{Risk factor} = 10{,}000 \times \text{Percentage price change for a 1 bp change in rates for the reference obligation}$$

By including the risk factor, this form of credit spread option overcomes the problem we identified with the credit spread option in which the underlying is a reference obligation: The payoff depends on both changes in the level of interest rates (the yield on the referenced benchmark) and the credit spread. Instead, it is only dependent upon the change in the credit spread. Therefore, fluctuations in the level of the referenced benchmark's interest rate will not affect the value of the option.

Notice that when the underlying for the credit spread option is the credit spread for a reference obligation over a referenced benchmark, a credit spread call option is used to protect against an increase in the credit spread. In contrast, when the underlying for the credit spread option is the reference obligation, a credit spread put option is used to protect against an increase in the credit spread.

To illustrate the payoff, suppose that the current credit spread for a credit spread call option is 300 basis points and the investor wants to protect against a spread widening to more than 350 basis points. Accordingly, suppose that a strike credit spread of 350 basis points is selected. Then assuming that the risk factor is 5 and the notional amount is $10 million, then the payoff for this option is:

$$\text{Payoff} = (\text{Credit spread at exercise} - 0.035) \times \$10{,}000{,}000 \times 5$$

If at the exercise date the credit spread is 450 basis points, then the payoff is:

$$\text{Payoff} = (0.045 - 0.035) \times \$10{,}000{,}000 \times 5 = \$500{,}000$$

The profit realized from this option is $500,000 less the cost of the option.

CREDIT DERIVATIVE APPLICATIONS

Credit derivatives have allowed market participants to separate and disaggregate credit risk, and then to trade this risk in a secondary market.[11] While our focus in this book is on controlling credit risk by using credit derivatives, as with other derivatives available in the financial markets they can be used to increase exposure and enhance portfolio returns.

Reducing Credit Exposure for Banks

A bank can reduce credit exposure either for an individual loan or a sectoral concentration, by buying a credit default swap (a type of credit derivative discussed later). This may be desirable for assets in the bank's portfolio that cannot be sold for client relationship or tax reasons. For fixed-income managers a particular asset or collection of assets may be viewed as favorable holdings in the long-term, but at risk from short-term downward price movement. In this instance, a sale would not fit in with long-term objectives, however short-term credit protection can be obtained via credit default swap.

Enhancing Portfolio Returns

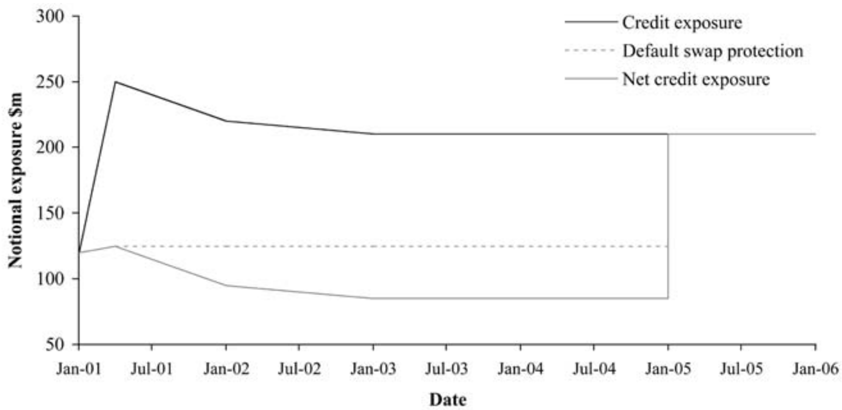
Asset managers can derive premium income by trading credit exposures in the form of derivatives issued with synthetic structured notes. The multi-tranching aspect of structured products enables specific credit exposures (credit spreads and outright default), and their expectations, to be sold to specific areas of demand. By using structured notes such as CLNs, tied to the assets in the reference pool of the portfolio manager, the trading of credit exposures is captured as added yield on the asset manager's fixed income portfolio. In this way the portfolio manager has enabled other market participants to gain an exposure to the credit risk of a pool of assets but not to any other aspects of the portfolio, and without the need to hold the assets themselves.

Reducing Credit Exposure for Fund Managers

Consider a portfolio manager that holds a large portfolio of bonds issued by a particular sector (say, utilities) and believes that spreads in this sector will widen in the short term. Previously, in order to reduce its credit exposure it would have to sell bonds; however, this may generate a mark-to-market loss and may conflict with its long-term investment strategy. An alternative approach would be to enter into a credit default swap, purchasing protection for the short term; if spreads do widen,

[11] For example, see Chapters 2–4 in Satyajit Das, *Credit Derivatives and Credit Linked Notes: Second Edition* (Singapore: John Wiley & Sons Ltd., 2000).

EXHIBIT 16.7 Reducing Credit Exposure



these swaps will increase in value and may be sold at a profit in the secondary market. Alternatively, the portfolio manager may enter into total return swaps on the desired credits. It pays the counterparty the total return on the reference assets, in return for Libor. This transfers the credit exposure of the bonds to the counterparty for the term of the swap, in return for the credit exposure of the counterparty.

Consider now the case of a portfolio manager wishing to reduce credit risk from a growing portfolio (say, one that has just been launched). Exhibit 16.7 shows an example of an unhedged credit exposure to an hypothetical credit-risky portfolio. It illustrates the manager's expectation of credit risk building up to $250 million as the portfolio is ramped up, and then reducing to a more stable level as the credits become more established. A 3-year credit default swap entered into shortly after provides protection on half of the notional exposure, shown as the broken line. The net exposure to credit events has been reduced by a significant margin.

Credit Switches and Zero-Cost Credit Exposure

Protection buyers utilizing credit default swaps must pay a premium in return for laying off their credit risk exposure. An alternative approach for an asset manager involves the use of credit switches for specific sectors of the portfolio. In a credit switch the portfolio manager purchases credit protection on one reference asset or pool of assets, and simultaneously sells protection on another asset or pool of assets.[12] So, for

[12] A pool of assets would be concentrated on one sector, such as utility company bonds.

example, the portfolio manager would purchase protection for a particular fund and sell protection on another. Typically the entire transaction would be undertaken with one investment bank, which would price the structure so that the net cash flows would be zero. This has the effect of synthetically diversifying the credit exposure of the portfolio manager, enabling it to gain and/or reduce exposure to sectors desired.

RISKS IN USING CREDIT DEFAULT SWAPS

As credit derivatives can be tailored to specific requirements in terms of reference exposure, term to maturity, currency, and cash flows, they have enabled market participants to establish exposure to specific entities without the need for them to hold the bond or loan of that entity. This has raised issues of the different risk exposure that this entails compared to the cash equivalent. A 2001 Moody's special report highlights the unintended risks of holding credit exposures in the form of credit default swaps and credit-linked notes.[13] Under certain circumstances it is possible for credit default swaps to create unintended risk exposure for holders, by exposing them to greater frequency and magnitude of losses compared to that suffered by a holder of the underlying reference credit.

In a credit default swap, the payout to a buyer of protection is determined by the occurrence of credit events. The definition of a credit event sets the level of credit risk exposure of the protection seller. A wide definition of "credit event" results in a higher level of risk. To reduce the likelihood of disputes, counterparties can adopt the ISDA Credit Derivatives definitions to govern their dealings. The Moody's special report states that the current ISDA definitions do not unequivocally separate and isolate credit risk, and in certain circumstances credit derivatives can expose holders to additional risks. The report appears to suggest that differences in definitions can lead to unintended risks being taken on by protection sellers. Two examples from the report are cited below as an illustration.

Extending Loan Maturity

The bank debt of Conseco, a corporate entity, was restructured in August 2000. The restructuring provisions included deferment of the loan maturity by three months, higher coupon, corporate guarantee, and additional covenants. Under the Moody's definition, as lenders

[13] Jeffrey Tolk, "Understanding the Risks in Credit Default Swaps," *Moody's Investors Service Special Report* (March 16, 2001).

received compensation in return for an extension of the debt, the restructuring was not considered to be a "diminished financial obligation," although Conseco's credit rating was downgraded one notch. However under the ISDA definition the extension of the loan maturity meant that the restructuring was considered to be a credit event, and thus triggered payments on default swaps written on Conseco's bank debt. Hence this was an example of a loss event under ISDA definitions that was not considered by Moody's to be a default.

Risks of Synthetic Positions and Cash Positions Compared

Consider two investors in XYZ, one of whom owns bonds issued by XYZ while the other holds a credit-linked note referenced to XYZ. Following a deterioration in its debt situation, XYZ violates a number of covenants on its bank loans, but its bonds are unaffected. XYZ's bank accelerates the bank loan, but the bonds continue to trade at 85 cents on the dollar, coupons are paid, and the bond is redeemed in full at maturity. However the default swap underlying the CLN cites "obligation acceleration" (of either bond or loan) as a credit event, so the holder of the CLN receives 85% of par in cash settlement and the CLN is terminated. However the cash investor receives all the coupons and the par value of the bonds on maturity.

These two examples from the Moody's report illustrate how, as credit default swaps are defined to pay out in the event of a very broad range of definitions of a "credit event," portfolio managers may suffer losses as a result of occurrences that are not captured by one or more of the ratings agencies rating of the reference asset. This results in a potentially greater risk for the portfolio manager compared to the position were it to actually hold the underlying reference asset. Essentially therefore it is important for the range of definitions of a "credit event" to be fully understood by counterparties, so that holders of default swaps are not taking on greater risk than is intended.

KEY POINTS

1. Credit derivatives are instruments designed to manage credit risk and trade credit as an asset class in itself.

2. Credit derivatives are governed by legal documentation describing their mechanics. A payout is triggered in the event of a pre-defined credit event.

3. A credit event can include bankruptcy, restructuring, interest coverage default or credit rating downgrade.

4. The advantages of credit derivatives include: (1) the issuer of the reference asset is not required to be a party to the credit transfer process; (2) the credit derivative can be tailor-made to meet the specific requirements of the protection buyer; (3) credit derivatives can be "sold short," without risk of a liquidity squeeze, and can be used to trade in otherwise illiquid assets such as bank loans; and (4) credit derivatives enable the trading of credit as an asset class, leading to efficient and transparent pricing of credit in its own right.

5. Credit derivatives are cash settled or physically settled.

6. The main credit derivatives are credit default swaps, total return swaps, and credit-linked notes.

7. A credit default swap is a bilateral contract in which a protection buyer pays a premium to a protection seller, in return for which the protection seller will pay the notional value to the buyer on occurrence of a specified credit event associated with the credit default swap reference asset.

8. A total return swap is an agreement between two parties to exchange the "total return" from a financial asset such as a bond or loan in return for a payment of Libor plus a spread. Total return is change in market value and interest.

9. Credit options can be classified as credit default options and credit spread options.

10. Credit default options have a payoff that is triggered by the occurrence of a credit event.

11. Credit spread options fall into two categories based on the underlying: (1) a financial obligation with a fixed credit spread or (2) a credit spread.

12. The strike price for a credit spread option in which the underlying is a financial obligation with a fixed credit spread is determined at the exercise date by spread over a benchmarked reference obligation.

13. A credit spread option in which the underlying is a credit spread has a payoff that is adjusted to eliminate interest rate risk.

14. A funded total return swap has been used in synthetic CDO structures.

15. A credit-linked note is a bond issued at par, whose principal and/or interest is linked to the performance of a reference asset. The buyer of the note is the protection seller and the issuer is the protection buyer.

16. Banks transfer credit risk away from their balance sheet using credit derivatives, without removing risky assets from the balance sheet itself.

17. Credit derivatives are used for a number of applications including capital structure arbitrage and exposure to specific market sectors and corporate credit.

18. The main applications of credit derivatives by fund managers are (1) enhancing portfolio return, (2) reducing credit exposure, and (3) entering into credit switches and zero-cost credit exposure trades.

19. There are risks associated with using credit derivatives, including counterparty risk.

CHAPTER 17

Credit Derivative Valuation

In the previous chapter we discussed credit derivatives and their applications. In this chapter, we look at the various approaches used in pricing and valuation of credit derivatives. We consider generic techniques and also compare prices obtained using different pricing models. In addition, we highlight the difference that one encounters in the market between the cash and synthetic spread levels for the same reference name, known as the *basis* and which is observed closely in the market.

The objectives of this chapter are to:

1. Introduce the concept of "fair value" pricing of credit derivatives and the no-arbitrage principle behind fair value pricing.
2. Explain the asset swap pricing approach to credit derivative pricing.
3. Introduce credit derivative pricing models, including structural models and reduced-form models.
4. Describe the Jarrow-Lando-Turnbull model and the Duffie-Singleton model.
5. Explain the concept of credit spread modeling.
6. Demonstrate pricing for a credit default swap.
7. Compare the different price results obtained from different pricing models.
8. Compare cash and synthetic markets.
9. Explain the difference in spread levels between asset swap and credit default swap prices on the same reference name.
10. Illustrate the concept of the basis using observations from the market.

This chapter was coauthored with Richard Pereira at Dresdner Kleinwort Wasserstein.

PRICING OF CREDIT DERIVATIVES

The pricing of credit derivatives should aim to provide a "fair value" for the credit derivative instrument. In this chapter we discuss the pricing models currently used by the industry. The effective use of pricing models requires an understanding of a model's assumptions, the key pricing parameters, and the limitations of a pricing model.

Issues to consider when carrying out credit derivative pricing include:

- Implementation and selection of appropriate modeling techniques.
- Parameter estimation.
- Quality and quantity of data to support parameters and calibration.
- Calibration to market instruments for risky debt.

For credit derivative contracts in which the payout is on credit events other than default, the modeling of the credit evolutionary path is critical. If however a credit derivative contract does not payout on intermediate stages between the current state and default, then the important factor is the probability of default from the current state.

We begin by considering the asset swap pricing method, which was commonly used at the inception of the credit derivatives market.

INTEREST RATE SWAP (ASSET SWAP) PRICING

Credit derivatives are commonly valued using the interest rate swap or asset swap pricing technique. In addition to its use by dealers, risk management departments that wish to independently price such swaps adopt this technique. The asset swap market is a reasonably reliable indicator of the returns required for individual credit exposures and provides a mark-to-market framework for reference assets as well as a hedging mechanism.

A *par asset swap* typically combines the sale of an asset such as a fixed-rate corporate bond to a counterparty (at par and with no interest accrued) with an interest rate swap. The coupon on the bond is paid in return for Libor, plus a spread if necessary. This spread is the *asset swap spread* and is the price of the asset swap. In effect the asset swap allows market participants that pay Libor-based funding to receive the asset swap spread. This spread is a function of the credit risk of the underlying bond asset, which is why it in effect becomes the cornerstone of the price payable on a credit default swap written on that reference asset.

The generic pricing is given by the following equation:

$$Y_a = Y_b - ir \tag{17.1}$$

where

Y_a = the asset swap spread
Y_b = the asset spread over the benchmark
ir = the interest rate swap spread

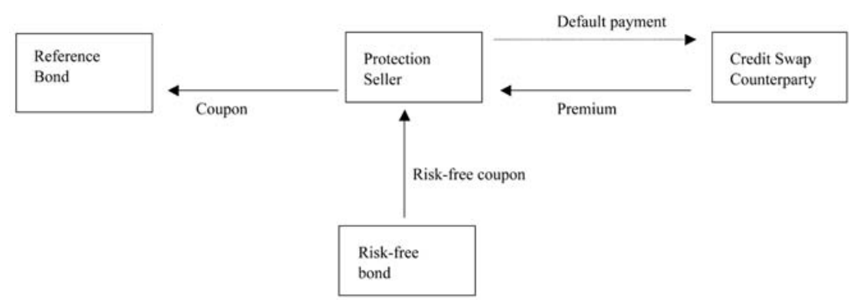
The asset spread over the benchmark is simply the bond (asset) redemption yield over that of the government benchmark. The interest rate swap spread reflects the cost involved in converting fixed-coupon benchmark bonds into a floating-rate coupon during the life of the asset (or credit default swap), and is based on the swap rate for that tenor X.

For example, XYZ Corp. is a Baa2 rated corporate. The 7-year asset swap for this entity is currently trading at 93 basis points; the underlying 7-year bond is hedged by an interest rate swap with an Aa2 rated bank. The risk-free rate for floating-rate bonds is Libid (the London interbank bid rate) minus 12.5 basis points (assume the bid offer spread is 6 basis points). This suggests that the credit spread for XYZ Corp. is 111.5 basis points. The credit spread is the return required by an investor for holding the credit of XYZ Corp. The protection seller is conceptually long the asset, and so would short the asset as a hedge of its position. This is illustrated in Exhibit 17.1. The price charged for the credit default swap is the price of the shorting the asset, which works out as 111.5 basis points each year.

Therefore we can price a credit default swap written on XYZ Corp. as the present value of 111.5 basis points for seven years, discounted at the interest rate swap rate of 5.875%. This computes to a credit default swap price of 6.25%.

There are a number of reasons why this approach is no longer applied except perhaps by risk managers or middle office staff as an

EXHIBIT 17.1 **Credit Default Swap and Asset Swap Hedge**



independent check.[1] These reflect the respective nature of asset swaps and credit default swaps as market instruments.

PRICING MODELS

We now consider a number of pricing models as used in the credit derivative markets. Pricing models for credit derivatives fall into two classes:

- Structural models
- Reduced form models

We discuss these models next.

Structural Models

Structural models are characterized by modeling the firm's value in order to provide the probability of a firm default. The Black-Scholes-Merton option pricing framework is the foundation of the structural model approach. The default event is assumed to occur when the firm's assets fall below the book value of the debt.

Merton applied option pricing techniques to the valuation of corporate debt.[2] By extension, the pricing of credit derivatives based on corporate debt may in some circumstances be treated as an option on debt (which is therefore analogous to an option on an option model).

Merton models have the following features:

- Default events occur predictably when a firm has insufficient assets to pay its debt.
- Firm's assets evolve randomly. The probability of a firm default is determined using the Black-Scholes-Merton option pricing theory.

Some practitioners argue that Merton models are more appropriate than reduced form models when pricing default swaps on high-yield bonds, due to the higher correlation of high-yield bonds with the underlying equity of the issuer firm.

The constraint of structural models is that the behavior of the value of assets and the parameters used to describe the process for the value of

[1] See Moorad Choudhry, "Some Issues in the Asset Swap Pricing of Credit Default Swaps," in Frank J. Fabozzi (ed.), *Professional Perspectives in Fixed Income Portfolio Management: Volume 4* (Hoboken, NJ: John Wiley & Sons, Inc., 2003).

[2] Robert C. Merton, "On the Pricing of Corporate Debt: The Risk Structure of Interest Rates," *Journal of Finance*, June 1974, pp. 449–470.

the firm's assets are not directly observable and the method does not consider the underlying market information for credit instruments.

Reduced Form Models

Reduced form models are a form of no-arbitrage model. These models can be fitted to the current term structure of risky bonds to generate no arbitrage prices. In this way the pricing of credit derivatives using these models will be consistent with the market data on the credit risky bonds traded in the market. These models allow the default process to be separated from the asset value and are more commonly used to price credit derivatives than structural models.

When implementing reduced form models, it is necessary to consider issues such as the illiquidity of underlying credit risky assets. Liquidity is often assumed to be present when we develop pricing models. However, in practice, there may be problems when calibrating a model to illiquid positions, and in such cases the resulting pricing framework may be unstable and provide the user with spurious results. Another issue is the relevance of using historical credit transition data, used to project future credit migration probabilities. In practice it is worthwhile reviewing the sensitivity of price to the historical credit transition data when using the model.

Recent reduced form models which provide a detailed modeling of default risk include those presented by Jarrow, Lando, and Turnbull,[3] Das and Tufano,[4] and Duffie and Singleton.[5] We consider these models in this section.

Jarrow-Lando-Turnbull Model

The Jarrow-Lando-Turnbull (JLT) model focuses on modeling default and credit migration. Its data and assumptions include the use of

- A statistical rating transition matrix which is based on historic data.
- Risky bond prices from the market used in the calibration process.
- A constant recovery rate assumption. The recovery amount is assumed to be received at the maturity of the bond.
- A credit spread assumption for each rating level.

[3] Robert Jarrow, David Lando, and Stuart Turnbull, "A Markov Model for the Term Structure of Credit Spreads," *Review of Financial Studies* 10(2) (1997), pp. 481–523.
[4] Darrell Duffie and Kenneth Singleton, "Modelling Term Structures of Defaultable Bonds," *Review of Financial Studies* 12(4) (1999), pp. 687–720.
[5] Sanjiv Das and Peter Tufano, "Pricing Credit Sensitive Debt when Interest Rate, Credit Ratings and Credit Spreads Are Stochastic," *Journal of Financial Engineering* 5(2) (1996), pp. 161–198.

It also assumes no correlation between interest rates and credit rating migration.

The statistical transition matrix is adjusted by calibrating the expected risky bond values to the market values for risky bonds. The adjusted matrix is referred to as the *risk-neutral transition matrix*. The risk-neutral transition matrix is key to the pricing of several credit derivatives.

The JLT model allows the pricing of default swaps, as the risk neutral transition matrix can be used to determine the probability of default. The JLT model is sensitive to the level of the recovery rate assumption and the statistical rating matrix. It has a number of advantages. As the model is based on credit migration, it allows the pricing of derivatives for which the payout depends on such credit migration. In addition, the default probability can be explicitly determined and may be used in the pricing of credit default swaps.

The disadvantages of the model include the fact that it depends on the selected historical transition matrix. The applicability of this matrix to future periods needs to be considered carefully, whether, for example, it adequately describes future credit migration patterns. In addition, it assumes all securities with the same credit rating have the same spread, which is a restrictive assumption. For this reason, the spread levels chosen in the model are a key assumption in the pricing model. Finally, the constant recovery rate is another practical constraint, as in practice the level of recovery will vary.

The Das-Tufano Model

The Das-Tufano (DT) model is an extension of the JLT model. The model aims to produce the risk-neutral transition matrix in a similar way to the JLT model, however this model uses stochastic recovery rates. The final risk-neutral transition matrix should be computed from the observable term structures. The stochastic recovery rates introduce more variability in the spread volatility. Spreads are a function of factors which may not only be dependent on the rating level of the credit, because in practice credit spreads may change even though credit ratings have not changed. Therefore, to some extent the DT model introduces this additional variability into the risk-neutral transition matrix.

Various credit derivatives may be priced using this model—for example, credit default swaps, total return swaps, and credit spread options. The pricing of these products requires the generation of the appropriate credit dependent cash flows at each node on a lattice of possible outcomes. The fair value may be determined by discounting the probability weighted cash flows. The probability of the outcomes would be determined by reference to the risk-neutral transition matrix.

The Duffie-Singleton Model

The Duffie Singleton modeling approach considers the three components of risk for a credit risky product, namely the risk-free rate, the hazard rate, and the recovery rate.

The *hazard rate* characterizes the instantaneous probability of default of the credit risky product underlying exposure. Because each of the components above may not be static over time, a pricing model may assume a process for each of these components of risk. The process may be implemented using a lattice approach for each component. The constraint on the lattice formation is that this lattice framework should agree to the market pricing of credit risky debt.

Here we demonstrate that the credit spread is related to risk of default (as represented by the hazard rate) and the level of recovery of the bond. We assume that a zero-coupon risky bond maturing in a small time element Δt where:

λ = the annualized hazard rate
φ = the recovery value
r = the risk-free rate
s = the credit spread

and where its price P is given by

$$P = e^{-r\Delta t}[(1 - \lambda\Delta t) + (\lambda\Delta t)\varphi] \tag{17.2}$$

Alternatively P may be expressed as

$$P \cong e^{-\Delta t(r + \lambda(1 - \varphi))} \tag{17.3}$$

However as the usual form for a risky zero-coupon bond is

$$P = e^{-\Delta t(r + s)} \tag{17.4}$$

Therefore we have shown that

$$s \cong \lambda(1 - \varphi) \tag{17.5}$$

This would imply that the credit spread is closely related to the hazard rate (that is, the likelihood of default) and the recovery rate.

This relationship between the credit spread, the hazard rate, and the recovery rate is intuitively appealing. The credit spread is perceived to be the extra yield (or return) an investor requires for credit risk assumed. For example:

- As the hazard rate (or instantaneous probability of default) rises, the credit spread increases.
- As the recovery rate decreases the credit spread increases.

Hazard Rate A "hazard rate" function may be determined from the term structure of credit spreads. The hazard rate function has its foundation in statistics and may be linked to the instantaneous default probability. The hazard rate function, $\lambda(s)$, can then be used to derive a probability function for the survival function $S(t)$:

$$S(t) = \exp^{-\int_0^t \lambda(s)\,ds} \tag{17.6}$$

The hazard rate function may be determined by using the prices of risky bonds. The lattice for the evolution of the hazard rate should be consistent with the hazard rate function implied from market data. An issue when performing this calibration is the volume of relevant data available for the credit.

Recovery Rates The recovery rate usually takes the form of the percentage of the par value of the security recovered by the investor. The key elements of the recovery rate include:

- Level of the recovery rate.
- Uncertainty of the recovery rate based on current conditions specific to the reference credit.
- Time interval between default and the recovery value being realized.

Generally recovery rates are related to the seniority of the debt. Therefore if the seniority of debt changes then the recovery value of the debt may change. Also recovery rates exhibit significant volatility.

CREDIT SPREAD MODELING

Although credit spreads may be viewed as a function of default risk and recovery risk, credit spread models do not attempt to break down the credit spread into its default risk and recovery risk components.

The pricing of credit derivatives which payout according to the level of the credit spread would require that the credit spread process is adequately modeled. In order to achieve this a stochastic process for the distribution of outcomes for the credit spread is an important consideration.

An example of the stochastic process for modeling credit spreads which may be assumed, includes a mean reverting process such as

$$ds = k(\mu - s)dt + \sigma s dw \qquad (17.7)$$

where

ds = the change in the value of the credit spread over an element of time (dt)
dt = the element of time over which the change in credit spread is modeled
s = the credit spread
k = the rate of mean reversion
μ = the mean level of the spread
dw = Wiener increment
σ = the volatility of the credit spread

In this model when s rises above a mean level of the spread, the drift term ($\mu - s$) will become negative and the spread process will drift towards (revert) to the mean level. The rate of this drift towards the mean is dependent on k the rate of mean reversion.

The pricing of a European credit spread option requires the distribution of the credit spread at the maturity (T) of the option. The choice of model affects the probability assigned to each outcome. The mean reversion factor reflects the historic economic features over time of credit spreads, to revert to the average spreads after larger than expected movements away from the average spread.

Therefore the European option price may be reflected as

$$\text{Option price} = E[e^{-rT}(\text{Payoff}(s, X))] = e^{-rT}\int_0^\infty f(s, X)p(s)ds \qquad (17.8)$$

where

X = the strike price of the credit spread option
$p(s)$ = the probability function of the credit spread

$E[\ ]$ denotes the expected value, and $f(s,X)$ is the payoff function at maturity of the credit spread.

More complex models for the credit spread process may take into account factors such as the term structure of credit spreads and possible correlation between the credit spread process and the interest process.

EXHIBIT 17.2 Comparison of Pricing Results for Spread Option Models

Expiry in 6 months Risk free rate = 10% Strike = 70 bps Credit spread = 60 bps Volatility = 20%	Mean Reversion Model Price	Standard Black Scholes Price	Difference Between Standard Black Scholes and Mean Reversion Model Price (%)
Mean level = 50 bps K = 0.2			
Put	0.4696	0.5524	17.63
Call	10.9355	9.7663	11.97
Mean level = 50 bps K = 0.3			
Put	0.3510	0.5524	57.79
Call	11.2031	9.7663	14.12
Mean level = 80 bps K = 0.2			
Put	0.8729	0.5524	58.02
Call	8.4907	9.7663	15.02
Mean level = 80 bps K = 0.3			
Put	0.8887	0.5524	60.87
Call	7.5411	9.7663	29.51

The pricing of a credit spread option is dependent on the underlying process. As an example we compare the pricing results for a spread option model including mean reversion to the pricing results from a standard Black-Scholes option pricing model in the Exhibit 17.2.

Exhibits 17.2 and 17.3 show the sensitivity on the pricing of a credit spread option to changes to the underlying process. Comparing the two exhibits shows the impact of time to expiry increasing by six months. In a mean reversion model, the mean level and the rate of mean reversion are important parameters which may significantly affect the probability distribution of outcomes for the credit spread, and hence the price.

Credit Default Swaps

The pricing of a credit default swap that has a payout on an underlying risky bond involves the following key factors when pricing:

EXHIBIT 17.3 Comparing Model Results After Selected Underlying Changes

Expiry in 12 months Risk free rate = 10% Strike = 70 bps Credit spread = 60 bps Volatility = 20%	Mean Reversion Model Price	Standard Black Scholes Price	Difference Between Standard Black Scholes and Mean Reversion Model Price (%)
Mean level = 50 bps K = 0.2			
Put	0.8501	1.4331	68.58
Call	11.2952	10.4040	8.56
Mean level = 50 bps K = 0.3			
Put	0.7624	1.4331	87.97
Call	12.0504	10.4040	15.82
Mean level = 80 bps K = 0.2			
Put	1.9876	1.4331	38.69
Call	7.6776	10.4040	35.51
Mean level = 80 bps K = 0.3			
Put	2.4198	1.4331	68.85
Call	6.7290	10.4040	54.61

- Risk-free interest rate term structure.
- Risky term structure. Ideally, we would determine this by considering the term of the bonds issued. However if a wide term is not available then the bonds of similar credit risky companies may be used to create a more complete term structure of credit.
- The recovery rate.

For the risk-free interest rate term structure the observable money market and swap curve may be the best choice; however, a risk-free interest term structure may also be built from government bond prices.

The risky term structure and the recovery rate can be used to estimate the risk-neutral probability of default.

$$rs_{\text{Risky}} = rs_{\text{Riskfree}}\{[(1-p)\times 1] + (p \times R)\} \tag{17.9}$$

where

rs_{Risky} = risky zero-coupon rate (from the risky term structure)
rs_{Riskfree} = risk-free zero-coupon rate (from the risk-free term structure)
p = the risk neutral default probability
R = the recovery rate

The unknown p can be implied from equation (17.9):

$$p = [1/(1-R)] \times [1-(rs_{\text{Risky}}/rs_{\text{Riskfree}})] \tag{17.10}$$

Equation (17.10) shows that the probability of default is related to the risky term structure, the risk-free term structure, and the recovery rate. In order to determine p we will make assumptions for a suitable recovery value to be used in equation (17.10).

The expected value of a single-period credit default swap may take the form of the expected payout:

$$CDS_t = rs_{\text{Riskfree}}\{[0 \times (1-p)] + [p \times (1-R)]\} \tag{17.11}$$

This reflects the fact that there is no payout on survival of the credit, that is in the event of no default.

Key issues in the pricing of credit default swaps include:

- Determining an appropriate assumption for the recovery rate.
- Selection of appropriate risky debt required for calibration and the necessary adjustments to allow for liquidity and embedded options of the risky debt.
- Selection of the appropriate risk-free curve.
- Allowance for the credit risk of the counterparty and correlation of the underlying credit with the counterparty. We would expect that the cost of credit protection is cheaper if it is purchased from a "high" risk counterparty.

KEY POINTS

1. The market uses two generic approaches in pricing credit derivatives: the asset swap technique and the stochastic pricing model technique.

2. The asset swap technique assumes that the asset swap spread on an issuer name is the price the market assigns to that name's credit risk above Libor risk.

3. Credit derivatives pricing is based on the no-arbitrage principle.

4. Credit derivatives isolate and trade credit as their sole asset, thus trade at a different level than the asset swap on the same reference asset. This difference is known as the basis.

5. Pricing models make assumptions about the reference asset probability of default and default correlation, and credit rating migration.

6. The effective use of pricing models requires an understanding of the model's assumptions, the key pricing parameters, and an understanding of the limitations of a pricing model.

7. Pricing models are defined as structural models or reduced-form models.

8. Issues to consider when carrying out credit derivative pricing include implementation and selection of appropriate modeling techniques, parameter estimation, quality and quantity of data to support parameters and calibration, and calibration to market instruments for risky debt.

9. Structural models are characterized by modeling the firm's value in order to provide the probability of a firm default. The Black-Scholes-Merton option pricing framework is the foundation of the structural model approach.

10. Reduced form models are a form of no-arbitrage model. They are fitted to the term structure of risky bonds to generate no arbitrage prices for credit derivatives.

11. The Jarrow-Lando-Turnbull reduced form model focuses on modeling default probability and credit migration

12. Pricing a credit default swap involves assessment of (1) the risk-free interest rate risk term structure, (2) the credit term structure, and (3) the recovery rate.

CHAPTER 18

Managing Credit Risk Using Structured Products

Previous chapters have introduced credit derivative instruments and shown how they are used to both manage and transfer credit risk. In this chapter we look at how credit derivatives can be combined with securitization techniques to create structured products, used by banks to manage credit risk and regulatory capital.

The objectives of this chapter are to:

1. Define credit risk transfer and regulatory capital relief as undertaken by commercial banks.
2. Describe how credit derivatives are used in the construction of structured products known as synthetic collateralized debt obligations (CDO).
3. Demonstrate how banks can use synthetic CDOs to manage and transfer credit risk.
4. Illustrate the advantages of the synthetic CDO structure to a bank using them for risk management purposes.
5. Illustrate how synthetic CDOs are structured.
6. Introduce the concept of the managed synthetic CDO used by portfolio managers for credit arbitrage and trading purposes.

MANAGING CREDIT RISK IN BANKING

For a commercial bank, the risks inherent in its core business are the traditional ones of credit risk, interest rate risk, and funding risk. All these risks are contained within its loan book. A bank will, in the normal course of business, seek to manage the risk exposures inherent in its loan book through a combination of risk management techniques. In this chapter we show how credit risk can be stripped out of the combined group of risks and managed in its own right. This is achieved through the use of credit derivatives, which enable credit risk to be traded as an asset class in its own right. By combining the use of credit derivatives with securitization techniques, a bank can manage its credit risk as well as its regulatory capital costs. Hence the rise in popularity of the static synthetic balance sheet collateralized debt obligation (CDO), which is the focus of this chapter. We also consider how fund managers can also mirror this technology but for a distinctly different purpose, namely credit trading and arbitrage in the corporate names cash and synthetic markets.

The reasons that banks originate static synthetic CDOs are twofold:

- *Transfer of credit risk*: A synthetic CDO structure enables the credit risk of a loan book to be separated from the interest rate risk and funding risk and managed on its own. The costs of transferring this risk away are a function of the CDO structure and related to the credit derivative pricing of the reference assets and whether these are funded or unfunded. With a *partially funded* structure, the issue amount is typically a relatively small share of the asset portfolio. This lowers substantially the credit default swap premium. Also, as the CDO investors suffer the first loss element of the portfolio, the super senior credit default swap can be entered into at a considerably lower cost than that on a fully funded CDO.
- *Capital relief*: Banks can obtain regulatory capital relief by transferring lower-yield corporate credit risk such as corporate bank loans off their balance sheet. Under Basel I rules, all corporate debt carries an identical 100% risk weighting. Therefore, with banks having to assign 8% of capital for such loans, higher-rated (and hence lower-yielding) corporate assets will require the same amount of capital but will generate a lower return on that capital. A bank may wish to transfer such higher-rated, lower-yield assets from its balance sheet, and this can be achieved via a CDO transaction. The capital requirements for a synthetic CDO are lower than for corporate assets. For example, the funded segment of the deal will be supported by high quality collateral such as government bonds and via a repo arrangement with an OECD bank would carry a 20% risk weighting, as does the super senior element.

This chapter analyzes the structure and use of the static synthetic balance sheet CDO for credit risk management purposes.

THE BALANCE SHEET SYNTHETIC CDO

A synthetic securitization structure is engineered so that the credit risk of a pool of assets held on the originator's own balance sheet is transferred from itself to investors by means of credit derivative instruments. The originator is in effect buying credit protection from investors who are the credit protection sellers. This credit risk transfer may be undertaken either directly or via a special purpose vehicle (SPV). Using this approach, underlying or reference assets are not necessarily moved off the originator's balance sheet. This makes the vehicle an ideal means by which to manage credit risk. Because the synthetic structure enables removal of credit exposure without asset transfer, commercial banks can use it risk management and regulatory capital relief purposes. For banking institutions it also enables loan risk to be transferred without selling the loans themselves, thereby allowing customer relationships to remain unaffected.

The Value of the Static Synthetic Balance Sheet CDO

A synthetic CDO can be seen as being constructed out of the following:

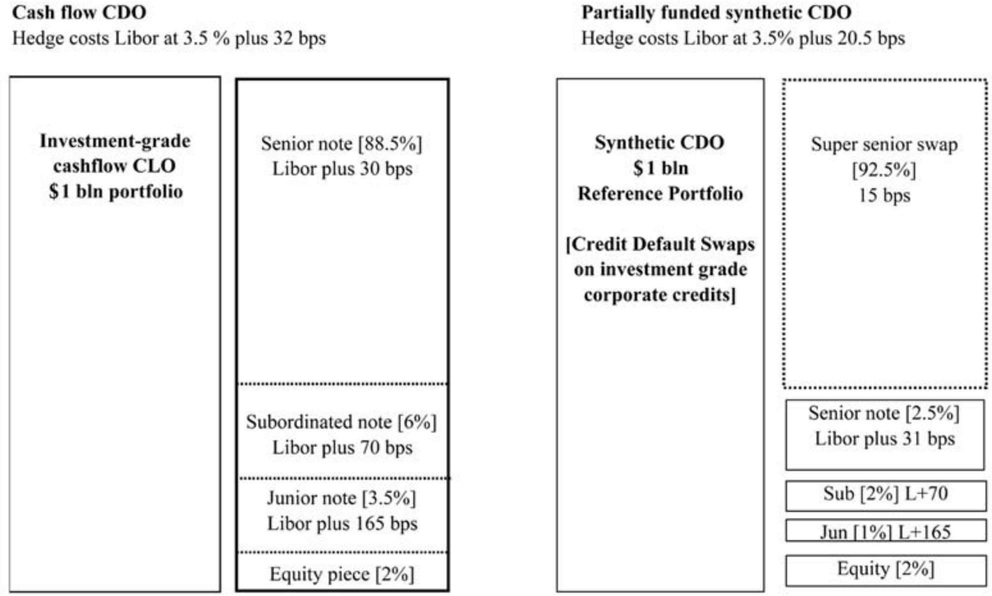
- A short position in a credit default swap (bought protection), by which the sponsor transfers its portfolio credit risk to the issuer.
- A long position in a portfolio of bonds or loans, the cash flow from which enables the sponsor to pay liabilities of overlying notes.

The economic advantage of issuing a synthetic versus a cash CDO can be significant. Put simply, the net benefit to the originator is the gain in regulatory capital cost, minus the cost of paying for credit protection on the credit default swap side. In a partially funded structure, a sponsoring bank will obtain full capital relief when note proceeds are invested in 0% risk-weighted collateral such as U.S. Treasuries or British gilts. The super senior swap portion will carry a 20% risk weighting.[1] In fact, a moment's thought should make clear to us that a synthetic deal would be cheaper. Where credit default swaps are used, the sponsor pays a basis point fee, which for a AAA security might be in the range of 10 to 30 bps, depending on the stage of the credit cycle. In a cash structure where bonds are issued, the cost to the sponsor would be the benchmark yield plus the credit spread, which would be considerably higher compared to the default swap premium.

This is illustrated in the example shown in Exhibit 18.1, where we assume certain spreads and premiums in comparing a partially funded synthetic deal with a cash deal. The assumptions are:

[1] This is as long as the counterparty is an OECD bank, which is invariably the case.

EXHIBIT 18.1 Cost Structure of Synthetic versus Cash Flow CDO



Regulatory capital relief
Cash CDO:
Capital charge on assets reduces from 8% (100% RW) to 2% (equity piece only now 100% RW)
Regulatory capital relief is 6%
Synthetic CDO:
Capital charge on assets reduces from 8% (100% RW) to 3.48% (equity piece plus super senior swap at 20% RW)
Regulatory capital relief is 4.52%

- That the super senior credit default swap cost is 15 bps and carries a 20% risk weight.
- The equity piece retains a 100% risk-weighting.
- The synthetic CDO invests note proceeds in sovereign collateral that pays sub-Libor.

Structuring Mechanics

A generic synthetic CDO structure is shown in Exhibit 18.2. In this generic structure, the credit risk of the reference assets is transferred to the issuer SPV and ultimately the investors, by means of the credit default swap and an issue of credit-linked notes. In the credit default swap arrangement, the risk transfer is undertaken in return for the swap premium, which is then paid to investors by the issuer. The note issue is invested in risk-free collateral rather than passed on to the originator in order to delink the credit ratings of the notes from the credit rating of the originator. If the collateral pool was not established, a downgrade of the sponsor could result in a downgrade of the issued notes.

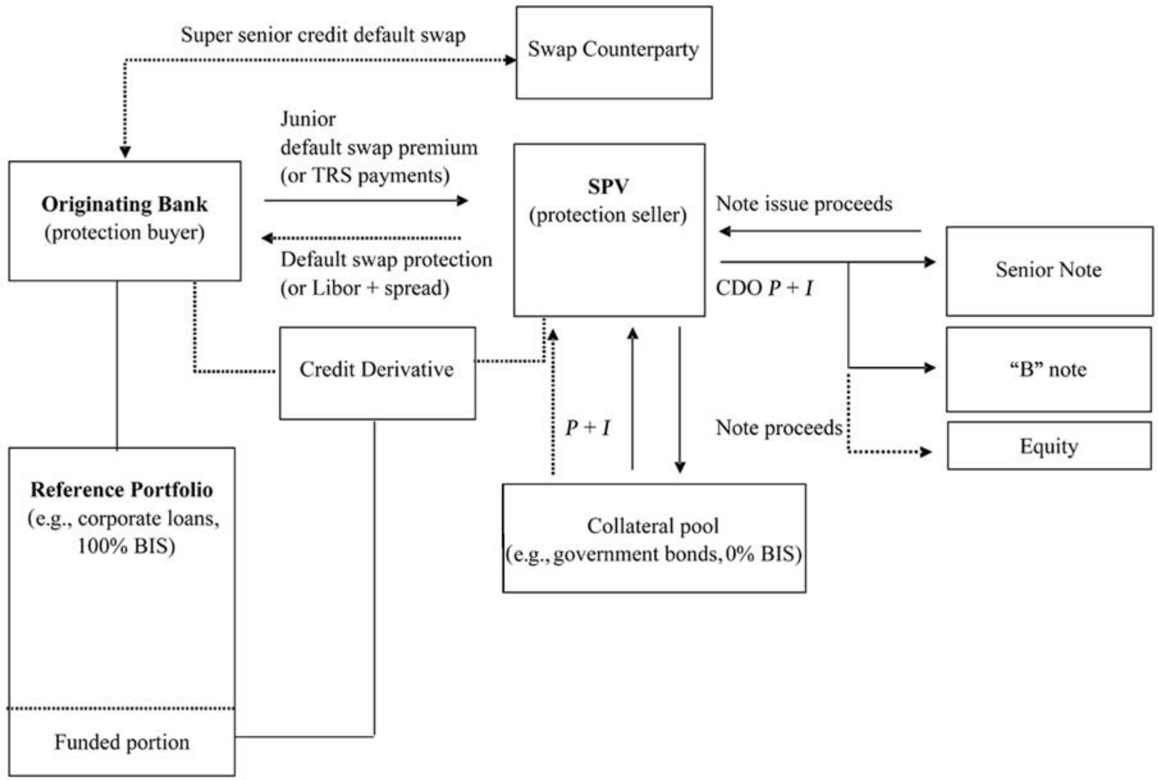
Investors in the notes are exposed to the credit risk of the reference assets, and if there are no credit events, they will earn returns at least the equal of the collateral assets and the credit default swap premium. If the notes are credit-linked, they will also earn excess returns based on the performance of the reference portfolio. If there are credit events, the issuer will deliver the assets to the swap counterparty and will pay the nominal value of the assets to the originator out of the collateral pool. Credit default swaps are unfunded credit derivatives, while CLNs are funded credit derivatives where the protection seller (the investors) fund the value of the reference assets upfront and will receive a reduced return on occurrence of a credit event.

Funding Mechanics

As the super-senior piece in a synthetic CDO does not need to be funded, this provides the key advantage of the synthetic mechanism compared to a cash flow arbitrage CDO. During the first half of 2002, for example, the yield spread for the AAA note piece averaged 45–50 bps over Libor,[2] while the cost of the super-senior swap was around 10–12 bps. This means that the CDO manager can reinvest in the collateral pool risk-free assets at Libor minus 5 bps, it is able to gain from a saving of 28–35 bps on each nominal $100 of the structure that is not funded. This is a considerable gain. If we assume that a synthetic CDO

[2] Averaged from the yield spread on seven synthetic deals closed during the first six months of 2002, yield spread at issue, rates data from Bloomberg.

EXHIBIT 18.2 Synthetic CDO Structure



is 95% unfunded and 5% funded, this is equivalent to the reference assets trading at approximately 26–33 bps cheaper in the market. There is also an improvement to the return on capital measure for the CDO manager. Since typically the manager retains the equity piece, if this is 2% of the structure and the gain is 33 bps, the return on equity will be improved by 16.5% (= 0.36/0.02).

Another benefit of structuring CDOs as synthetic deals is their potentially greater attraction for investors (protection sellers). Often, selling credit default swap protection on a particular reference credit generates a higher return than going long the underlying cash bond. In general this is because the credit default swap price is greater than the asset swap price for the same name, for a number of reasons.[3] For instance, during 2001 the average spread of the synthetic price over the cash price was 15 basis points in the 5-year maturity area for BBB-rated credits. The two main reasons why default swap spreads tend to be above cash spreads are:

- The credit risk covered by the default swap includes trigger events that are not pure default scenarios, such as restructuring.
- On occurrence of a credit event, the amount of loss is calculated assuming that the reference security was at an initial price of par, whereas in the cash market that security may have been bought at a discount to par. Assume we buy a security at a price discount to par of x, and that the obligor defaults: the physical security can be sold at the new defaulted-price of y, where $x > y$, resulting in a loss of $(x - y)$. If the investor had instead sold a credit default swap on the same name, the investor would pay the difference between par and y, which is a greater loss. Therefore the credit default swap price is higher to compensate for this.

Note however the existence of ongoing counterparty risk for the seller of a credit default swap is a factor that suggests that its price should be below the cash price!

Advantages of Synthetic Structures for Originators

For the purposes of asset-backed credit risk management, balance sheet synthetic securitization vehicles present certain advantages over traditional cash flow structures. These include:[4]

[3] See Moorad Choudhry, "Issues in the Asset Swap Pricing of Credit Default Swaps," in Frank J. Fabozzi, (ed.), *Professional Perspectives on Fixed Income Portfolio Management: Volume 4* (Hoboken, NJ: John Wiley & Sons, 2003).

[4] See Laurie S. Goodman and Frank J. Fabozzi, *Collateralized Debt Obligations: Structures and Analysis* (Hoboken, NJ: John Wiley and Sons, Inc., 2002).

- A synthetic transaction can, in theory, be placed in the market sooner than a cash deal, and the time from inception to closure can be as low as four weeks, with average execution time of 6–8 weeks compared to 3–4 months for the equivalent cash deal.
- No requirement to fund the super-senior element.
- For many reference names, the credit default swap is frequently cheaper than the same name underlying cash bond.
- Transaction costs such as legal fees can be lower as there is no necessity to set up an SPV.
- Banking relationships can be maintained with clients whose loans need not be actually sold off the sponsoring entity's balance sheet.
- The range of reference assets that can be covered is wider, and includes undrawn lines of credit, bank guarantees and derivative instruments that would give rise to legal and true sale issues in a cash transaction.
- The use of credit derivatives introduces greater flexibility to provide tailor-made solutions for credit risk requirements.
- The cost of buying protection is usually lower as there is little or no funding element and the credit protection price is below the equivalent-rate note liability.

For this reason synthetic structures are increasingly preferred by commercial banking Treasury and ALM desks.

VARIATIONS IN BALANCE SHEET SYNTHETIC CDOS

A balance sheet synthetic CDO is employed by banks that wish to manage credit risk and regulatory capital. In a balance sheet CDO, the SPV enters into a credit default swap agreement with the originator with the specific collateral pool designated as the reference portfolio. The SPV receives the premium payable on the credit default swap and thereby provides credit protection on the reference portfolio.

There are three types of CDO within this structure. A *fully synthetic CDO* is a completely *unfunded* structure, which uses credit default swaps to transfer the entire credit risk of the reference assets to investors who are protection sellers. In a *partially funded CDO*, only the highest credit risk segment of the portfolio is transferred. The cash flow that would be needed to service the synthetic CDO overlying liability is received from the AAA rated collateral that is purchased by the SPV with the proceeds of an overlying note issue. An originating bank obtains maximum regulatory capital relief by means of a partially funded structure, through a combination of the synthetic CDO and what is known as a *super-senior* *swap* arrangement with an OECD banking counterparty. A super-senior swap provides additional protection to that part of the portfolio, the senior segment, that is already protected by the funded portion of the transaction. The sponsor may retain the super-senior element or may sell it to a monoline insurance firm or credit default swap provider.

A *fully funded CDO* is a structure where the credit risk of the entire portfolio is transferred to the SPV via a credit default swap. In a fully funded (or just "funded") synthetic CDO, the issuer enters into the credit default swap with the SPV, which itself issues credit-linked notes to the entire value of the assets on which the risk has been transferred. The proceeds from the notes are invested in risk-free government or agency debt or in senior unsecured bank debt. Should there be a default on one or more of the underlying assets, the required amount of the collateral is sold and the proceeds from the sale paid to the issuer to recompense for the losses. The premium paid on the credit default swap must be sufficiently high to ensure that it covers the difference in yield between that on the collateral and that on the notes issued by the SPV.

Fully funded CDOs are relatively uncommon. One of the advantages of the partially funded arrangement is that the issuer will pay a lower premium compared to a fully funded synthetic CDO because it is not required to pay the difference between the yield on the collateral and the coupon on the note issue (the unfunded part of the transaction). The downside is that the issuing bank will receive a reduction in risk weighting for capital purposes to 20% for the risk transferred via the super-senior default swap.

The fully *unfunded* CDO uses only credit derivatives in its structure. The swaps are rated in a similar fashion to notes, and there is usually an "equity" piece that is retained by the originator. The reference portfolio will again be commercial loans, usually 100% risk-weighted, or other assets. The credit rating of the swap tranches is based on the rating of the reference assets, as well as other factors such as the diversity of the assets and ratings performance correlation. As well as the equity tranche, there will be one or more junior tranches, one or more senior tranches and super-senior tranche. The senior tranches are sold on to AAA rated banks as a portfolio credit default swap, while the junior tranche is usually sold to a an OECD bank. The credit default swaps are not single-name swaps, but are written on a class of debt. The advantage for the originator is that it can name the reference asset class to investors without having to disclose the name of specific loans. Credit default swaps are usually cash-settled and not physically settled, so that the reference assets can be replaced with other assets if desired by the sponsor.

As we noted earlier, synthetic deals may be either static or managed. Static deals hold the following advantages:

- There are no ongoing management fees to be borne by the vehicle.
- The investor can review and grant approval to credits that are to make up the reference portfolio.

The disadvantage is that if there is a deterioration in credit quality of one or more names, there is no ability to remove or offset this name from the pool and the vehicle continues to suffer from it. During 2001, for example, a number of high profile defaults in the market meant that static pool CDOs performed below expectation. This explains partly the rise in popularity of the managed synthetic deal, which we consider next.

CASE STUDY: ALCO 1 LIMITED

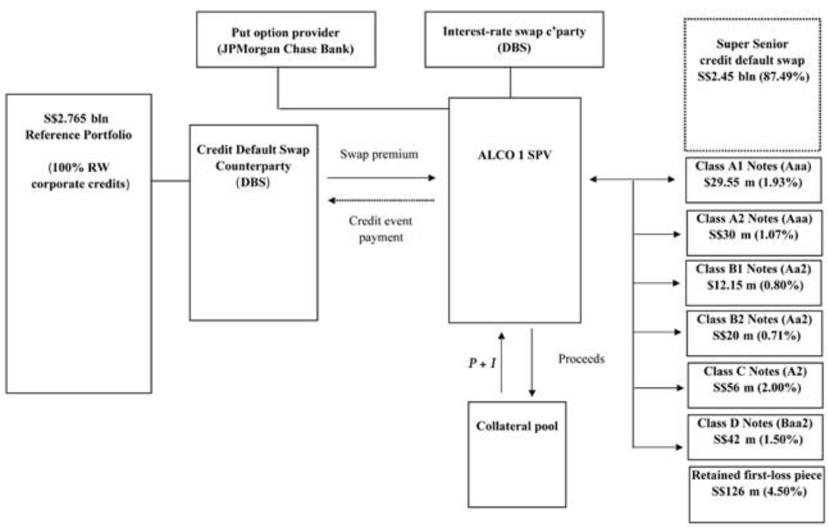
To illustrate the concept of the static balance sheet synthetic CDO and its application in credit risk management, we consider now the ALCO 1 structure, originated by the Development Bank of Singapore and closed in December 2001. According to Moody's, the ALCO 1 CDO is the first rated synthetic balance sheet CDO from a non-Japanese bank. It is a S$2.8 billion structure sponsored and managed by the Development Bank of Singapore (DBS).

The structure allows DBS to shift the credit risk on a S$2.8 billion reference portfolio of mainly Singapore corporate loans to a SPV, ALCO 1, using credit default swaps. It is illustrated in Exhibit 18.3. As a result DBS can reduce the risk capital it has to hold on the reference loans, without physically moving the assets from its balance sheet. The structure is S$2.45 billion super-senior tranche—unfunded credit default swap—with S$224 million notes issue and S$126 million first-loss piece retained by DBS. The notes are issued in six classes, collateralized by Singapore government T-bills and a reserve bank account known as a "GIC" account. There is also a currency and interest-rate swap structure in place for risk hedging, and a put option that covers purchase of assets by arranger if the deal terminates before expected maturity date. The issuer enters into credit default swaps with specified list of counterparties. The credit default swap pool is static, but there is a substitution facility for up to 10% of the portfolio. This means that under certain specified conditions, up to 10% of the reference loan portfolio may be replaced by loans from outside the vehicle. Other than this though, the reference portfolio is static.

The first rated synthetic balance sheet deal in Asia, ALCO 1-type structures have subsequently been adopted by other commercial banks in the region. The principal innovation of the vehicle is the method by which the reference credits are selected. The choice of reference credits on which swaps are written must, as expected with a CDO, follow a

EXHIBIT 18.3 ALCO 1 structure and tranching

Name	ALCO 1 Limited
Originator	Development Bank of Singapore Ltd
Arrangers	JPMorgan Chase Bank
	DBS Ltd.
Trustee	Bank of New York
Closing date	15 December 2001
Maturity	March 2009
Portfolio	S$2.8 billion of credit default swaps
Reference assets	199 reference obligations (136 obligors)
Portfolio Administrator	JPMorgan Chase Bank Institutional Trust Services



Class	Amount	Percent	Rating	Interest Rate
Super senior swap	S$2.450 m	87.49%	NR	N/A
Class A1	US$29.55 m	1.93%	Aaa	3 m USD Libor + 50 bps
Class A2	S$30 m	1.07%	Aaa	3 m SOR + 45 bps
Class B1	US$12.15 m	0.80%	Aa2	3 m USD Libor + 85 bps
Class B2	S$20 m	0.71%	Aa2	3 m SOR + 80 bps
Class C	S$56 m	2.00%	A2	5.20%
Class D	S$42 m	1.50%	Baa2	6.70%

Source: Moody's Investors Service

number of criteria set by the ratings agency, including diversity score, rating factor, weighted average spread, geographical, and industry concentration, among others.

The issuer enters into a portfolio credit default swap with DBS as the CDS counterparty to provide credit protection against losses in reference portfolio. The credit default swaps are cash settled. In return for protection premium payments, after aggregate losses exceeding the S$126 million "threshold" amount, the issuer is obliged to make protection payments to DBS. The maximum obligation is the S$224 million note proceeds value. In standard fashion associated with securitized notes, further losses above the threshold amount will be allocated to overlying notes in their reverse order of seniority. The note proceeds are invested in a collateral pool comprised initially of Singapore Treasury bills.

During the term of the transaction, DBS as the CDS counterparty is permitted to remove any eliminated reference obligations that are fully paid, terminated early or otherwise no longer eligible. In addition DBS has the option to remove up to 10% of the initial aggregate amount of the reference portfolio, and substitute new or existing reference names.

For this structure, credit events are defined specifically as[5]

- Failure to pay
- Bankruptcy

The reference portfolio is an Asian corporate portfolio, but with small percentage of loans originated in Australia. The portfolio is concentrated in Singapore (80%). The weighted average credit quality is Baa3/Ba1, with an average life of three years. The Moody's diversity score is low (20), reflecting the concentration of loans in Singapore. There is a high industrial concentration. The total portfolio at inception was 199 reference obligations amongst 136 reference entities (obligors). By structuring the deal in this way, DBS obtains capital relief on the funded portion of the assets, but at lower cost and less administrative burden than a traditional cash flow securitization, and without having to have a true sale of the assets.

The case study we have considered here is an innovative structure and a creative combination of securitization technology and credit derivatives. Analysis of the ALCO 1 vehicle shows clearly how a commercial bank can utilize the arrangement to effectively manage its credit risk exposure and optimize balance sheet capital, as well as provide attractive returns for investors. The most flexible vehicles in theory allow more efficient portfolio risk management when compared to static or more restrictive deals.

[5] This differs from European market CDOs where the list of defined credit events is invariably longer, frequently including restructuring and credit rating downgrade.

KEY POINTS

1. Credit derivatives such as credit default swaps are used in the construction of structured products known as *synthetic collateralized debt obligations*.

2. Synthetic CDOs are used by commercial banks to manage and transfer credit risk from their balance sheets without transferring assets themselves.

3. Transferring credit risk using synthetic CDOs enables banks to reduce their regulatory capital costs.

4. For a generic CDO structure, the credit risk of the bank's assets is transferred to the issuer, a special purpose vehicle, and then investors, by means of credit default swaps and an issue of credit-linked notes.

5. In the credit default swap arrangement, the risk transfer is undertaken in return for the swap premium, which is then paid to investors by the issuer.

6. Investors in the credit-linked notes expose themselves to the credit risk of the originator's assets, which are referenced to the notes. If there are no credit events investor coupon on notes.

7. A synthetic CDO is a completely unfunded structure which uses credit default swaps to transfer the entire credit risk of the reference assets to investors who are protection sellers. In a partially funded CDO, only the highest credit risk segment of the portfolio is transferred.

8. A fully funded CDO is a structure where the credit risk of the entire portfolio is transferred to the SPV via a credit default swap. In a fully funded synthetic CDO the issuer enters into the credit default swap with the SPV, which itself issues credit-linked notes to the entire value of the assets on which the risk has been transferred.

9. The fully unfunded CDO uses only credit derivatives in its structure.

10. Investors seek access to the credit portfolio of a bank static synthetic CDO, and the credit trading expertise of portfolio managers, when considering to invest in static CDO or managed synthetic CDO.

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